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An EOQ Model For Deteriorating Items with Varying Different Time-Dependent Demand Rates

Prachi Swain[1], Trailokyanath Singh[2*], Chittaranjan Mallick[3], Manzoor Hussain [4]

[1]Department of Mathematics, C. V. Raman Global University, Bhubaneswar – 752054, Odisha, India

E-mail: prachiswain21@gmail.com

[2]Department of Mathematics, C. V. Raman Global University, Bhubaneswar – 752054, Odisha, India

*E-mail: trailokyanaths108@gmail.com (Corresponding Author)

https://orcid.org/0000-0003-3483-791X

[3]Department of Basic Science (Mathematics), Parala Maharaja Engineering College, Berhampur – 761003, Odisha, India

E-mail: cmallick75@gmail.com

[4]Indus University, Karachi Pakistan

E-mail: manzoor.hussain@indus.edu.pk

Abstract: In this work, the Economic Order Quantity (EOQ) inventory model for deteriorating products has been expanded. A two-component type deterministic pattern is considered here. It is stable in the first part of the cycle due to the introduction of items into the system, and then in the second half of the inventory cycle, it becomes a quadratic function of time. Here the deterioration rate of the items is constant and partial backlogging and shortages are prohibited here. To determine the absolute period of the cycle, optimum cost as well as initial order quantity, the theoretical and mathematical expressions are developed. To calculate the above quantities a simple algorithm is used. The results can be defined by two numerical examples given at the end of this paper. Finally, a sensitivity study based on different parameters along with decision variables is derived.

Keywords: Constant deterioration, time-dependent quadratic demand, EOQ, Inventor

1. Introduction

During the last three decades, most of the inventory research has been carried out on deteriorating items for the control and maintenance of inventory. Firstly, Wilson (1934) [1] laid the foundation of inventory research by introducing the Economic Order Quantity (EOQ) formula for the computation of system costs. The physical loss of an item depends on several natural phenomena like decay, depletion, evaporation, radiation or spoilage which is termed "deterioration". Nowadays, it's very essential fact to preserve the inventory of items appropriately so that the demand for items can be fulfilled effectively for future needs. By using the following differential equation Ghare and Schrader (1963) first develop a mathematical representation of the EOQ model along with the exponential deterioration rate [2].

$$\frac{dI(t)}{dt}+Z(t)I(t)+D(t)=0, \qquad 0\le t\le T \tag{1}$$

The inventory's cycle length is indicated by T here. In this case, $I(t)$, $D(t)$ and $Z(t)$ indicate the inventory level, proposed demand pattern and proposed deterioration rate at any instant, respectively. By applying the calculus mechanism, Donaldson (1977) presented a review of the inventory literature by focusing on degrading items [3], where the linear demand pattern is considered and shortages are prohibited. Aggarwal (1978) suggested an order-level inventory model with a continuous deterioration rate and completely back-ordered shortages [4].

2. Statement of the Problem

The EOQ inventory model is established depending upon different time-varying, realistic demands and different time-proportional deteriorations rates. The most applicable demand pattern is time-dependent quadratic demand and it can be characterized as $D(t) = a + bt + ct^2, a \geq 0, b \geq 0, c > 0 \,\&\, c \neq 0$. If we substitute $c = 0$ and $b = c = 0$ in the quadratic demand pattern, hence the linear demand pattern and the constant demand are obtained respectively. Khanra and Chaudhuri (2003) create an EOQ inventory model for deteriorating items, by taking the rate of deterioration constant [5]. Here, demand is quadratic and time-dependent, and shortages are completely back-ordered. Singh et al. (2017) introduced a deterministic inventory model with time-proportional deterioration when shortages are prohibited [6].

3. Related Work

Dave and Patel (1981) extended an EOQ inventory model by considering the ongoing development of demand for deteriorating elements [7]. Here constant is the deterioration rate and also shortage is not taken into consideration. By taking shortages as a new factor Sachan (1984) [8] expanded the EOQ model introduced by Dave and Patel (1981). Most of the inventory modellers formulated their model by concentrating on time. For deteriorating items Goswami and Chaudhuri (1991) [9] as well as Chakrabarti T [2], Chaudhuri KS (1997) proposed replenishment policies including continuous modification in the demand rate and the entire cycles shortages are occurring [10]. By considering the demand pattern of deteriorating items corresponding to time Chung and Ting (1994) fostered one EOQ model [11]. A replenishment arrangement including the time-changing demand along with shortage was brought up by Wee (1995) [12]. By analyzing the literature, we conclude that depending on time mainly the modellers are concentrated on two different categories of demand, such as linear demand and exponential demand. Sometimes in the case of real market situations, it has been detected that the demand rate of the component which changes continuously with time is determined by the uniformly varying demand rate.

The models discussed above concentrated on the stable deterioration rate. Here shortages are taken into consideration. Srivastava and Gupta (2007) extended the model in consideration of both constant and linearly increasing demand rates [13]. Here shortages are not allowed. Singh and Pattnayak (2014) created an inventory problem involving two warehouses by considering the linearly increasing demand rate as well as the payment, which is an acceptable order delay [14].

4. Background

When we consider the classical inventory models, it has been observed that both the deterioration rate along with the demand pattern are considered constant. It is not always true because of the introduction of new items or seasonal items. Therefore it increases or decreases according to the usefulness of the items. Promote inventory model time is an important factor. In a certain period deficiency rate of a few items is developed, and then the rate of deterioration is developed according to that time. Thus to establish an EOQ model, the deterioration rate corresponding to the time factor is more reasonable. Srivastava and Gupta (2007) [13] established the optimal inventory model by considering a two-component demand pattern, shortages and a constant deterioration rate. When the inventory cycle begins at this point, demand is act as stable because of the introduction of new items or seasonal items and then, it increases linearly according to the usefulness of the items [14-20]. In the first stage of the cycle, there is no deterioration, however, it is constant in the second stage of the inventory cycle. Moreover, the rate of deterioration is also proportional to the corresponding time. A situation like this frequently arises in the market. The proposed model is validated for newly launched items such as branded mobile phones, automobiles, dress materials, cosmetics, etc [21-23]. The demand for this type of product increases which is constant at the beginning. Reducing average cost by maximizing the length of the inventory cycle is the major objective of the current EOQ inventory model. The optimal order quantity is similarly calculated in this model by substituting the value of the decision variable. To explain this model, the method of solutions is provided with numerical examples [24-30]. Finally, the sensitivity of different parameters is explained in this model.

The remainder of this paper is examined in the section below Section II represents the assumptions along with the notations that regulated the present problem. Section III represents the establishment of the present model and its method of solution. Section IV represents two numerical examples and the sensitivity study of Example No.1. Finally, in Section V the summary together with the future scope of research are given.

5. Motivation

Moreover, time is an essential component and it acts a significant role to extend the inventory model. Depending on that period, the deficiency rate rises for a few components, and the deterioration rate is then calculated. As the deterioration rate is time-proportional, hence most of the inventory models are formulated according to the use of different Weibull distributions of time. Singh et al. (2017) fostered an EOQ model without shortages, with changing rates of deterioration, and time-dependent demand patterns [15]. To promote this model, the deterioration rate associated with the cycle is more responsible. In this model, an effort has been analysed to reduce the average total cost by maximizing the inventory cycle's time. Finally optimal order quantity is also determined by substituting the decision variable of the inventory problem.

6. Notations and Assumptions

Based on the following notations and essential assumptions, the EOQ inventory model is studied:

6.1. Notations

Several notations remain adopted for illustration of the present model:

$\theta(t)=\theta_0,\ 0<\theta_0<<1$: Deterioration rate.

$D(t)$: The demand pattern that changes with time, i.e.,

$$D(t)=\begin{cases} a_0, & 0\le t\le\gamma \\ a_0+b_0(t-\gamma)+c_0(t-\gamma)^2, & \gamma\le t\le T \end{cases}.$$

Here the demand pattern is constant i.e. a_0 units /units in the time interval $[0,\gamma]$ and it is quadratic within the interval $[\gamma,T]$.

$I(t)$: the number of goods in stock at any particular time.

T: inventory cycle length.

I_S: Ordered quantity at the start of the cycle.

C_o: Cost for each unit ordered.

h_c: Cost/item/unit time is currently being held.

d_c: cost per unit/item/time.

γ: time at which the items start deteriorating.

$ATC(T)$: Average cost for each item/unit of time.

T^*, I_S^* & $ATC(T^*)$: Optimal values of T, I_S and $ATC(T)$, respectively.

6.2 Assumptions

We consider the following hypotheses for the derivation of the model.

(i) The system is based on a single typed item.
(ii) In the first half of the cycle, the rate of deterioration is zero, whereas, in the second portion, it is constant.
(iii) Here the deterministic demand pattern is remaining considered. Depending on the time horizon, the demand pattern contains two components: stable at the start of the cycle and thereafter a quadratic function of time.

(iv) Shortages and backlogs are strictly forbidden.
(v) The replenishment period is rapid whereas the distributing period is negligible.

(vi) There is an infinite scope of arrangement. Only a common procedure of arrangement is recognized and all other remaining series are exact.
(vii) At the time of cycle duration, the deteriorated items are neither recovered nor adjusted.
(viii) During the cycle period the ordering price, holding price as well as unit price remain stable.

7. Model formulation and Solution

The model as well as the method of solution is given below:

7.1. Model formulation

The time $t = 0$ and lot-size I_S units enter the system when the inventory cycle begins. The amount of stock, let it be a_0 units, falls due to the constant demand in the period $[0, \gamma]$. Thereafter, the stock reduces continuously over the period $t = \gamma$, because of the combined effects of quadratic demand and ongoing deterioration the reduction is developed and it arrives towards an end, when $t = T$. Fig. 1 depicts the relation between the inventory system and time.

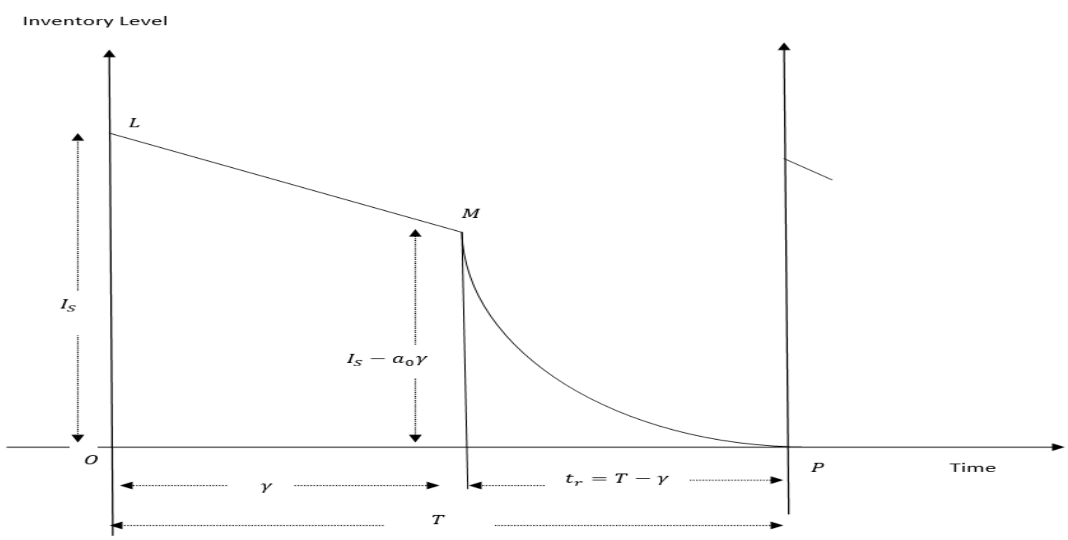

Fig. 1: Representation of "Inventory level" as "Time".

This model's objective is to compute the decision variable T as well as the average total cost $ATC(T)$ across the time interval $[0,T]$.

The item's demand rate is constant across the time interval $[0,\ \gamma]$, that is, a_0 units.

Therefore, the total demand (ITD)during this interval $[0,\ \gamma]$ is given by

$$ITD = a_0\,\gamma\,. \tag{2}$$

Then, the remaining inventory level (RIL) during the interval $[\gamma,T]$ is

$$RIL = I_S - a_0\,\gamma\,. \tag{3}$$

The interval $[\gamma,T]$ can be defined simply as

$$t_r = T - \gamma\,. \tag{4}$$

The differential equation shown below defines the instantaneous inventory level $I(t)$ at a particular time t .

$$\frac{dI(t)}{dt} + \theta(t)I(t) + D(t) = 0, \qquad 0 \le t \le t_r \tag{5}$$

where $\theta(t) = \theta_0\ \ (0 < \theta_0 << 1)$ and $D(t) = a_0 + b_0(t-\gamma) + c_0(t-\gamma)^2$.

Using the fundamental assumptions and the boundary condition, $I(0) = I_S - a_0\,\gamma$, the outcomes of Eq. (4.4) are given below

$$I(t) = \left[I_S - a_0\gamma + \frac{a_0 - b_0\gamma + c_0\gamma^2}{\theta_0} - \frac{b_0 - 2c_0\gamma}{\theta_0^2} + \frac{2c_0}{\theta_0^3}\right]e^{-\theta_0 t}$$
$$-\frac{a_0 + b_0(t-\gamma) + c_0(t-\gamma)^2}{\theta_0} + \frac{b_0 + 2c_0(t-\gamma)}{\theta_0^2} - \frac{2c}{\theta_0^3}, \qquad 0 \le t \le t_r\,. \tag{6}$$

The Eq. (4.5) $I(t_r) = 0$ is given by

$$I_S = a_0\,\gamma - \frac{a_0 - b_0\gamma + c_0\gamma^2}{\theta_0} + \frac{b_0 - 2c_0\gamma}{\theta_0^2} - \frac{2c_0}{\theta_0^3}$$
$$+\left[\frac{a_0 + b_0(t-\gamma) + c_0(t-\gamma)^2}{\theta_0} - \frac{b_0 + 2c_0(t-\gamma)}{\theta_0^2} - \frac{2c_0}{\theta_0^3}\right]e^{\theta_0\, t_r}\,. \tag{7}$$

Using the relation $t_r = T - \gamma$, Eq. (4.6) can be written as

$$I_S = a_0\,\gamma - \frac{a_0 - b_0\gamma + c_0\gamma^2}{\theta_0} + \frac{b_0 - 2c_0\gamma}{\theta_0^2} - \frac{2c_0}{\theta_0^3}$$

$$+\left[\frac{a_0 + b_0\left(T-2\gamma\right) + c_0\left(T-2\gamma\right)^2}{\theta_0} - \frac{b_0 + 2c_0\left(T-2\gamma\right)}{\theta_0^2} + \frac{2c_0}{\theta_0^3}\right]e^{\theta_0(T-\gamma)}. \tag{8}$$

The overall average cost can be calculated by adding the costs listed below.

I. The ordering cost (SOC) is denoted as:

$$SOC = C_0. \tag{9}$$

II. The holding cost (SHC) is computed as follows:

1. In the interval $\left[0,\gamma\right]$, the holding cost of the first sub-system (SHC_1)is computed as:

$h_c \times$ area of trapezium LMNO, i.e.,

$$h_c \times \left[\frac{\left(LO + MN\right).ON}{2}\right]$$

$$= \gamma h_c \left[\left(I_0 - a_0\,\gamma\right) + \frac{a_0\,\gamma}{2}\right] \quad (4.9)$$

and

2. The holding cost of the second sub-system (SHC_2), in the interval $\left[0,t_r\right]$ is computed as:

$h_c \times$ area of triangle MNP, i.e.,

$$h_c \times \left[\frac{1}{2}.PN.MN\right]$$

$$= h_c \times \left[\frac{1}{2}.t_r.\left(I_S - a_0\,\gamma\right)\right]. \tag{10}$$

Therefore, the inventory holding cost (SHC) becomes:

$$SHC = \gamma h_c \left[\left(I_S - a_0\,\gamma\right) + \frac{a_0\,\gamma}{2}\right] + h_c \times \left[\frac{1}{2}.t_r.\left(I_S - a_0\,\gamma\right)\right]$$

$$= h_c\left[\frac{a_0\gamma^2}{2} + \left(I_0 - a_0\,\gamma\right)\left(\gamma + \frac{t_r}{2}\right)\right]. \tag{11}$$

III. The deterioration cost (SDC) is calculated in the time interval $[0, T]$ as follows:

$$SDC = d_c\left[I_S - a_0\gamma - \int_0^{t_r}\left[a_0 + b_0(t-\gamma) + c_0(t-\gamma)^2\right]dt\right]$$
$$= d_c\left[I_S - a_0\gamma - a_0 t_r - \frac{b_0(t_r-\gamma)^2}{2} - \frac{c_0(t_r-\gamma)^3}{3} + \frac{b_0\gamma^2}{2} - \frac{c_0\gamma^3}{3}\right]. \quad (12)$$

ATC(T) stands for the average total cost of the system per unit of time in the time interval $[0, T]$ and is defined as follows:

$$ATC(T) = \frac{1}{T}\left[SOC + SHC + SDC\right]$$

$$= \frac{1}{T}\left[\begin{array}{l} C_0 + h_c\left[\frac{a_0\gamma^2}{2} + (I_S - a_0\gamma)\left(\gamma + \frac{t_r}{2}\right)\right] + \\ d_c\left[I_S - a_0\gamma - a_0 t_r - \frac{b_0(t_r-\gamma)^2}{2} - \frac{c_0(t_r-\gamma)^3}{3} + \frac{b_0\gamma^2}{2} - \frac{c_0\gamma^3}{3}\right]\end{array}\right]$$

$$= \frac{1}{T}\left[\begin{array}{l}\left[\frac{a_0 + b_0(T-2\gamma) + c_0(T-2\gamma)^2}{\theta_0} - \frac{b_0 + 2c_0(T-2\gamma)}{\theta_0^2} + \frac{2c_0}{\theta_0^3}\right]e^{\theta_0(T-\gamma)} \\ -\frac{a_0 - b_0\gamma + c_0\gamma^2}{\theta_0} + \frac{b_0 - 2c_0\gamma}{\theta_0^2} - \frac{2c_0}{\theta_0^3}\end{array}\right]\left[h_c\left(\frac{T+\gamma}{2}\right) + d_c\right]$$

$$+\frac{1}{T}\left[C_0 + \frac{h_c a_0\gamma^2}{2} - d_c\left[a_0(T-\gamma) + \frac{b_0(T-2\gamma)^2}{2} + \frac{c_0(T-2\gamma)^3}{3} - \frac{b_0\gamma^2}{2} + \frac{c_0\gamma^3}{3}\right]\right], \quad (13)$$

(utilizing the equations (4.3) and (4.4)).

The goal of the above-discussed model is to determine T^*, which is the representation of the optimal value of T as a result of which $ATC(T)$ is minimum.

The optimal value $ATC(T)$ is defined as follows:

$$\frac{d\left(ATC(T)\right)}{dT} = 0 \quad (14)$$

and

$$\frac{d^2\left(ATC(T)\right)}{dT^2} = 0 \quad . \quad (15)$$

Eq. (4.15) gives us

$$\frac{d\left(ATC(T)\right)}{dT}=\frac{1}{T}\left[h_c\left(\frac{T+\eta}{2}\right)+d_c\right]\left[a_0+b_0\left(T-2\gamma\right)+c_0\left(T-2\gamma\right)^2\right]e^{\theta_0(T-\gamma)}$$

$$+\frac{h_c}{2T}\left[\begin{array}{l}\left[\dfrac{a_0+b_0\left(T-2\gamma\right)+c_0\left(T-2\gamma\right)^2}{\theta_0}-\dfrac{b_0+2c_0\left(T-2\gamma\right)}{\theta_0^2}+\dfrac{2c_0}{\theta_0^3}\right]e^{\theta_0(T-\gamma)}\\-\dfrac{a_0+b_0\gamma+c_0\gamma^2}{\theta_0}+\dfrac{b_0+2c_0\gamma}{\theta_0^2}+\dfrac{2c_0}{\theta_0^3}\end{array}\right]$$

$$-\frac{1}{T}\left[d_c\left[a_0+b_0\left(T-2\gamma\right)+c_0\left(T-2\gamma\right)^2\right]+\left(ATC(T)\right)\right]=0\text{,(4.16)}$$

subject to the sufficient condition

$$\frac{d^2\left(ATC(T)\right)}{dT^2}>0.$$

(See Appendix).

Special case:

Taking $c_0=0$, the pattern of demand $D(t)=\begin{cases}a_0, & 0\le t\le\gamma\\ a_0+b_0\left(t-\gamma\right)+c_0\left(t-\gamma\right)^2, & \gamma\le t\le T\end{cases}$, the current model's demand pattern is equivalent to that of Srivastava and Gupta (2007).

The solution approach for the current model is given below.

7.2. Method of Solving: algorithms

We have to follow the following procedure to measure the optimum values of $ATC(T)$ and I_S.

Step I: Consider the appropriate parameter value.

Step II: Using Eq. (4.17), calculate the value of T

Step III: Comparing the replenishment cycle's fixed length, T to γ

(i) When $\gamma<T$ the solution T is feasible and is noted as T^*. Then, we proceed to the next step i.e. Step IV.

(ii) Whenever $\gamma>T$ the solution T is impossible to implement.

Step IV: By putting the values T^* in Eqs. (4.13) and (4.7), we obtain the values of $ATC\left(T^*\right)$ and I_S^* respectively.

8. Numerical Examples and sensitivity studies

In this section, two numerical examples including the sensitivity study of Example -1 are illustrated.

8.1. Numerical Examples

We provide the following numerical examples, which are based on the deteriorating item with two components of demand.

Example - 1: The values of parameters are given below:

Let $a_0 = 20\ units$, $b_0 = 0.2$, $c_0 = 100$, $\theta_0 = 0.002$, $\gamma = 0.4\ days$, $C_0 = \$80$, $d_c = \$18/unit$ and $h_c = \$0.5/unit/day$.

The optimal cycle can be derived by solving Eq. (4.16). i.e. $T^* = 1.64493$ days and that satisfies the essential criteria, i.e., $\frac{d^2\left(ATC(T)\right)}{dT^2} = 88.6032 > 0$.

By putting $T^* = 1.88613$ in Eq. (4.13) and (4.7), we obtain the optimal values $\left(ATC\left(T^*\right)\right) = 66.3267$ and $I_S^* = 61.8532$ units.

Example - 2: The values of parameters are given below:

Let $a_0 = 24\ units$, $b_0 = 0.2$, $c_0 = 200$, $\theta_0 = 0.2$, $\gamma = 0.5\ days$, $C_0 = \$80$, $d_c = \$18/unit$ and $h_c = \$0.5/unit/day$.

The optimal cycle can be derived by solving Eq. (4.16). i.e. $T^* = 1.71108$ days and that satisfies the essential criteria, i.e. $\frac{d^2\left(ATC(T)\right)}{dT^2} = 309.83 > 0$.

We get $\left(ATC\left(T^*\right)\right) = 100.242$ $I_S^* = 83.3453$ units, by substituting $T^* = 1.71108$ in Eq. (4.13) and Eq. (4.7).

To discuss the effect of the sensitivity of the system, the following analysis of data is provided.

8.2 Sensitivity Analysis

In this paper, the changing effect of different parameters a_0, b_0, c_0, γ, θ_0, C_0, d_c and h_c on the system, optimum cost and optimum order quantity are studied. Then the sensitivity analysis is done simultaneously by taking changes in the value of each parameter by +50%, +20%, +10%, -10%, -20% and -50% and keeping other parameters fixed.

By analyzing Example 1, the following points are detected and the outcomes are presented in Table 1.

(i) Here T^* decreases with ascending value of $ATC\left(T^*\right)$ & I_S^* the parameter a_0. Moreover T^*, $ATC\left(T^*\right)$ & I_S^* are not affected by changing the value a_0.

(ii) Here T^*, $ATC\left(T^*\right)$ & I_S^* decrease with the change of the values b_0. Moreover T^*, $ATC\left(T^*\right)$ & I_S^* are not affected by the changing value b_0.

(iii) Here T^* decreases as the parameter value of $ATC(T^*)$ & I_S^* increases. Furthermore $T^*, ATC(T^*)$ & I_S^* are quite insensitive to changes in the value of C_0

(iv) Here T^* & I_S^* decrease with a rising value of $ATC(T^*)$ and the parameter θ_0 & h_c. Moreover T^*, $ATC(T^*)$ & I_S^* are moderately sensitive to changing values θ_0 & h_c.

(v) Here T^*, $ATC(T^*)$ & I_S^* all increase with the ascending value γ. Moreover T^*, $ATC(T^*)$ & I_S^* are moderately sensitive to changing values γ.

(vi) Here T^*, $ATC(T^*)$ & I_S^* all increase with the change of the values c_0 & d_c. Moreover T^*, $ATC(T^*)$ & I_S^* are all insensitive by changing their values c_0 & d_c.

Table 1: Study of the sensitivity of different system parameters

Parameter	% Change in the values of parameters	T^*	$\left(ATC(T^*)\right)$	% Change in the values of $\left(ATC(T^*)\right)$	I_S^*	% Change in the values of I_S^*
θ_0	+50	1.10012	110.993	+10.725	37.4907	-2.228
	+20	1.14415	104.718	+4.465	37.5155	-2.164
	+10	1.16013	102.513	+2.265	37.9097	-1.135
	-10	1.19524	97.8953	-2.341	38.8349	+1.276
	-20	1.21497	95.4611	-4.769	39.3976	+2.744
	-50	1.28904	87.4393	-12.771	41.8557	+9.154
a_0	+50	1.09874	111.925	+11.654	49.4914	+29.067
	+20	1.14697	105.124	+4.870	43.0535	+12.278
	+10	1.16222	102.717	+2.469	40.7385	+6.241
	-10	1.19153	97.7006	-2.535	35.8763	-6.438
	-20	1.20561	95.0941	-5.135	33.3356	-13.064
	-50	1.24572	86.8962	-13.313	25.3115	-33.990
b_0	+50	1.17691	100.235	-0.006	38.3282	-0.044
	+20	1.17701	100.239	-0.002	38.3384	-0.017
	+10	1.17704	100.241	0	38.3417	-0.009
	-10	1.17711	100.243	0	38.3486	+0.008
	-20	1.17714	100.245	+0.002	38.3518	+0.016
	-50	1.17724	100.249	+0.006	38.3620	+0.043
c_0	+50	1.16155	103.799	+3.548	42.2488	+10.179
	+20	1.16996	101.675	+1.429	39.8838	+4..122
	+10	1.17333	100.961	+0.717	39.1098	+1.993
	-10	1.18126	99.5186	-0.721	37.5914	-1.966
	-20	1.18600	98.7894	-1.449	36.8517	-3.895
	-50	1.20508	96.5516	-3.681	34.7527	-9.369
γ	+50	1.54040	103.763	+3.512	67.7095	+76.578
	+20	1.32066	98.4939	-1.743	47.9456	+25.036
	+10	1.24838	98.8219	-1.416	42.8114	+11.647
	-10	1.10694	102.811	+2.562	34.5142	-9.991
	-20	1.03818	106.611	+6.353	31.2914	-18.395
	-50	0.842014	126.748	+26.442	25.0967	-34.550
c_0	+50	1.25151	133.122	+32.800	41.2068	+7.462
	+20	1.21072	113.638	+13.363	39.5657	+3.182
	+10	1.19473	106.987	+6.728	38.9720	+1.634
	-10	1.15727	93.3888	-6.836	37.6745	-1.749

	-20 -50	1.13457 1.02929	86.409 64.3516	-13.799 -35.803	36.9436 33.9217	-3.655 -11.536
d_c	+50 +20 +10 -10 -20 -50	1.10870 1.14756 1.16185 1.19346 1.21131 1.27850	110.249 104.424 102.367 98.0398 95.7488 88.156	+9.982 +4.171 +2.119 -2.196 -4.482 -12.056	36.1528 37.3573 37.8267 38.9259 39.5882 42.4055	-5.717 -2.576 -1.352 +1.514 +3.241 +10.588
h_c	+50 +20 +10 -10 -20 -50	1.16312 1.17140 1.17422 1.17997 1.18289 1.19191	105.540 102.369 101.307 99.1749 98.1051 94.8796	+5.285 +2.121 +1.062 -1.064 -2.131 -5.349	37.8692 38.1496 38.2464 38.4460 38.5484 38.8699	-1.241 -0.510 -0.257 +0.262 +0.529 +1.368

Table 2 Sensitivity on the time interval γ

The results of altering the value of γ the optimal solution are given below.

γ	Change (%) in the value of γ	T^*	Correlation among γ & T^*	$ATC(T^*)$	I_S^*
0.600	(+20.0000)	1.32066 (+12.198)	$\gamma < T^*$	98.4939 (-1.74388)	47.9456 (+25.0364)
1.000	(+100.000)	1.9322 (+64.154)	$\gamma < T^*$	133.326 (+33.0041)	118.601 (+208.515)
1.500	(+200.000)	2.8062 (+138.404)	$\gamma < T^*$	282.594 (+181.192)	313.884 (+718.572)
1.700	(+240.000)	3.14176 (+166.911)	$\gamma < T^*$	383.437 (+282.511)	436.419 (+1038.13)
1.701	(+240.200)	3.54089 (+200.82)	$\gamma < T^*$	378.904 (+277.989)	452.165 (+1079.19)
1.702	(+240.400)	3.54315 (+201.012)	$\gamma < T^*$	379.454 (+278.538)	452.884 (+1081.07)
1.703	(+240.600)	3.54542 (+201.205)	$\gamma < T^*$	380.006 (+279.089)	453.605 (+1082.95)
1.704	(+240.800)	3.54769 (+201.398)	$\gamma < T^*$	380.558 (+279.639)	454.326 (+1084.83)
1.705	(+241.000)	3.54995 (+201.590)	$\gamma < T^*$	381.111 (+280.312)	455.048 (+1086.71)
1.706	(+241.200)	3.55222 (+201.782)	$\gamma < T^*$	381.664 (+280.742)	455.772 (+1088.6)
1.707	(+241.400)	3.55448 (+201.974)	$\gamma < T^*$	382.218 (+281.295)	456.495 (+1090.48)
1.708	(+241.600)	1.51131 (+28.3948)	$\gamma > T^*$	...	...
1.709	(+241.800)	1.51215 (+28.4662)	$\gamma > T^*$	...	...
1.800	(+260.000)	1.58911 (+35.0044)	$\gamma > T^*$	...	...
2.000	(+300.000)	1.75985 (+49.5098)	$\gamma > T^*$	...	...

Here the symbol '...' indicates the infeasible solution.

By observing Table 2, we conclude that

(i) When the value of γ is increased by 1543.5%, the initial order quantity is increased by 23493.5%, increasing the decision variable by 839.699% and the average total cost by 28693.9%.
(ii) Furthermore, increasing the parameter value γ by 1543.75% reduces the decision variable value by -50.7245%.
(iii) An increase in the parameter value γ after 6.574 results in an infeasible solution due to the condition $\gamma > T^*$.

9. Conclusions

The present study is developed for newly launched items or seasonal items with realistic features such as two-staged demand and constant deterioration. In this model, the demand pattern is of two types. In the cycle's first section, it is constant, while in the second, it is a quadratic function of time. Shortages and partial backlog are not included in this model. Because of the newly introduced items like new Android mobile phones, laptops, automobiles, dress materials, cosmetics, etc., the constant, as well as time-dependent quadratic demand rates, are appropriate in these two stages. At the beginning of the first part of the cycle, the demand for this product is constant for some time and consequently, it is developed as a result of the demand for the products in the market. The purpose of this research is to optimize the decision variable to reduce total system cost and compute optimal order quantity.

10. Discussion

The demand pattern in the model stated here consists of two main different stages. It is constant at the first stage and time-varying quadratic demand at the second stage. The justification for considering the quadratic demand pattern has been clarified in the literature part. In general, researchers assume the demand pattern to be a linear function of time. Such a category of demand pattern can be represented as $D(t) = a + bt,\ a \geq 0 \,\&\, b \neq 0$. It increases when $b > 0$ and decreases when $b < 0$ it is not easily applicable to any item. The exponential demand pattern is represented by the formula $D(t) = ae^{bt},\ a > 0 \,\&\, b \neq 0$. It increases when $b > 0$ and decreases when $b < 0$. Most of the time, these two demand patterns are discarded because of the rapid change in demand rate. However, accelerated growth as well as retarded growth; and also accelerated decline and retarded decline in demand rate are seen in the quadratic demand pattern. Such situations are suitable in the case of fashion items, cosmetics, aircraft, android mobiles, laptops, machines and spare parts, etc. The quadratic demand pattern can be expressed as follows $D(t) = a + bt + ct^2,\ a \geq 0,\ b \neq 0 \ \&\ c \neq 0$. The conditions that are both necessary and sufficient for the quadratic demand pattern are $\frac{dD(t)}{dt} = b + 2ct$ and $\frac{d^2D(t)}{dt^2} = 2c$, respectively. The different cases of change in the quadratic demand pattern are obtained by changing the signs of the parameters $b \ \&\ c$.

11. Scope for further research:

The above-discussing model is utilised in different industries like textile industries, cosmetics, healthcare industries, automobile industries, etc. The model studied here can be further extended in terms of the generalized demand patternlike time, price and stock-dependent demand, fuzzy environment, stochastic demand and use of the preservation technology. The deterioration rate can also be generalized to many types of Weibull distribution deterioration rates, such as the two-parameter Weibull distribution, three-parameter Weibull distribution, and Gamma distribution., etc. Shortages and different backlogging options can also be considered as one of the major factors for the development of the model.

Appendix

$$\frac{d^2ATC(T)}{dT^2}=\frac{1}{T}\left[\begin{array}{l}\left[\begin{array}{l}\left[b_0+2c_0(T-2\gamma)\right]\left[h_c\left(\frac{T+\gamma}{2}\right)+d_c\right]\\+h_c\left[a_0+b_0(T-2\gamma)+c_0(T-2\gamma)^2\right]\\+a_0\theta_0+b_0(T-2\gamma)+c_0(T-2\gamma)^2\left[h_c\left(\frac{T+\gamma}{2}\right)+d_c\right]\end{array}\right]e^{\theta_0(T-\gamma)}\\-d_c\left[b_0+2c_0(T-2\gamma)\right]\end{array}\right]$$

$$-\frac{2}{T^2}\left[\begin{array}{l}\left[h_c\left(\frac{T+\eta}{2}\right)+d_c\right]\left[a_0+b_0(T-2\gamma)+c_0(T-2\gamma)^2\right]e^{\theta_0(T-\gamma)}\\+\frac{h_c}{2}\left[\begin{array}{l}\left[\frac{a_0+b_0(T-2\gamma)+c_0(T-2\gamma)^2}{\theta_0}-\frac{b_0+2c_0(T-2\gamma)}{\theta_0^2}+\frac{2c_0}{\theta_0^3}\right]e^{\theta_0(T-\gamma)}\\-\frac{a_0+b_0\gamma+c_0\gamma^2}{\theta_0}+\frac{b_0+2c_0\gamma}{\theta_0^2}+\frac{2c_0}{\theta_0^3}\end{array}\right]\\-d_c\left[a_0+b_0(T-2\gamma)+c_0(T-2\gamma)^2\right]-\left(ATC(T)\right)\end{array}\right]$$

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