

Optimal Discrete Coefficients of IIR Digital Filters Using Gasa Algorithm

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Abstract -The aim of this paper is to determine optimal coefficients of Infinite Impulse Response(IIR) digital filters using Genetic Algorithm and Simulaed Annealing (GASA). IIR filter is fundamentally a digital filter with Recursive response. As the error surface of IIR digital filters is usually nonlinear and multimodal, comprehensive optimization techniques are essential in order to avoid the local minima. It offers a novel hybrid approach for IIR digital filters. Unliketraditional attempts, this paper uses heuristic types which are hybridized to design digital filters that satisfies prescribed specifications. Further, this methodology broadens the scope of hybrid optimization algorithms. The design of Lowpass digital IIR filter is projected to provide evaluation of transition band. The paper finds the minimum mean-square-error through the Coefficients of IIR digital filter by GASA. It is found that the calculated values are more optimal than fda tool available for the design of filter in MATLAB and individual heuristic techniques. The simulation effects of the engagedcases shows an improvement on transition band and mean-square-error. The Pole-Zero position is alsodisplayed to define the stability of considereddigital filters.

Key words- Digital Filter, Infinite-Impulse Response (IIR), Genetic Algorithm (GA), Simulated Annealing (SA), Optimization.

Introduction

A short time ago, the field of Digital Signal Processing (DSP) has grown significantly both theoretically and technologically. In DSP, there are two main types of Systems. The primary type of systems implements signal filtering in time domain and hence it is known as Digital filters. The another type of systems delivers signal representation in frequency domain and are called Spectrum Analyzer. Digital filtering is one of the most commanding tools of the DSP. Digital filters are proficient of specifications that would, at best, be extremely difficult, if not impossible, to achieve an analog implementation. In addition, the features of a digital filter can be easily altered under software control. Digital filters are categorized either as Finite duration impulse response (FIR) digital filters or Infinite duration impulse response (IIR) digital filters, depending on the form of impulse response of the system. In the FIR digital filters, the impulse response sequence is of finite duration, i.e., it has a finite number of non-zero terms. Digital infinite-impulse-response (IIR) filters can often provide a much better performance and less computational cost than their equivalent finite-impulse-response (FIR) digital filters and have become the target of growing interest.

However, because the error surface of digital IIR filters is ordinarily nonlinear and multimodal, conventional gradient centered design methods may easily get stuck in local minima of error surface. Therefore, some scientists have tried to develop design methods based on recent heuristic optimization algorithms such as genetic algorithm (GA), simulated annealing (SA), tabu search (TS) etc.

Iterative or analytic methods generally lead to sub-optimal designs. Subsequently, there is a necessity of optimization methods (heuristic type) that can be use to design digital filters that would satisfy prescribed specifications. Goldberg presented a detailed mathematical model of Genetic Algorithm [1]. Benvenuto et al. (1992) described the salient features of using simulated annealing (SA) algorithm in the context of designing digital filters with linear phase digital filter. The algorithm is then applied to the design of FIR digital filter and the simulation results were not impressive. Moreover, it is computationally very expensive. Liang et al. (2003) used genetic algorithm to design 1-D IIR filter with canonical-signed-digit coefficients restricted to low-pass filter. Ahmad and Antoniou (2006) explored FIR filters and

equalizers through the use of GA. Consequently, GAs requires a large amount of computation. Oliveira et al. (2007) presented a new approach for designing linear FIR filters by using nonlinear stochastic global optimization based on simulated annealing techniques. Jung et al. (2008) originated the design method of a linear phase finite word length finite-duration impulse response (FIR) filter using SA. Apaydin (2009) presented a development of reduced-delay FIR digital filters with amplitude requirements using equi-ripple passband, peak constrained least squared stopband, and nearly constant group delay in passband for real time applications. Adaptive IIR system identification based on PSO with quantum infusion is proposed in Luitel and Venayagamoorthy (2010). Liang and Tao (2011) presented a hybrid algorithm to solve scheduling problems in flexible production environment, where time, cost and equipment utilization rate are all concerned. Coelho et al. (2012) included "swarm intelligence" based algorithms and distributed problem solvers inspired by the collective behavior of insect colonies and other animal societies. Deb and Jain (2014) recognized a few recent efforts and discuss a number of viable directions for developing a potential evolutionary multiobjective optimization algorithm for solving many-objective optimization problems. Bolea et al. (2015) explored digital filters multiple access for passive type optical networks (PONs), where digital orthogonal filtering is used. Choi et al. (2016) mentioned a hybrid procedure recreated on the GA method and evolutionary strategy (ES) for electromagnetic optimization problem. Cheng et al. (2016) presented non-dominated sorting GA with multi-objective optimization process for large array thinning. Silva and Attux (2018) demonstrated a challenging combinatorial optimization task on SA, through objective function based on an entropy. Wang et al. (2018) discussed scheme which realized photonic ADC and digital photonic filtering for microwave signals. Dong et al. (2019) proposed a hybrid transformation technique for passive optical network created on digital signal processing. The drawback of preceding design methods is that the computation time is quite long and single algorithm is used for optimization. To test the

optimization procedure, the proposed algorithm is implemented in Matlab and results are found to be very encouraging.

This Paper is organized as follows: In the Section 2, IIR type of digital filter design aspects are described. In section 3, heuristic approaches are concisely stated. The Genetic Algorithm and Simulated Annealing (GASA) related to calculation of discrete coefficients are projected in Section 4. The achieved simulation results of few examples used is briefly presented in Section 5. The Conclusion & future scope mentioned in the last section.

IIR Digital Filter Design Issues

Normally, Digital filters are classified as Recursive and Non-Recursive types of filters. The response of Recursive or IIR digital filters is dependent on one or more of its past output. If such filter subjected to an impulse then its output need not essentially become zero. This specifies that the system is prone to feedback and instability. Consider the IIR digital filter with the input-output relationship directed by equation as:

$$y(k) + \sum_{i=1}^M b_i y(k-i) = \sum_{i=0}^L a_i x(k-i) \quad (1)$$

where $x(k)$ and $y(k)$ are the filter's input and output, respectively, $M (\geq L)$ is the filter order. The transfer function of this IIR digital filter can be depicted as:

$$H(z) = \frac{A(z)}{B(z)} = \frac{\sum_{i=0}^L a_i z^{-i}}{1 + \sum_{i=1}^M b_i z^{-i}} \quad (2)$$

The specifications $a_0, a_1, a_2, \dots, a_L, b_1, b_2, \dots, b_M$ appearing in equation (1) and equation (2) are termed as the filter coefficients. These define the characteristics of the designed filter. The Digital filters have various types of stages in their design as presented in fig. 1. Hence, the design of digital filter can be considered as one of optimization problem for cost function $J(w)$ stated as the following:

$\min J(w)$, where $w = [a_0, a_1, a_2, \dots, a_L, b_1, b_2, \dots, b_M]$ is the filter coefficient vector.

The objective is to minimize the cost function $J(w)$ by adjusting w . The cost function is generally

expressed as the time-averaged cost function termed as:

$$J(w) = \frac{1}{N} \sum_{k=1}^N (d(k) - y(k))^2 \quad (3)$$

where $d(k)$ and $y(k)$ are the desired and actual responses of the designed filter, respectively.

Further, N is the number of samples used for the calculation of cost function.

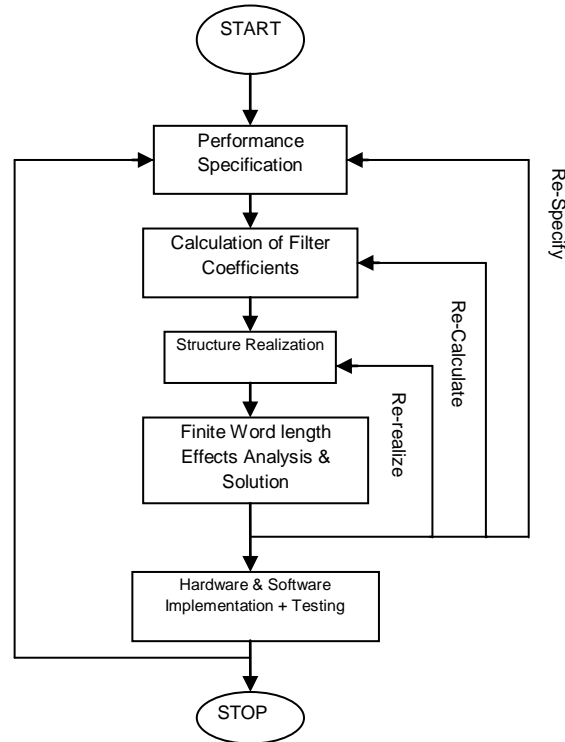


Figure 1 Basic Steps to Design IIR digital filters

Heuristic Algorithms

Simulated Annealing

In 1983 Kirkpatrick et al. created Simulated annealing (SA), a local search technique first used in combinatorial optimisation issues. The statistical mechanics-based simulated annealing process is simply an iterative random search procedure with adaptive motions along the coordinate axes. It allows incline movements controlled by a probabilistic criterion. Simulated annealing is a desirable approach for optimising functions with numerous variables and, consequently, numerous local minima because of its ability to cross "hills". Combinatorial minimization and statistical mechanics have been demonstrated to be closely related. The study specifically demonstrates how the natural process that occurs for matter to reach its low temperature state might offer practical reduction approaches. The work also demonstrates that, in reality, merely identifying a

low-temperature condition does not ensure that the underlying matter is in one of its lowest energy states. The substance might be in a low-temperature condition but still be unable to sustain equilibrium there. The starting point for experiments to determine a substance's low temperature state is to heat the substance up to its melting point, which is a very high temperature. The temperature is then gradually dropped so that the chemical has enough time to reach thermal equilibrium at each temperature value. The procedure is repeated until the substance is cold, or until there are no more energy changes. A group of atoms in thermal equilibrium at a specific temperature can be simulated using the Metropolis algorithm, which has been widely utilised in simulated annealing. A modest random movement of an atom is applied at each step of this process, and the consequent change in the system's energy (E) is computed. The method of

simulated annealing consists of three functional relationships.

1. $g(x)$: Probability density of state-space of D parameters $x=\{x_i; i=1, \dots, D\}$.
2. $h(\Delta E)$: Probability for acceptance of new cost-function given the just previous value.
3. $T(k)$: schedule of "annealing" the "temperature" T in annealing-time steps k , i.e., of changing the volatility or fluctuations of one or both of the two previous probability densities. The new displacement is accepted subject to the criterion: If $\Delta E \leq 0$, the acceptance probability is based on the chances of obtaining a new state with "energy" E_{k+1} relative to a previous state with "energy"

$$\begin{aligned}
 h(\Delta E) &= \frac{\exp(-E_{k+1}/T)}{\exp(-E/T) + \exp(-E_k/T)} \\
 &= \frac{1}{1 + \exp(\Delta E/T)} \\
 &= \exp(-\Delta E/T)
 \end{aligned}
 \tag{4}$$

where ΔE represents the "energy" difference between the present and previous values of the energies (considered here as cost functions). This essentially is the Boltzmann distribution contributing to the statistical mechanical partition function of the system. Uniform random deviates in the interval (0, 1) are a convenient means of implementing the random part $h(\Delta E)$. The accepted rearrangement is then used as the starting point of the next step. By repeating the

step many times, the algorithm simulates the thermal motion of atoms in thermal contact with a heat bath at temperature T . At very high values of temperature T , in the above expressions, virtually any rearrangement will be accepted. As the temperature falls to low values, rearrangements that increase the change in energy will be likely be rejected. This strategy allows for controlled "uphill" excursions to be taken and hence to seek "hidden" arrangements with lower energy levels. Using the Metropolis algorithm, simulated annealing has been used for discrete optimization problems. Although the algorithm does not guarantee to find the global optimum, if the function has many good near optimal solutions it will find one. The simulated annealing design process consists of first "melting" the system being optimized at a high temperature and then lowering the temperature in slow stages until the system is "cold", that is until no further changes occur. As in statistical mechanics, the simulation must proceed long enough for the system to attain a steady state at each temperature. An annealing schedule is the sequence of temperatures and rearrangements of the coefficient values attempted to reach steady state at each temperature. An important feature of simulated annealing is that the "gross" features of the function appear at high values of temperature and the "finer" detail at lower values. The summarized pseudo code of SA is shown in fig 2.

```

Procedure SA
    Initialization;
    while condition not met
        new_model=random
        delta_E=E(new_model) – E(current_model)
        if(delta_E<0) current_model=new_model
    else probability  $P = \exp^{-\text{delta\_E}/\text{alpha\_T}}$  : currnt_model=new_model
    end while
end procedure
    
```

Figure 2 Pseudo code of Simulated Annealing

Genetic Algorithm

Genetic algorithms, which are computer programmes that imitate the natural evolutionary process proposed in the early 1970s, Holland proposed g. In 1970, Goldberg published a

comprehensive mathematical model of GA, and D. Jong expanded the GAs to functional optimisation. Genetic algorithms (GA) are stochastic search techniques that can be used to look for the best answer to an optimisation problem's evolution

function. Every generation, GAs influence a population of people. Population of people in each generation (iteration), where each person, referred to as a chromosome, stands in for one potential solution to the issue. Fit individuals in the population live to reproduce, recombining their genetic material to create new individuals as progeny. Some data structure, most frequently a finite-length of properties, models the genetic material. Similar to how evolution works, selection offers the impetus for better solutions to prevail. Each solution has a fitness value assigned to it that describes how effective it is when compared to other solutions in the population. A crossover

mechanism that switches out chunks of data strings between the chromosomes simulates the recombination process. Additionally, mutations that result in arbitrary changes to the strings add new genetic material to the genome. These genetic operations only occur frequently if certain predetermined probability are met. The basic pseudo code of GA is made up of the selection, crossover, and mutation processes, which are repeated until a set of predetermined criteria are met (see fig. 3). Through this process, successively better and better individuals of the species are generated.

```
Procedure GA
  Initialization;
  while condition not met
    Evaluate fitness for each individual
    prune population
    Selection
    Crossover
    Mutation
  end while
end procedure
```

Figure 3 Pseudo code of Genetic Algorithm

Hybrid Algorithm

Hybrid algorithm states to combining two search algorithms to solve a given problem. It is also referred as memetic algorithms. As we have discussed in previous section, both SA and GA are powerful heuristic technique for optimization, but SA falls easily into a local solution and the GA lack in fine local tuning capabilities. The main drawback of SA algorithm is the computation time and limited convergence behavior. In strong co-operation with GA, the problem of premature convergence of the best individuals of the population to a local optimum has been cancelled.

This paper presents a hybrid scheme (GASA) to calculate optimum coefficients of IIR digital filters using GA and SA. This is often a population based search such as GA with local searches performed by other algorithms i.e. SA. The solution from GA is used as an initial point for SA that is faster and more efficient for local search. The GASA approach is summarized as follows:

Step 1) *Initialization*: This includes initial parameters for GA and SA.

Step 2) *Data Preparation using GA*: Generate population and evaluate fitness of each individual. Select some of the best fit chromosome according to some selection (*stochastic universal sampling, tournament method etc.*) criteria. Now cross over (*single point cross over, double point crossover etc.*) partial solution between pair of selected solution with some probability (*cross over probability*) to generate child solution. Mutate the value an allele of child with small probability (*mutation probability*) value. Evaluate child population and replace parent population. Repeat the process (GA operator) until termination criteria satisfies.

Step 3) *Solution using SA*: The best solution from GA is used as initial point for SA. Determine the *acceptance probability* and produce new solution and evaluate. Reach to maximum temperature and replace the current solution with best solution so far. Decrease the temperature until stopping criteria is reached.

The fitness value of a solution i in the population is determined by using fitness formula given as:

$$fit(i) = \frac{1}{k + J(w)i} \quad (5)$$

where $J(w)$ is the cost function value computed for i and k is the number of poles outside the unit circle. The hybridization of GA and SA is shown in the flow chart given below in fig. 4.

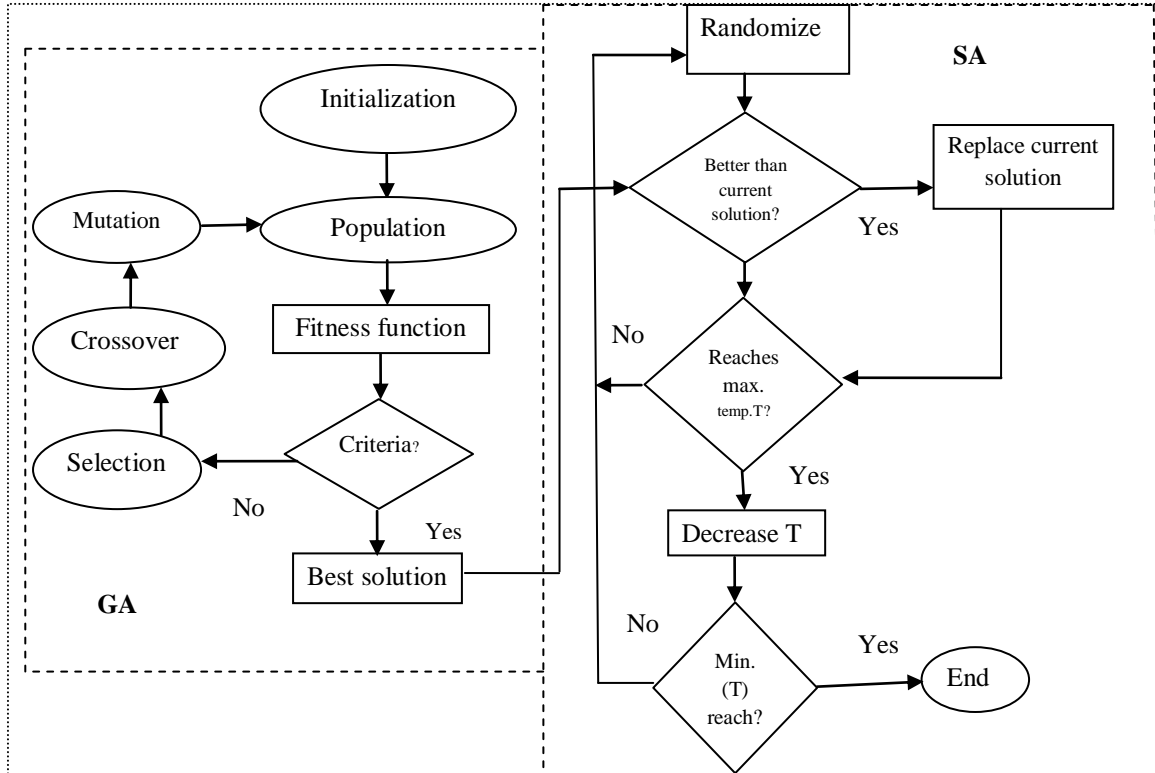


Figure 4 Flow chart of GASA

Results And Discussion

Simulation studies are carried for a well-known IIR filter (see Problem at Appendix), which has been used by many authors as a “benchmark filter” for comparison purpose. The examples were

performed on with Pentium IV 2.80 GHz CPU and 1GB of RAM. Simulation study is carried out in MATLAB to demonstrate the potentiality of GASA for design of IIR digital filters. The parameters of GA are:

Number of Population = 50
 Number of Generation = 500
 Selection = stochastic universal sampling
 Crossover = double point crossover
 Crossover Probability = 0.8
 Mutation Probability = 0.2

Moreover, SA is done with probability of 20%, and other parameters are given below:

Temperature Gradient = 100
 Initial Temperature = 100
 Temperature update = exponential temperature update
 Annealing Function = fast annealing
 Function tolerance = exp^{-6}
 Finishing Condition = $100*5$

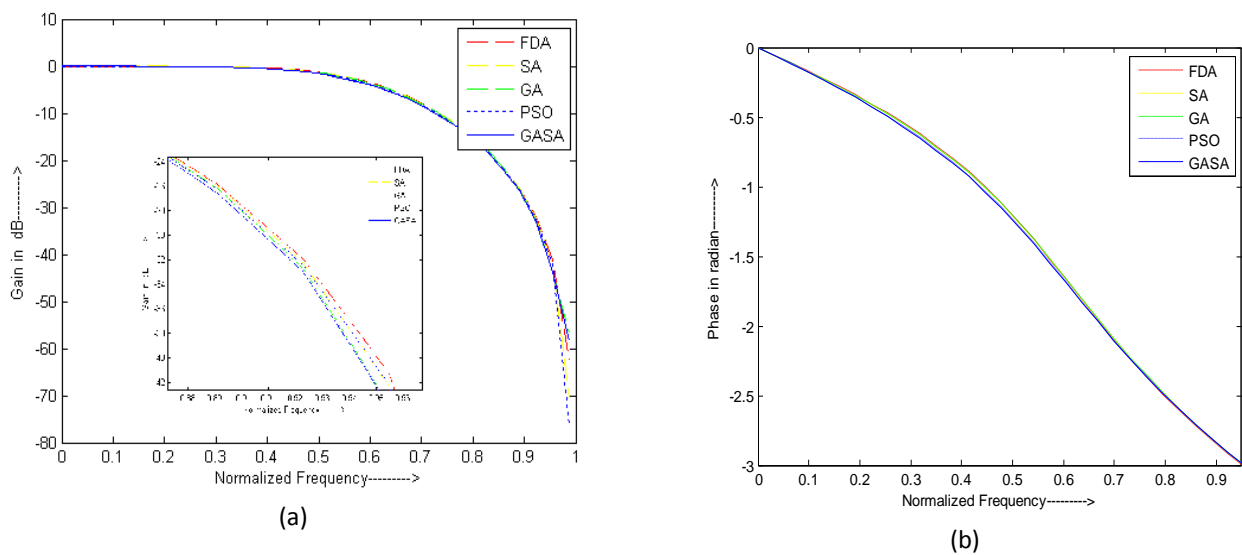


Figure 5 Response of Low pass filter: (a) Magnitude response; (b) Phase response.

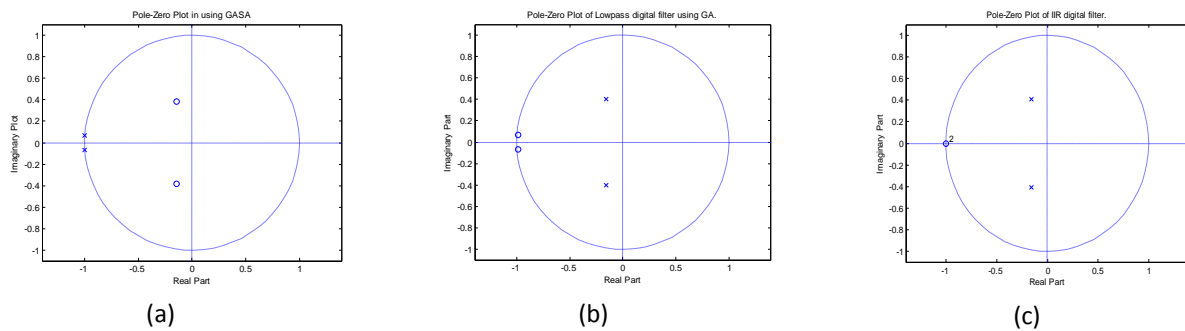


Figure 6 Pole-Zero Position of Low pass filter using: (a) GASA; (b) GA; (c) fda

The magnitude and phase response of low pass filters is shown in fig. 5 in which a zoomed in curve for transition band is included comparing with other heuristic techniques. It is observed from the magnitude responses that the gain is -62.61 at 0.9868 without using hybrid method, but using proposed method, this gain occurs at 0.9746 . This gives us better transition band compared to other technique. It is clear from phase response that phase is identical to filter design analysis (*fda*) tool response and other individual heuristic ways. In fig. 6, we have summarized Pole-Zero behavior of Low pass filter using *fda*, GA and GASA. It can be

realized that the poles and zeros of thecalculated filter falls within the unit circle. This shows that the designed filter is also stable using proposed method. The mean-square-error (MSE) and resulting values of the low-pass filter in term of coefficients (Numerator and Denominator) are shown in Table 1 and Table 2. It is observed from table 1 that the proposed method gives optimal values of coefficients with minimum mean-square-error (MSE).Table 2 provides a comparison of coefficients with the *fda*, simulated annealing (SA), genetic Algorithm (GA) and particle swarm optimization (PSO).

TABLE 1 Results of MSE of Low-pass filter

Name of Method	Order of filter	MSE
<i>fda</i> tool	2	0.3296
SA	2	0.3284

GA	2	0.3275
PSO	2	0.3195
Proposed	2	0.3177

TABLE 2 Coefficients of Low-pass filter designed with 2nd order

Coefficients	fda tool	SA	GA	PSO	Proposed method
a_0	0.3752	0.3667	0.3739	0.3659	0.3649
a_1	0.7504	0.7213	0.7459	0.7313	0.7281
a_2	0.3752	0.3525	0.3739	0.3659	0.3649
b_1	1	1	1	1	1
b_2	0.3120	0.3116	0.3116	0.2916	0.2851
b_3	0.1888	0.1790	0.1789	0.1723	0.1691

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Conclusion

We have applied a hybrid GA and SA to determine optimal discrete coefficients of IIR digital filters. Design example with the same specifications as those given in the literature are provided, and the design result shows that the procedure under the provided initial points has obtained better filter with optimal coefficients. It is observed that MSE is 0.3177 which is least observed as compared to other heuristic techniques. The large computation cost of the genetic algorithms and simulated annealing can be significantly reduced by the proposed algorithm, which is especially effective for higher order filters. The position of Pole-Zero of the designed low pass filter using optimal coefficients gives us stable filters. Thus it is assumed that the suggested algorithm is capable of fast response, high performance and stability. The projected method can also be extended to random magnitude response specifications and multiband.

Appendix

Problem: Calculate the responses of a low-pass digital filter with subsequent specifications: Stop/Pass band ripples 15dB/1dB, and band edges 400Hz /200Hz with a sampling frequency of 1000Hz.

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