Journal of Harbin Engineering University
ISSN: 1006-7043

Vol. 44 No. 8
August 2023

Inventory Model for Time-Dependent Linear Demand with Three Levels of Production with Shortage

Hariom[1], Kailash Chandra Sharma[1], Krapal Singh[1], Dharamender Singh[1*]

[1,] Department of Mathematics, Maharani Shri Jaya Government College Bharatpur affiliated by Maharaja Surajmal Brij University, Bharatpur, Rajasthan, India
([1]E-mail: hariomdips@gmail.com,Email: ks.pragya@gmail.com, E-mail: devkraha@gmail.com)

[1*] Department of Mathematics, Maharani Shri Jaya Government College Bharatpur affiliated by Maharaja Surajmal Brij University, Bharatpur, Rajasthan, India
(Corresponding E-mail**:** dharamender.2015dr7@am.ism.ac.in)

Abstract: An ongoing production stock issue for a three-level confronting a steady crumbling rate for a solitary item. The demand rate is a straight capacity of time, and the crumbling rate is constant. The multiplying rate is thought to be subject to the length of hanging tight for the next renewal. The point of this examination is to limit the complete stock expenses and boost total income. We also checked that the production time and ordinarily ideal time used for mathematical frameworks were investigated with a consistent creation model. We scientifically study the perfect creation time and renewal choices at the retailer. The calculations for this model have been creating in Mathematica Software 9.0. We logically study the ideal creation time and recharging choices at the retailer and the effects of different difficulty boundaries on the ideal choices. We likewise lead broad some mathematical investigations to look at changed beginning. Toward the end, affectability examination occurred for to contemplate the aftereffect of significant boundaries with the whole expense of this model.

2010 AMS Classification: 90B05, 90C30 90B035

Keywords and phrases: Inventory; Deteriorating item; Shortage; EPQ; Time Dependent Demand; Production.

Article type: Research article

1. Introduction and Literature Review

Deterioration is characterized as harm, a rot of waste value of an article that lessens practically from extraordinary ones. Blood, fish, berries and root vegetable, alcohol, fuel, radioactive chemicals, drugs, etc., lose their advantage concerning time. For this situation, a rebate markdown value strategy applies to advance sales by the providers of these items. In the essential food item retailing industry, mass merchandisers and storeroom clubs have become low-value pioneers by purchasing in colossal volume. However, supermarkets work with shallow edges and rely upon volume to create benefits. Helpless evaluation and stock choices can undoubtedly harm the productivity of deteriorating things in a retailer. Singh et al. [1] have examined an optimal system with time-proportional deterioration and time-subordinate direct interest. Shaikh et al. [2] have clarified a stock model for single deteriorating things with two particular storage spaces (individual and leased stockrooms) because of a limited limit of the current stockpiling, i.e., own distribution center seeing allowable postponement in instalment. Singh et. al [3] fostered a stock model for a consistent deteriorating thing with selling cost and recurrence of promotion subordinate interest of an item under the blended kind financial exchange credit strategy. Bai et al.4] speared an EOQ model with direct ward interest and diverse holding cost capacities. Banerjee and Agrawal [5] have clarified a stock framework when the request rate for a deteriorating thing depends initially on its selling cost and later on the newness condition. Mishra et al. [6] analyzed an EOQ stock model that considers the interest rate as an element of the stock ward while deficiency is adequate. Singh And Mukherjee [7] have fostered a stock model for finding the methodology for a firm that retails a seasonal thing throughout a finite booking time. Kumar and Datta [8] San-jos. et al. [9 have explored a stock model for non-prompt deteriorating things under inflationary conditions though partially multiplied flaws are allowed. Tiwari et al. [10] have explored a stock model with inconsistent stockpiles while each part may have irregular parts of undersupplied things with known dispersion. Accordingly, thing assessment is essential in all circumstances, significantly when things are deteriorating.

At the current age, everything relies upon a time. Time is a significant factor for fruitful business, and without appropriate using time effectively, we cannot go on. A portion of the researchers examined time-subordinate interest, and many; However, most existing models in this stock of writing of joint estimating with reference value impacts neglect to consider the vital interest part of retail inventories. De and Mehta [11], Singh et. al. [12] have fostered a deterministic stock model for time subordinate interest though thought to be steady holding cost with partial multiplying. Compared to consistent or deterministic interest rates, the showed stock level emphatically affects sales and benefits for specific things. It happens because massive stock measures are exceptionally noticeable than small ones, and this expanded perceivability invigorates requests by making strategically pitching and spur-of-the-moment purchase openings. Mishra and Sahib [13] have discussed a stock model for deteriorating things with time-subordinate interest and time depending on holding cost under partial multiplying. Sarkar et al. [14] have fostered an EMQ (economies, producing amount) model with price and time-subordinate interest beneath the impact of dependability and expansion rate. Tripathi and Mishra [15] examined an EOQ (monetary request amount) model with direct time-subordinate interest and diverse holding cost capacities. Singh et al. [16] have referenced an optimal strategy for deteriorating things under time-proportional deterioration and time-subordinate direct interest rates.

In a couple of studies on production, stock writing is talked about; Lian [17] has considered a ceaseless audit stock precarious for a retailer confronting steady client interest for a solitary creation. Viji and Karthikeyan [18] showed a financial creation amount model for three degrees of invention with Weibull conveyance deterioration and lack. This retailer is expected to review the notable, broadly utilized request up-to strategy in making renewal decisions and arrange from two suppliers who contrast inconstancy and expenses. Some researchers Sivashankari, and Panayappan [19], Khanra [20] talked about a creation stock model for constantly deteriorating things with two unique paces of creations and deficiencies. They have fostered a creation stock model for two-level design with declining things and identified with defects. Shah [21], Khedlekar, and Namdeo [22] inferred a creation stock model with disturbance rate time proportional interest and thinking about lack. In this paper, we join the reference value impact into a stock model of optimal estimating for deteriorating things when the request rate relies upon both stock level and selling cost. All sale incomes and relative stock expenses are communicated in start of-period cash units whereby a one-period rebate factor rediscounts incomes are happening in resulting periods. We plan to research how stock level and buyers' value assumptions reflect in managerial evaluating choices when clients have repetitive long-haul associations with the retailer. Lee and Dye [23] discussed a stock model for deteriorating items under stock-subordinate interest controllable deterioration rate. Soroush [24] has tended to various multicriteria single machine booking difficulties while a task handling time is either a raised or straight capacity of the span of asset allotted to work. Learning and maturing are work ward and position-based. Mishra et al. [25] have considered a stock model under cost and stock ward interest for a controllable deterioration rate with deficiencies utilizing conservation innovation speculation.

Besides, there is convincing empirical proof that the reference cost and the measure of showed stock influence interest; however, few specialists focus on these impacts simultaneously. Presently a day's protection procedure assumes a significant part in stock administration and real-life things. Safeguarding strategies control the deterioration rate and improve thing's validity. A portion of the researchers talked about; We also study the communications among assessing, deteriorating rate, shape parameter of stock level and reference cost, and analyzing the initial reference value means for the retailer's optimal estimating methodology. The expression "overabundance" utilizes in money/creation. A couple of deals with deficiency and multiplying are discussed: Begum et al. [26], Kumar et al. [27] discussed how we benefit from expansion creation, stock models with time-subordinate interest with partial accumulation. Sarkar and Sarkar [28] have dealt with an improved stock model time-differing deterioration rate and stock-subordinate interest with partial multiplying. Tai [29] had tended to a stock model where things are audited through various transmission measures on different quality attributes already conveyance to clients. Each screening method on a solitary quality particular has an autonomous screening rate and flawed rate. Deficiency delay purchasing is also reasonable in the model. Wu et al. [30] have doled out two stock frameworks with trapezoidal-type request rate and time-subordinate deterioration rate with partial accumulating. Sadeghi et al. [31] have dealt with a multi-thing financial creation amount model with fluffy uniform interest is created in which deficiencies are allowed, and the distribution center space is confined. Samanta [32] has zeroed in on the investigation of partial multiplying stock issues. Chowdhary et al [33] Mahapatra et al. [34] dealt with a stock issue for deteriorating things with time and dependability subordinate interest and partial delay purchase.

This paper adds two streams of writing: (1) the limited arranging limit stock model for deteriorating things with direct/straight time-subordinate interest, and (2) shortage is allowed evaluating with reference value impacts. The principal stream of the writing expects that the completion stock levels are limited to nothing and that interest relies just upon the direct pattern level and current cost. The subsequent stream typically disregards the impact of the deterioration rate upon demand. The paper is coordinated as follows: Section 2 gives the fundamental documentation and suspicions for the proposed stock model. Area 3 talks about the mathematical models for the stock framework. Area 4 determines theoretical outcomes for optimal arrangements. Segment 5 presents a numerical guide to validate the proposed model. Area 6 gives an affectability analysis of the significant parameter and a graphical analysis. Lastly, section 7 talked about end with ideas for future exploration work utilization of this paper.

2.0 Assumptions and Notations

2.1: The assumptions of an inventory model are as follows:

1. The production rate is well known and constant.
2. The demand rate is a linear function of time and nonnegative.
3. Three rates of production are considered.
4. The item is a solo product; it does not associate with any other inventory items.
5. The production rate is always greater than or equal to the sum of the demand rate.
6. The lead time assumes zero or negligible.
7. Replenishment rate is finite.

8. Shortages are allowed and partial backlogging.

2.2: Notations:

(a) p – Production rate is per unit's time.

(b) D – Demand rate per unit time $D=(a+bt)$, where a and b are a constant parameter and non-negative.

(c) θ – Deterioration rate without preservation technique.

(d) S_1 – Maximum inventory level at a time T_1.

(e) S_2 – Maximum inventory level at a time T_2.

(f) S_3 – Maximum inventory level at a time T_3.

(g) B – Maximum shortage level.

(h) c_p – Production cost per unit.

(i) h_c – Holding cost per unit time.

(j) C_0 – Set-up cost per production cycle.

(k) $\theta_1(\xi)$ – Deterioration rate after investing in preservation technology.

(l) ξ – Cost of preservation technology per unit time where ξ > 0.

(m) ϕ – The sensitive parameter of investment to the deterioration rate.

(n) s_c – Shortage cost per unit/per unit time.

(o) l_c – Lot sale cost per unit time.

(p) T – Length of the inventory cycle.

(q) T_i – Unit time in periods $i=1,2,3\,\&\,4$.

(r) TC – Total cost.

3. Mathematical Model and Solution

Each production starts with first opening/initial stage and stops with the last closing stage. Demand rate is time dependent. Let us assume that the production started at the time $t=0$ and finished at the time $t=T$. Let during the time interval $[0,T_1]$, the production rate be 'p' and demand rate be '$D=(a+bt)$' where a,b are a positive value and D is less than p. The stock attains a maximum inventory level S_1 at time $t=T_1$. Again during the time intervals $[T_1,T_2]$ and $[T_2,T_3]$, the rate of growth to be considered as $c(p-D)$ and $d(p-D)$ where c and d are constants.

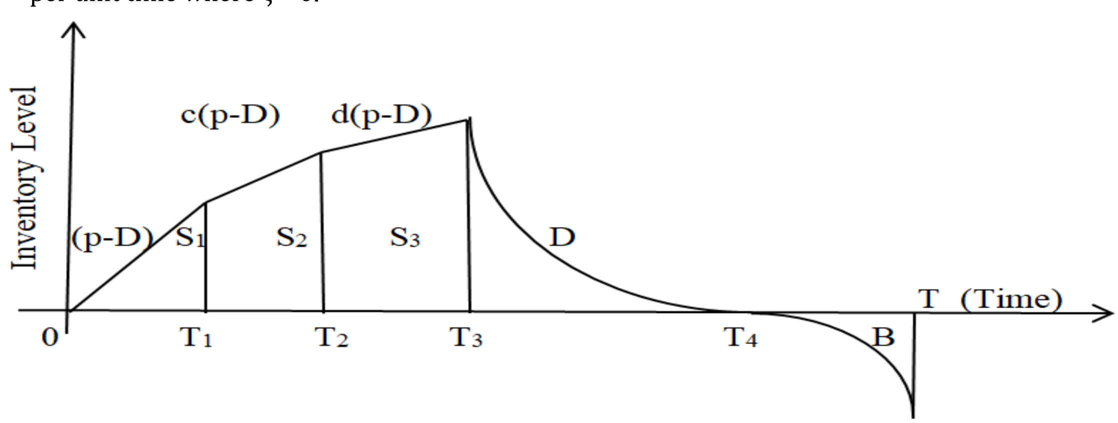


Figure-1: Geometry of production inventory model with shortage

The inventory level attains maximum inventory levels S_2 and S_3 at the time T_2 and T_3 respect. During the decline time, the product becomes technologically obsolete. The inventory level decreases due to demand at a rate of D. When inventory reduces to zero total time of a cycle

$t = T$ the shortage will reach its maximum values. The manufacturer practically backlogs the insufficient demand via a subcontract. Let $I(t)$ denote the inventory level of the system at the time T .

The following differential equations controlling the inventory system in the time interval $[0,T]$ given are as follow:

Phase-1: When Shortage is not allowed;

$$\frac{dI(t)}{dt} + \theta I(t) = p - (a + bt) \quad ; \qquad 0 \le t \le T_1 \tag{1}$$

$$\frac{dI(t)}{dt} + \theta I(t) = c\left\{p - (a + bt)\right\}; \quad \mathrm{T}_1 \le t \le T_2 \tag{2}$$

$$\frac{dI(t)}{dt} + \theta I(t) = d\left\{p - (a + bt)\right\}; \quad \mathrm{T}_2 \le t \le T_3 \tag{3}$$

$$\frac{dI(t)}{dt} + \theta I(t) = -(a + bt) \; ; \qquad \mathrm{T}_3 \le t \le T_4 \tag{4}$$

Phase-2: When shortage allowed

$$\frac{dI(t)}{dt} = -\frac{(a + bt)}{1 + \delta(T - t)}; \qquad \mathrm{T}_4 \le t \le T \tag{5}$$

With boundary conditions $I(0) = 0$, $I(T_1) = S_1$, $I(T_2) = S_2$, $I(T_3) = S_3$, $I(T_4) = 0$ & $I(T) = -B$ (6)

A solution of the phase-1 from differential equations (1), (2), (3) & (4) using boundary condition (6) are as follow:

$$\mathrm{I(t)} = \frac{-e^{-t\theta}\left[b - be^{t\theta} - a\theta + a\theta e^{t\theta} + p\theta - p\theta e^{t\theta} + bt\theta e^{t\theta}\right]}{\theta^2} \tag{7}$$

$$\mathrm{I(t)} = \frac{-ce^{-t\theta}\left[b - be^{t\theta} - a\theta + a\theta e^{t\theta} + p\theta - p\theta e^{t\theta} + bt\theta e^{t\theta}\right]}{\theta^2} \tag{8}$$

$$\mathrm{I(t)} = \frac{-de^{-t\theta}\left[b - be^{t\theta} - a\theta + a\theta e^{t\theta} + p\theta - p\theta e^{t\theta} + bt\theta e^{t\theta}\right]}{\theta^2} \tag{9}$$

$$\mathrm{I(t)} = \frac{-e^{-t\theta}\left[-be^{t\theta} + be^{T_4\theta} + a\theta e^{t\theta} - a\theta e^{T_4\theta} + bt\theta e^{t\theta} - bT_4\theta e^{T_4\theta}\right]}{\theta^2} \tag{10}$$

A solution of the phase-2 differential equations (5) using boundary condition (6) is as follow:

$$\mathrm{I(t)} = \frac{1}{\delta}\left[bt - bT - B\delta^2 + (b + a\delta + bT\delta)(T_4 - t) + b(T - t)\right] \tag{11}$$

Maximum inventory level S_1 : The maximum inventory level during time T_1 is solve as follow from equation (6) & (7), $I(T_1) = S_1$

$$S_1=\frac{-e^{-T_1\theta}\left[b-be^{T_1\theta}-a\theta+a\theta e^{T_1\theta}+p\theta-p\theta e^{T_1\theta}+bT_1\theta e^{T_1\theta}\right]}{\theta^2}$$

Expanding the exponential term and overlooking the third span and higher power of θ for the small value of θ then we get

$$S_1=\left[\frac{bT_1^2}{2}+aT_1-\frac{a\theta T_1^2}{2}-pT_1+\frac{p\theta T_1^2}{2}\right] \quad (12)$$

Maximum inventory S_2: The maximum inventory level throughout time T_2 is solve as from equation (6) & (8), $I(T_2)=S_2$

$$S_2=\frac{-ce^{-T_2\theta}\left[b-be^{T_2\theta}-a\theta+a\theta e^{T_2\theta}+p\theta-p\theta e^{T_2\theta}+bT_2\theta e^{T_2\theta}\right]}{\theta^2}$$

Expanding the exponential term and ignoring the third term and higher power of θ for the small value of θ then we get

$$S_2=c\left[\frac{bT_2^2}{2}+aT_2-\frac{a\theta T_2^2}{2}-pT_2+\frac{p\theta T_2^2}{2}\right] \quad (13)$$

Maximum inventory S_3: The maximum inventory level during time T_3 is solve as follow from equation (7) & (9), $I(T_3)=S_3$

$$S_3=\frac{-de^{-T_3\theta}\left[b-be^{T_3\theta}-a\theta+a\theta e^{T_3\theta}+p\theta-p\theta e^{T_3\theta}+bT_3\theta e^{T_3\theta}\right]}{\theta^2}$$

Expanding the exponential term and neglecting the third span and higher power of θ for the small value of θ then we get

$$S_3=d\left[\frac{bT_3^2}{2}+aT_3-\frac{a\theta T_3^2}{2}-pT_3+\frac{p\theta T_3^2}{2}\right] \quad (14)$$

4. Cost calculation of inventory model

1. Production cost $PC=Dc_p$ (15)

2. Set up cost $TS_c=C_0$ (16)

3. Holding cost of per unit time TH_c

$$TH_c=h_c\int_0^{T_4}I(t)dt=h_c\left[\int_0^{T_1}I(t)dt+\int_{T_1}^{T_2}I(t)dt+\int_{T_2}^{T_3}I(t)dt+\int_{T_3}^{T_4}I(t)dt\right]$$

$$= h_c \left[\begin{array}{l} \frac{e^{-T1\theta}\left\{-2\theta(a-p)\left\{1+e^{T_1\theta}(-1+T_1\theta)\right\}+b\left\{2-e^{T_1\theta}(2+T_1\theta(-2+T_1\theta))\right\}\right\}}{2\theta^3} + \\ \frac{e^{-(T_1+T_2)\theta}\left\{2c(e^{T_1\theta}-e^{T_2\theta})(b-a\theta+p\theta)+ce^{(T_1+T_2)\theta}(T_1-T_2)\theta\left(2a\theta-2p\theta-2b+b(T_1+T_2)\theta\right)\right\}}{2\theta^3} + \\ \frac{e^{-(T_2+T_3)\theta}\left\{2d(e^{T_2\theta}-e^{T_3\theta})(b-a\theta+p\theta)+de^{(T_2+T_3)\theta}(T_2-T_3)\theta\left\{2a\theta-2p\theta-2b+b(T2+T3)\theta\right\}\right\}}{2\theta^3} \\ + \frac{2a\theta(-1+e^{(-T_3+T_4)\theta}+T_3\theta-T_4\theta)+b(2-2T_3\theta+(T_3-T_4)(T_3+T_4)\theta^2+2e^{(-T_3+T_4)\theta}(-1+T_4\theta))}{2\theta^3} \end{array} \right]$$

Expanding the exponential functions and ignoring second and higher power of θ then

$$TH_c = h_c \left[\frac{bT_1^2 - c(T_1^2 - T_2^2)\left(b - 2a\theta + 2p\theta\right) - d(T_2^2 - T_3^2)\left(b - 2a\theta + 2p\theta\right) + b\left(T_3 - T_4\right)^2}{2\theta} \right] \tag{17}$$

4. Deterioration cost of finished products TD_c

$$TD_c = c_d\theta\int_0^{T_4} I(t)dt = c_d\theta\left[\int_0^{T_1} I(t)dt + \int_{T_1}^{T_2} I(t)dt + \int_{T_2}^{T_3} I(t)dt + \int_{T_3}^{T_4} I(t)dt\right]$$

Expanding the exponential functions and overlooking second and higher power of θ then

$$TD_c = \theta c_d \left[\frac{bT_1^2 - c(T_1^2 - T_2^2)\left(b - 2a\theta + 2p\theta\right) - d(T_2^2 - T_3^2)\left(b - 2a\theta + 2p\theta\right) + b\left(T_3 - T_4\right)^2}{2\theta} \right] \tag{18}$$

5. Preservation Cost $PT_c = \xi T$ (19)

6. Shortage Cost $TS_c = -s_c\int_{T_4}^{T} I(t)dt$

$$TS_c = -s_c\left[BT - \frac{aT^2}{2} - \frac{bT^3}{2} - BT_4 + aTT_4 + bT^2T_4 - \frac{aT_4^2}{2} - \frac{bTT_4^2}{2}\right] \tag{20}$$

7. Lot sale cost $TL_s = l_s\int_{T_4}^{T}(a+bt)\left(1 - \frac{1}{1+\delta(T-t)}\right)dt$

$$TL_s = \frac{l_s\delta}{6}\left[(T-T_4)^2(3a + bT + 2bT_4)\right] \tag{21}$$

The total cost of this model $TC = \left[PC + TS_c + TH_c + TD_c + PT_c + TS_c + TL_s\right]$

$$TC=\begin{bmatrix} C_0+c_p(a+bT_4\delta)+\frac{T_4^2(h_c+c_d\theta)}{2\theta}\left\{bT_1^2-c(T_1^2-T_2^2)\left(b-2a\theta+2p\theta\right)-d(T_2^2\right. \\ \left.-T_3^2)\left(b-2a\theta+2p\theta\right)+b\left(T_3-T_4\right)^2\right\}+\xi T+\frac{\delta l_s(T-T4)^2}{6}(3a+bT+2bT_4) \\ -c_s\left(BT-\frac{aT^2}{2}-\frac{bT^3}{2}-BT_4+aTT_4+bT^2T_4-\frac{aT_4^2}{2}-\frac{bTT_4^2}{2}\right) \end{bmatrix} \quad (22)$$

Let $T_1=\alpha T_4;\ T_2=\beta T_4;\ T_3=\gamma T_4\ \&\ T=\lambda T_4$ therefore the whole cost will be

$$TC=\begin{bmatrix} C_0+c_p(a+\lambda bT_4)-+\frac{T_4^2(h_c+c_d\theta)}{2\theta}\left\{b\left(c(\beta^2-\alpha^2)+d(\gamma^2-\beta^2)+(1-\alpha^2)\right.\right. \\ \left.\left.+\gamma(2-\gamma)\right)+2(p-a)\left(c(\beta^2-\alpha^2)+d(\gamma^2-\beta^2)\right)\theta\right\}+\xi\lambda T_4+\frac{\delta l_s(\lambda-1)^2}{6} \\ (3aT_4^2+2bT_4^3+\delta bT_4^3)-\frac{T_4(\lambda-1)c_s}{2}\left\{-2B+(\lambda-1)(aT_4+b\lambda T_4^2)\right\} \end{bmatrix} \quad (23)$$

Objective

The objective of the study is to regulate the optimal value of preservation cost ξ^* for this model that minimizes the total cost TC is as follows Put $\theta=\theta_1 e^{-\phi\xi}$ then equation reduces to TC follow as;

$$TC=\begin{bmatrix} C_0+c_p(a+bT_4\lambda)+\frac{T_4^2(h_c+c_d\theta_1 e^{-\phi\xi})}{2\theta_1 e^{-\phi\xi}}\left\{b\left(c(\beta^2-\alpha^2)+d(\gamma^2-\beta^2)+(1-\alpha^2)\right.\right. \\ \left.\left.\gamma(2-\gamma)\right)+2(p-a)\left(c(\beta^2-\alpha^2)+d(\gamma^2-\beta^2)\right)\theta_1 e^{-\phi\xi}\right\}+\xi\lambda T_4+\frac{\delta l_s(\lambda-1)^2}{6} \\ (3aT_4^2+2bT_4^3+\lambda bT_4^3)-\frac{T_4(\lambda-1)c_s}{2}\left\{-2B+(\lambda-1)(aT_4+b\lambda T_4^2)\right\} \end{bmatrix} \quad (24)$$

Differentiating cost function TC concerning ξ

$$\frac{dTC}{d\xi}=\left[\frac{T_4^2 h_c\phi\theta_1 e^{\phi\xi}b}{2}\begin{pmatrix} c(\beta^2-\alpha^2)+d(\gamma^2-\beta^2) \\ +(1-\alpha^2)+\gamma(2-\gamma) \end{pmatrix}+\lambda T_4-\phi T_4^2 c_d\theta_1(p-a)e^{-\phi\xi}\right] \quad (25)$$

Again, differentiating cost function concerning ξ

$$\frac{d^2TC}{d\xi^2}=\phi^2T_4^2\theta_1\left\{\frac{h_c e^{\phi\xi}b}{2}\begin{pmatrix} c(\beta^2-\alpha^2)+d(\gamma^2-\beta^2) \\ +(1-\alpha^2)+\gamma(\gamma-2) \end{pmatrix}+c_d(p-a)e^{-\phi\xi}\right\} \quad (26)$$

$$\frac{d^2TC}{d\xi^2}>0\,if\,,(p-a),(\beta^2-\alpha^2),(\gamma^2-\beta^2),(1-\alpha^2),(2-\gamma);$$

The optimal value $\xi*$ will be calculated using Mathematica-software-9 from equation (25). The next objective of the study is to determine the optimal value of T_4^*, therefore the value of T_4^*, which minimizes TC as surveys then differentiate concerning T_4.

$$\frac{dTC}{dT_4}=\left[\begin{array}{l} c_p b\lambda+\frac{T_4(h_c+c_d\theta)}{\theta}\left\{b\left(c(\beta^2-\alpha^2)+d(\gamma^2-\beta^2)+(1-\alpha^2)+\gamma(2-\gamma)\right)+\right. \\ \left.2(p-a)\left(c(\beta^2-\alpha^2)+d(\gamma^2-\beta^2)\right)\theta\right\}+\frac{(\lambda-1)^2 l_s\delta}{6}(6aT_4+3bT_4^2(2+\lambda)) \\ +\xi\lambda-\frac{(\lambda-1)c_s}{2}\left\{-2B+(\lambda-1)(2aT_4+3b\lambda T_4^2)\right\} \end{array}\right] \quad (27)$$

Again, differentiate cost function concerning T_4 then equation becomes

$$\frac{d^2TC}{dT_4^2}=\left[\begin{array}{l} \frac{(h_c+c_d\theta)}{\theta}\left\{b\left(c(\beta^2-\alpha^2)+d(\gamma^2-\beta^2)+(1-\alpha^2)+\gamma(2-\gamma)\right)\right. \\ \left.+2(p-a)\left(c(\beta^2-\alpha^2)+d(\gamma^2-\beta^2)\right)\theta\right\}+(\lambda-1)^2 a(l_s\delta-c_s) \\ +(\lambda-1)^2 bT_4\left\{(\lambda+2)l_s\delta-3\lambda c_s\right\} \end{array}\right] \quad (28)$$

$$\frac{d^2TC}{dT_4^2}>0\, if\ (p-a),(\lambda-1),\left\{(\lambda+2)l_s\delta-3\lambda c_s\right\},(l_s\delta-c_s)$$

5. Numerical Analysis

The following data has been considered an illustration to validate the proposed model. Production rate is \$200, the demand rate of parameter $a=5, b=4, c=2\ \&\ d=2.5$, unit, production cost is \$10; deterioration cost \$2, set-up cost \$200, holding cost \$0.3 per unit, $\alpha=0.6, \beta=0.7, \gamma=0.8\ \&\ \lambda=2.1$ deterioration rate is \$0.2, backlogging cost \$2.2 lot sale parameter \$1.6. The optimal value of preservation technique ξ^*, the optimal value of $T_1^*, T_2^*, T_3^*, T_4^*\ \&\ T^*$, maximum inventory of $S_1^*, S_2^*\ \&\ S_3^*$ and an optimal total cost TC^*, with preservation technology, respectively have been calculated with the help of equation (25, 27,12, 13,14 & 24), shown in table -1:

p	ξ^*	T_1^*	T_2^*	T_3^*	T_4^*	T^*	S_1^*	S_2^*	S_3^*	TC^*
200	5.11	12.04	14.04	16.05	20.06	42.13	768.28	3004.32	6022.85	65635.3
400	6.29	23.63	27.57	31.51	39.39	82.72	1258.99	5276.48	10878.9	78497.4

Table-1 (Optimal Value of Different Parameters)

6. Sensitivity Analysis

Effect of changes in the different parameter of the proposed model, the sensitivity analysis is performed by considering 15% and 30% increase or decrease in each one of the above parameters keeping all other parameters the same. The sensitivity analysis is carried out by changing the specified parameter $p, a, b, c, d, c_p, c_d, c_s \, \& \, l_s$ Table 2 shows the sensitiveness of the various parameters on optimal value of $\xi^*, T_1^*, T_2^*, T_3^*, T_4^*, T^*, S_1^*, S_2^* \, \& \, S_3^*$ and total cost TC^* the study manifested the following facts: Table-2

Parameter		ξ^*	T_1^*	T_2^*	T_3^*	T_4^*	T^*	S_1^*	S_2^*	S_3^*	TC^*
p	30	8.81	29.09	29.09	29.02	29.09	-22.53	19.16	22.69	24.19	16.36
	15	4.70	14.55	14.57	14.52	14.59	-12.73	9.58	11.34	12.09	8.18
	-15	-5.47	-14.69	-14.70	-14.81	-14.72	17.27	-9.58	-11.34	-12.09	-8.18
	-30	-11.93	-29.56	-29.58	-29.63	-29.57	42.01	-19.16	-22.69	-24.19	-16.36
a	30	-0.26	-2.89	-2.88	-2.92	-2.86	2.94	-0.48	-0.57	-0.60	0.79
	15	-0.13	-1.40	-1.38	-1.49	-1.41	1.43	-0.24	-0.28	-0.30	0.40
	-15	0.12	1.43	1.47	1.38	1.43	-1.42	0.24	0.28	0.30	-0.40
	-30	0.25	2.84	2.82	2.75	2.82	-2.54	0.48	0.57	0.60	-0.79
b	30	-15.65	-20.34	-20.32	-20.42	-20.35	25.55	11.64	8.26	6.82	12.12
	15	-8.21	-11.45	-11.49	-11.51	-11.48	12.96	5.98	4.32	3.61	5.79
	-15	9.79	15.47	15.42	15.39	15.43	-13.36	-5.34	-3.56	-2.80	-6.85
	-30	21.34	33.40	31.35	30.31	32.36	-27.21	-11.00	-7.50	-6.01	-13.18
c	30	-1.56	12.23	12.22	12.21	12.24	-10.90	34.97	30.00	43.84	21.52
	15	-0.78	6.08	6.10	6.05	6.11	-5.76	17.42	14.97	21.03	11.72
	-15	0.79	-6.13	-6.15	-6.22	-6.15	6.56	-16.55	-15.00	-19.42	-13.52
	-30	1.77	-12.36	-12.35	-12.39	-12.33	14.06	-33.16	-30.00	-38.18	-28.61
d	30	-2.34	17.63	17.63	17.57	17.63	-14.98	66.71	66.16	50.00	-31.68
	15	-1.17	8.82	8.87	8.79	8.85	-8.13	32.94	34.06	25.05	-14.03
	-15	1.18	-8.87	-8.86	-8.96	-8.89	9.75	-33.74	-31.76	-25.00	14.08
	-30	2.55	-17.84	-17.83	-17.87	-17.81	21.67	-65.18	-59.14	-51.00	30.66
c_p	30	8.62	-0.73	-0.74	-0.74	-0.72	0.73	-3.60	-2.72	-2.35	0.04
	15	4.51	-0.40	-0.38	-0.43	-0.37	0.37	-1.85	-1.40	-1.21	0.02
	-15	-5.47	0.35	0.33	0.26	0.33	-0.32	1.68	1.27	1.09	-0.03
	-30	-11.54	0.68	0.68	0.63	0.68	-0.68	3.46	2.60	2.25	-0.06
c_d	30	0.00	19.21	19.20	19.12	19.17	-16.09	0.00	0.00	0.00	9.41
	15	0.00	9.57	9.58	9.53	9.60	-8.76	0.00	0.00	0.00	4.67
	-15	0.00	-9.70	-9.64	-9.71	-9.64	10.67	0.00	0.00	0.00	-4.67
	-30	0.00	-19.34	-19.32	-19.42	-19.36	20.00	0.00	0.00	0.00	-9.35
c_s	30	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-42.39
	15	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-20.55
	-15	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	21.19
	-30	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	42.39
l_s	30	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	55.15
	15	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	27.57
	-15	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-27.57
	-30	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-55.15

Table-2 (Sensitivity Analysis of Different Parameter)

The optimal value of ξ^* the slight change in the value of parameters $a, c \& d$, moderately $c_p \& p$ and highly with b.

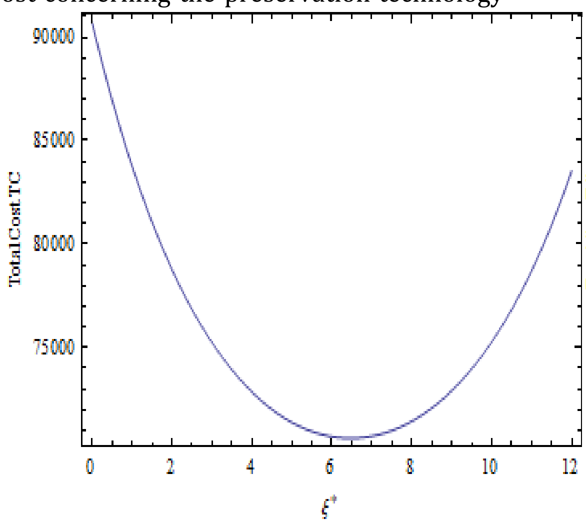
The optimal value of $T_1^*, T_2^*, T_3^*, T_4^* \& T$ slightly change in the value of parameters $a \& c_p$, moderately c highly with $p, b, d \& c_d$.

The optimal value of $S_1^*, S_2^* \& S_3^*$ slightly change in the value of parameters $a \& c_p$, moderately b highly with $p, c \& d$.

The optimal value of TC^* moderately changes in the value of parameters $p, b \& c_d$ highly with the value of $c, d, c_s \& l_s$ whereas slightly with $a \& c_p$

6.1. Graphical Analysis

The graphical representation of the optimal total cost concerning the preservation technology is the convexity of TC^* concerning ξ^* and an optimal time shown in figure-2 and figure-3.



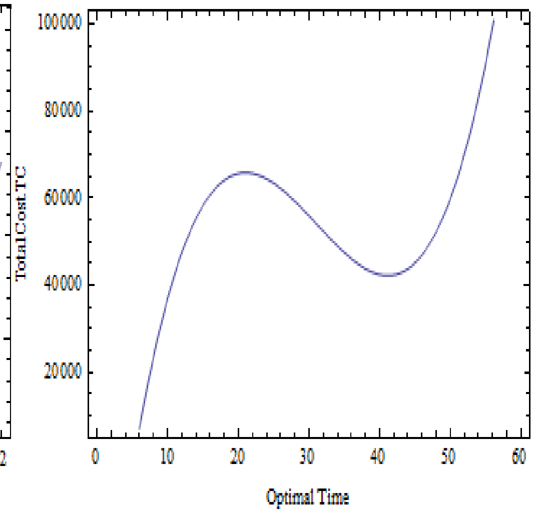


Figure-2: Total cost and preservation technique **Figure-3:** Total cost and optimal time T_4^*

7. Conclusion:

The proposed model is ideal for a recently dispatched item with a steady example to a certain degree in time-subordinate interest. In the present circumstance, the model is alluring since beginning at a low pace of creation, a massive quantum of a load of assembling things. Because of this, we will get customer fulfilment and procure more potential profit. Here we set up a mathematical model and an answer for this. In this model, a numerical model and its affectability analysis are given. The proposed stock model can precisely help the maker and retailer determine the optimal request amount, process duration, and total stock expense. We could generalize the model for the time value of cash. Also, we could consider the unit buy cost, time-subordinate stock holding cost, and others as time-subordinate. For future exploration, this model can be stretched out differently. For example, we may expand the interest for a wide range of criteria. We could generalize the model for the time value of cash. Consequently, our model has another managerial knowledge that aides an assembling framework/industry to acquire the optimal benefit level. The numerical model gives us a promising outcome, practically speaking.

Future Scope- This model can be extended from numerous points of view for additional exploration, including diverse interest rates like steady, quadratic, cubic, Weibull deterioration with three and more parameters, time-limiting, and modify of wrong things. A possible future examination issue is to look at the impact of time-moving damage on the ideal course of action. Another charming and testing component are thinking about a multi-thing.

Funding source: This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

Conflict of interest statement: The authors state that there is no conflict of interest.

Acknowledgement: The researchers would like to thank the Maharaja Surajmal Brij University Bharatpur, Rajasthan India for providing the facilities to conduct this study.

The authors would like to thank the editor and anonymous reviewers for their valuable and constructive comments, which have led to a significant improvement in the manuscript.

Reference

[1] Singh, T., Mishra, P. J., Pattanayak, H, An optimal policy for deteriorating items with time-proportional deterioration rate and constant and time-dependent linear demand rate. Journal of Industrial Engineering International. 1(9) (2017). DOI: 10.1007/s40092-017-0198-6

[2] Shaikh, A. A., Mashud, A. H. M., Uddin, M. S., Khan, M. A. A, Non-instantaneous deterioration inventory model with price and stock dependent demand for fully backlogged shortages under inflation. International Journal of Business Forecasting and Marketing Intelligence. 3(2) (2017), 152-164. DOI: 10.1504/IJBFMI.2017.084055

[3] Singh, D., Alharbi, M. G., Jayswal, A., & Shaikh, A. A, Analysis of inventory model for quadratic demand with three levels of production. Intelligent Automation And Soft Computing, 32(1) (2022), 167-182. DOI: 10.32604/iasc.2022.021815S

[4] Bai, H. G., Ismail, A. M., Kaviya, D., & Muniappan, P.: An EOQ model for deteriorating products with backorder and fixed transportation cost. In Journal of Physics: Conference Series (Vol. 1770, No. 1, p. 012100) (2021). IOP Publishing. DOI: 10.1088/1742-6596/1770/1/012100

[5] S. Banerjee, S. Agrawal, Inventory Model for Deteriorating Items with Freshness and Price Dependent Demand, Optimal Discounting and Ordering Policies, Applied Mathematical Modelling (2017), DOI: 10.1016/j.apm.2017.07.020

[6] Mishra, U., Cárdenas-Barrón, L. E., Tiwari, S., Shaikh, A. A., Treviño-Garza, G, An inventory model under price and stock dependent demand for controllable deterioration rate with shortages and preservation technology investment. Annals of Operations Research. 254(1-2), (2017), 165-190. DOI: 10.1007/s10479-017-2419-1

[7] Singh, D., & Mukherjee, B, A deterministic inventory model for time-varying demand and holding cost with partial backlogging. In Proceedings of the Jangjeon Mathematical Society, 21(4), (2018), 667-678. DOI: 10.17777/pjms2018.21.4.667

[8] Kumar, P., & Dutta, D, A deteriorating inventory model with uniformly distributed random demand and completely backlogged shortages. In Numerical Optimization in Engineering and Sciences, 2020, 213-223. Springer, Singapore. DOI: 10.1007/978-981-15-3215-3_20

[9] A. San-Jos, J. Sicilia, M. Gonzlez-De-la-Rosa, J. Febles-Acosta, Optimal inventory policy under power demand pattern and partial backlogging, Applied Mathematical Modelling. 46 (2017), 618-630. DOI: 10.1016/j.apm.2017.01.082

[10] Tiwari, S., Wee, H. M., Sarkar, S.: Lot-sizing policies for defective and deteriorating items with time-dependent demand and trade credit. European Journal of Industrial Engineering. 11(5), 683-703 (2017) DOI: 10.1504/EJIE.2017.087694

[11] De, S. K., & Mahata, G. C.: A production inventory supply chain model with partial backordering and disruption under triangular linguistic dense fuzzy lock set approach. Soft Computing, 24(7), 5053-5069 (2020). DOI: 10.1007/s00500-019-04254-2

[12] Singh, D., Jayswal, A., Alharbi, M. G., & Shaikh, A. A, An investigation of a supply chain model for co-ordination of finished products and raw materials in a production system under different situations. Sustainability, 13(22) (2021), 12601. DOI: 10.3390/su132212601

[13] Mishra, V. K., & Sahab Singh, L. (2011). Deteriorating inventory model for time-dependent demand and holding cost with partial backlogging. International Journal of Management Science and Engineering Management, 6(4) (2011), 267-271. DOI: 10.1080/17509653.2011.10671172

[14] Sarkar, B., Mandal, P., & Sarkar, S, An EMQ model with price and time-dependent demand under the effect of reliability and inflation. Applied Mathematics and Computation, 231 (2014), 414-421. DOI: 10.1016/j.amc.2014.01.004

[15] Tripathi, R. P., & Mishra, S. M, EOQ model with linear time-dependent demand and different holding cost functions. International Journal of Mathematics in Operational Research, 9(4) (2016), 452-466. DOI: 10.1504/IJMOR.2016.079804

[16] Singh, T., Mishra, P. J., & Pattanayak, H, An optimal policy for deteriorating items with time-proportional deterioration rate and constant and time-dependent linear demand rate. Journal of Industrial Engineering International, 1-9 (2017), DOI: 10.1007/s40092-017-0198-6

[17] Lian. qi, A continuous-review inventory model with random disruptions at the primary supplier. European Journal of Operational Research, 225(1) (2013), 59-74. DOI: 10.1016/j.ejor.2012.09.035

[18] Viji, G., & Karthikeyan, K, An economic production quantity model for three levels of production with Weibull distribution deterioration and shortage. Ain Shams Engineering Journal, 9(4) (2018), 1481-1487. DOI: 10.1016/j.ejor.2012.09.035

[19] Sivashankari, C. K., & Panayappan, S, Production inventory model for two-level production with deteriorative items and shortages. The International Journal of Advanced Manufacturing Technology, 76(9-12) (2015), 2003-2014. DOI: 10.1007/s00170-014-6259-8

[20] Khanra, S, A production-inventory model for a deteriorating item with shortage and time dependent demand. Yugoslav Journal of Operations Research, 21(1) (2016), DOI: 10.1111/j.1475-3995.2008.00676.x

[21] Shah, N. H, Three-layered integrated inventory model for deteriorating items with quadratic demand and two-level trade credit financing. International Journal of Systems Science: Operations & Logistics, 4(2), (2017), 85-91. DOI: 10.1080/23302674.2015.1075078

[22] Khedlekar, U. K., & Namdeo, A, Production Inventory Model with Disruption Considering Shortage and Time Proportional Demand. Yugoslav Journal of Operations Research, 27(2) (2017), DOI: 0.2298/YJOR161118008K

[23] Lee, Y. P., & Dye, C. Y, An inventory model for deteriorating items under stock-dependent demand and controllable deterioration rate. Computers & Industrial Engineering, 63(2), (2012) 474-482. DOI: 10.1016/j.cie.2012.04.006

[24] Soroush, H. M, Multicriteria scheduling with controllable processing times, setup times, and learning, ageing and deterioration effects on a single machine. European Journal of Industrial Engineering, 10(4) (2016), 455-478. DOI: 10.1504/EJIE.2016.078140

[25] Mishra, U., Cárdenas-Barrón, L. E., Tiwari, S., Shaikh, A. A., & Treviño-Garza, G, An inventory model under price and stock dependent demand for controllable deterioration rate with shortages and preservation technology investment. Annals of Operations Research, 1-26. (2017), DOI: 10.1007/s10479-017-2419-1

[26] Kumar, V., Singh, S. R., & Sharma, S, Profit Maximization production inventory models with time-dependent demand and partial backlogging. Int. J. Oper. Res. Optim, 1(2), (2010), 367-375.

[27] Begum, R., Sahu, S. K., & Sahoo, R. R, An EOQ model for deteriorating items with Weibull distribution deterioration, unit production cost with quadratic demand and shortages. Applied Mathematical Sciences, 4(6) (2010), 271-288.

[28] Sarkar, B., & Sarkar, S, An improved inventory model with partial backlogging, time-varying deterioration and stock-dependent demand. Economic Modelling, 30, (2013), 924-932. DOI: https://doi.org/10.1016/j.econmod.2012.09.049

[29] Tai, A. H, An EOQ model for imperfect quality items with multiple quality characteristic screening and shortage back ordering, European Journal of Industrial Engineering, 9(2) (2015), 261-276. DOI: /10.1504/EJIE.2015.068657

[30] Wu, J., Skouri, K., Teng, J. T., & Hu, Y, Two inventory systems with trapezoidal-type demand rate and time-dependent deterioration and backlogging. Expert Systems with Applications, 46, (2016), 367-379. DOI: 10.1016/j.eswa.2015.10.048.

[31] Sadeghi, J., Niaki, S. T. A., Malekian, M. R., & Sadeghi, S, Optimising multi-item economic production quantity model with trapezoidal fuzzy demand and back ordering: two tuned meta-heuristics. European Journal of Industrial Engineering, 10(2) (2016), 170-195. DOI: 10.1504/EJIE.2016.075847.

[32] Chowdhury, R. R., Ghosh, S. K., & Chaudhuri, K. S, An order-level inventory model for a deteriorating item with time-quadratic demand and time-dependent partial backlogging with shortages in all cycles, American Journal of Mathematical and Management Sciences, 33(2), (2014), 75-97. DOI: 10.1080/01966324.2014.881173.

[33] Mahapatra, G. S., Adak, S., Mandal, T. K., & Pal, S, An inventory model for deteriorating items with time and reliability dependent demand and partial backorder, International Journal of Operational Research, 29(3) (2017), 344-359. DOI: 10.1504/IJOR.2017.084340.