Journal of Harbin Engineering University
ISSN: 1006-7043

Vol 44 No. 8
June 2023

On the K-Metro Domination Number ofTriangular Ladder Graph

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Abstract: A dominating set $\boldsymbol{D}$ of a graph $\boldsymbol{G} = \boldsymbol{G}(\boldsymbol{V}, \boldsymbol{E})$ is called Metro dominating set of $\boldsymbol{G}$. If for every pair of verities $\boldsymbol{u}, \boldsymbol{v}$ there exists a vertex $\boldsymbol{w}$ in $\boldsymbol{D}$ such that $\boldsymbol{d}(\boldsymbol{u}, \boldsymbol{w}) \neq \boldsymbol{d}(\boldsymbol{v}, \boldsymbol{w})$. The K-metro domination number of triangular ladder graph $(\boldsymbol{\gamma}_{\boldsymbol{\beta}_{\boldsymbol{k}}}[\boldsymbol{TL}_{\boldsymbol{n}}])$, is the order of smallest K-dominating set of $[\boldsymbol{TL}_{\boldsymbol{n}}]$ which serves as a metric set. In this paper we calculate K-metro domination number of triangular ladder graph $(\boldsymbol{\gamma}_{\boldsymbol{\beta}_{\boldsymbol{k}}}[\boldsymbol{TL}_{\boldsymbol{n}}])$.

Keywords: Dominating set, K- Dominating set, Domination number, Locating dominating set, Metric dimension, Metro domination set**.**

1 Introduction

Every graph considered here are simple, finite, undirected and connected. A graph $G = (V, E)$ and $u, v \epsilon V, d_G(u, v)$ is denoted as distance between u and v in G. We refer [5,6,7,8,9,11,13] for the works on metro domination.

2 Known results

Definition 2.1: A set D of vertices in a graph G is called a dominating se G. If every vertex in $V - D$ is adjacent to some vertex in D. The domination number $\gamma(G)$ of a graph G is the minimum cardinality of a dominating set in G[1],[2].

Definition 2.2: A set $S \subseteq V$ is called resolving set if for every $u, v \epsilon V$ there exist $w \epsilon S$, such that the distance between vertices $u, v \in V$ is represented as $d(u, w) \neq d(v, w)$**.** A set of vertices $S \subseteq V(G)$ resolves G, then S is a resolving set of G and its minimum cardinality is a metric basis of G,and its cardinality is the metric dimension and it is represented by $\beta(G)$.

Definition 2.3: Metro domination number introduced by B.Sooryanarayan and Raghunath.P[4]. A dominating set D of $V(G)$ which is both dominating set as well as resolving set is called the metro dominating set of G. The minimum cardinality of metro dominating set of $G$is called metro domination number of G, and denoted by $\gamma_\beta(G)$.

Definition 2.4: A ladder graph L_n is defined by $L_n = P_n \, x \, K_2$ where P_n is a path with n vertices and x denotes the K_2 is a complete graph with two vertices

Definition 2.5: A triangular ladder $TL_n, n \geq 2$ is a graph obtained from L_n by adding the edges $u_i v_{i+1,} 1 \leq i \leq n-1$. The vertices of L_n are u_i and v_i. u_i and v_i are the two paths in the graph L_n where $i = 1,2,3, \dots n$.

Corollary 2.6: For any integer n, $\beta[TL_n] = 2$

Corollary 2.7: For any integer $n \geq 3$, the distance irregularity strength from the triangular ladder

graph L_n is $dis(L_n) = n$

3 Main Results

Theorem 3.1: For any integer n, $\gamma_\beta[TL_n] = \left\lceil \frac{2n+3}{5} \right\rceil, n \geq 4$

Proof: Let $G = T(L_n)$ be a triangular ladder graph on $2n$ vertices with

$V(TL_n) = \{u_1 v_1 | 1 \leq i \leq n\}$

and $E[TL_n] = \{u_1 u_{i+1} \mid i < n\} \cup \{v_1 v_{i+1} \mid i < n\} \cup \{u_i u_{i+1} \mid i < n\}$

By using the corollary 2.6, and [12], since a metro dominating set D also a dominating set.

Thus $\gamma_\beta[TL_n] \geq \left\lceil \frac{2n+3}{5} \right\rceil$ (1)

To prove the reverse inequality, we find a metro dominating set of cardinality $\left\lceil \frac{2n+3}{5} \right\rceil$

We define a set D as follows

$D_1 = \{u_{5l-1} : l \geq 1\}, n \equiv 4(mod\ 5)$

$D_2 = \{v_{5l-4} : l \geq 1\}, n \equiv 1(mod\ 5)$

We note that D is also dominating set for $T(L_n)$ and also D will serves as metric set of $T(L_n)$ as in

corollary 2.6.

Thus $\gamma_\beta[TL_n] \leq \left\lceil \frac{2n+3}{5} \right\rceil$ (2)

From (1) and (2),

$\gamma_\beta[TL_n] = \left\lceil \frac{2n+3}{5} \right\rceil$

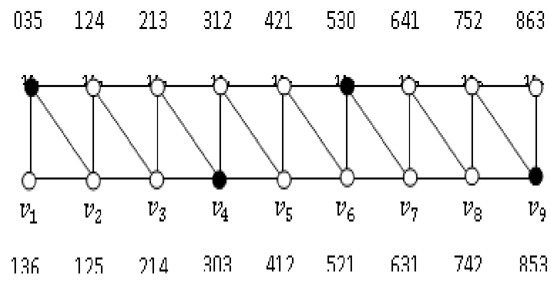

Figure 3.1: $\gamma_\beta[TL_8] = 4$

Theorem 3.2: For any integer n, $\gamma_{\beta_2}[TL_n] = \left\lceil \frac{2n+5}{9} \right\rceil, n \geq 7$

Proof: Let $G = T(L_n)$ be a triangular ladder graph on $2n$ vertices with

$V(TL_n) = \{u_1 v_1 | 1 \leq i \leq n\}$ and

$E[TL_n] = \{u_1 u_{i+1} \mid i < n\} \cup \{v_1 v_{i+1} \mid i < n\} \cup \{u_i u_{i+1} \mid i < n\}$

With for each i, u_i, v_i the only edges between two paths. $W = V - D$, now each $v_i \epsilon\, W$ is either adjacent to any of the vertex D (or) at least at distance of 2 form at least one of the vertex D. Any vertex $v_k \epsilon D$, will dominate at least 5 vertices including itself. Since a metric dimension of triangular ladder graph is 2, D itself serves as metric set.

Thus $\gamma_{\beta_2}[TL_n] \geq \left\lceil \frac{2n+5}{9} \right\rceil$ (1)

We define a set D as follows

$D_1 = \{u_{9l-3} : l \geq 1\}, n \equiv 6(mod\ 9)$

$D_2 = \{v_{9l-8} : l \geq 1\}, n \equiv 1(mod\ 9)$

We note that D is also dominating set for $T(L_n)$ and also D will serves as metric set of $T(L_n)$ as in

corollary 2.6.

Thus $\gamma_{\beta_2}[TL_n] \leq \left\lceil \frac{2n+5}{9} \right\rceil$ (2)

From (1) and (2)

$$\gamma_{\beta_2}[TL_n] = \left\lceil \frac{2n+5}{9} \right\rceil$$

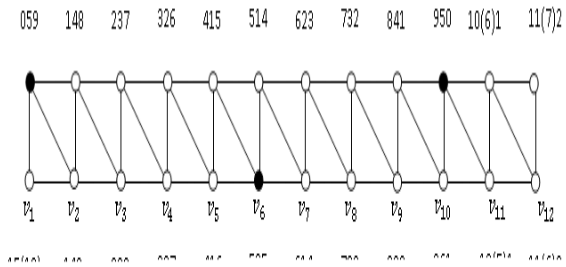

Figure 3.2: $\gamma_{\beta_2}[TL_{11}] = 3$

Theorem 3.3: For any integer n, $\gamma_{\beta_k}[TL_n] \leq \left\lceil \frac{2(n+k)+1}{4k+1} \right\rceil, n \geq 3k+1$

Proof: Let $G = T(L_n)$ be a triangular ladder graph on $2n$ vertices with

$V(TL_n) = \{u_1 v_1 | 1 \leq i \leq n\}$ and

$E[TL_n] = \{u_1 u_{i+1} \mid i < n\} \cup \{v_1 v_{i+1} \mid i < n\} \cup \{u_i u_{i+1} \mid i < n\}$

With for each i, u_i, v_i the only edges between two paths. $W = V - D$, now each $v_i \epsilon W$ is either adjacent to any of the vertex D (or) at least at distance of 2 from at least one of the vertex D. Any vertex $v_k \epsilon D$, will dominate at least $3k+1$ vertices including itself. The lower bound of $T(L_n)$ of order $n = (3k+1)l$ for some $l \geq 1$.

We define a set D as follows

$D_1 = \{u_{(4k+1)l-2k+1} : l \geq 1\}$, $n \equiv 2(k+1)(mod\ (4k+1))$

$D_2 = \{v_{(4k+1)l-4k} \quad : l \geq 1\}$, $n \equiv 1(mod\ (4k+1))$

We note that D is also dominating set for $T(L_n)$ and also D will serves as metric set of $T(L_n)$ as in

corollary 2.6.

Thus $\gamma_{\beta_k}[TL_n] \leq \left\lceil \frac{2(n+k)+1}{4k+1} \right\rceil$ $, n \geq 3k+1$

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