

## Relativistic Gravitational Force between Black Holes and Light Particles in XRBs

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### Abstract:

The present paper discusses the dependence on spinning velocity and wavelength in the case of spinning black holes and provides a model for the gravitational field of spinning black holes due to relativistic correction. This concludes that the relativistic gravitational force attains the maximum values for the lower masses and minimum value for higher masses as well as the spinning velocity is mainly responsible for increasing / decreasing the gravitational field of black holes.

### 1. Introduction:

Isaac Newton gave the universal laws of gravitation stated as all the bodies in the universe attract to each other. This force of attraction is called gravitational force depending upon the product of their masses and distance between them (Newton, 1687). On the other hand, in 1915 Einstein gave the generalised theory of gravitation as per general relativity for which the Newton's law of gravitation holds good for certain limitations (Bergmann, 1969). Mahto et al. studied the gravitational force between the black holes and light particles in XRBs (Mahto et. al, 2013) and also analysed this force regarding the black holes in the AGN (Nadeem, 2014)

The present work gives a model for relativistic gravitational force of spinning black holes and light particles passing near the event horizon coming from outer source of light in the universe.

### 2. Theoretical Treatment:

The gravitational force of spinning black holes and light particles passing near the event horizon coming from outer source of light in the universe is given by the following equation (Mahto et. al, 2013).

$$M = \frac{M_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Where

$$F = \frac{hc^3}{GM\lambda}$$

(1)

This equation consists of three constants like G, h and c. These three constants have the vital role in the general relativity and have their own significances as discussed in the reference (Dabholkar, 2006).

According to Newtonian theory, the mass of anybody remains the same, but special relativity states that the mass varies with velocity (Bergmann, 1969). For the velocity of a body comparable to the velocity of light, the Newtonian mass does not hold good.

The equation (1) can be written by the following equation.

$$F = \frac{hc^3}{G\lambda} (M)^{-1}$$

(2)

The black holes have velocity from 50% to 99% of the velocity of light. Hence if we consider BHs spinning with 50% to 99.9% of the velocity of light (Reicch, 2013), then we can apply it as below (Bergmann, 1969) and we have

(3)

$M_0$  = Rest mass of the body taken into consideration.

$v$  = Velocity of the body.

$c$  = Velocity of light.

Or

$$M = M_0 \left( 1 - \frac{v^2}{c^2} \right)^{-\frac{1}{2}} \quad (4)$$

$$(M)^{-1} = \left[ M_0 \left( 1 - \frac{v^2}{c^2} \right)^{-\frac{1}{2}} \right]^{-1} \quad (5)$$

Or

$$(M)^{-1} = (M_0)^{-1} \left[ \left( 1 - \frac{v^2}{c^2} \right)^{-\frac{1}{2}} \right]^{-1} \quad (6)$$

Or

$$(M)^{-1} = (M_0)^{-1} \left[ \left( 1 - \frac{v^2}{c^2} \right)^{\frac{1}{2}} \right] \quad (7)$$

Or

Using binomial theorem and neglecting higher power of terms, we have

$$(M)^{-1} = (M_0)^{-1} \left( 1 - \frac{v^2}{2c^2} \right) \quad (8)$$

Or

$$(M)^{-1} = \frac{1}{M_0} \left( 1 - \frac{v^2}{2c^2} \right) \quad (9)$$

Or

Putting (9) in the equation (5), we have

$$F_{rel} = \frac{hc^3}{GM_0\lambda} \left( 1 - \frac{v^2}{2c^2} \right) \quad (10)$$

The equation (10) gives a model for the relativistic force of spinning black holes attracting light particles coming from other sources passing near the black holes. This depends on  $\lambda$  of radiation. This model represents that the relativistic force is inversely proportional to  $\lambda$ . We know that the light is an electromagnetic radiation consisting of many radiations like  $\gamma$ -radiation, X-ray, Ultraviolet

ray, Visible rays, Infra red, Microwave, Radio wave and Hawking radiation.

Out of eight parts of electromagnetic radiation, only the radiations of visible region (VIBGYOR) are responsible to see any things and other components of the same electromagnetic spectrum have no role for visibility. Hence the study of visible part of spectrum has its own significances to explain the basic definition of black holes.

For convenience, putting  $G = c = h = 1$  in the equation (10), we have

$$F_{rel} = \frac{1}{M_0\lambda} \left( 1 - \frac{v^2}{2c^2} \right) \quad (11)$$

The above equation can be written as

$$F_{rel} = \frac{1}{M_0\lambda} - \frac{v^2}{2M_0\lambda c^2} \quad (12)$$

The equation (12) consists of two terms as follows:

$$(i) \frac{1}{M_0\lambda} \quad \text{and} \quad (ii) \frac{v^2}{2M_0\lambda c^2}$$

The first term is the same as **gravitational force** and light particles derived by the non-relativistic treatment. The second term gives the negative

contributions to **relativistic gravitational force** and light particles passing nearby the event horizon produced due to relativity.

**3. Table:**

**3.1. Table:**

Parameters	Violet	Blue	Cyan	Green	Yellow	Orange	Red
Frequency(Hz)	7.50 x10 <sup>14</sup>	6.66 x10 <sup>14</sup>	5.99x10 <sup>14</sup>	5.45 x 10 <sup>14</sup>	5.16 x 10 <sup>14</sup>	5.00 x 10 <sup>14</sup>	4.30 x 10 <sup>14</sup>
Wavelength(m)	4.00 x 10 <sup>-7</sup>	4.50 x 10 <sup>-7</sup>	5.00x10 <sup>-7</sup>	5.50 x10 <sup>-7</sup>	5.80 x 10 <sup>-7</sup>	6.00 x 10 <sup>-7</sup>	7.00 x 10 <sup>-7</sup>

**3.2. Table:**

Relativistic gravitational force of spinning BHs of wavelength (4.00 x 10 <sup>-7</sup> ) of violet colour in XRBS						
S. N.	Speed of BHs(%)	Speed of BHs (x 10 <sup>8</sup> m/s)	$F_{rel} = \frac{1}{M_0\lambda} \left(1 - \frac{v^2}{2c^2}\right) \times 10^{-26}$			
			For M=5 M <sub>0</sub> $\frac{1}{M_0\lambda} = 0.25 \times 10^{-24}$	For M=10 M <sub>0</sub> $\frac{1}{M_0\lambda} = 0.12 \times 10^{-24}$	For M=15 M <sub>0</sub> $\frac{1}{M_0\lambda} = 0.073 \times 10^{-24}$	For M=20M <sub>0</sub> $\frac{1}{M_0\lambda} = 0.062 \times 10^{-24}$
1.	50	1.50	21.87	10,50	6.38	5.42
2.	55	1.65	21.21	10,18	6.19	5.26
3.	60	1.80	20.50	9,84	5.98	5.08
4.	65	1.95	19.71	9.46	5.75	4.80
5.	70	2.10	18.87	9.06	5.51	4.68
6.	75	2.25	17.96	8.62	5.24	4.45
7.	80	2.40	17.00	8.16	4.96	4.21
8.	85	2.55	15.96	7.66	4.66	3.96
9.	90	2.70	14.87	7.14	4.34	3.68
10.	95	2.85	13.71	6.58	4.00	3.40
11.	99	2.97	12.74	6.11	3.72	3.16

**3.3. Table:**

Relativistic F <sub>rel</sub> of SBHs of wavelength (4.50 x 10 <sup>-7</sup> ) of blue colour in XRBS						
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S. N.	(% speed of black holes)	Speed of black holes (x 10 <sup>8</sup> m/s)	$F_{rel} = \frac{1}{M_0 \lambda} \left(1 - \frac{v^2}{2c^2}\right) \times 10^{-26}$			
			For M=5 M <sub>0</sub> $\frac{1}{M_0 \lambda} = 0.22 \times 10^{-24}$	For M=10 M <sub>0</sub> $\frac{1}{M_0 \lambda} = 0.11 \times 10^{-24}$	For M=15 M <sub>0</sub> $\frac{1}{M_0 \lambda} = 0.074 \times 10^{-24}$	For M=20M <sub>0</sub> $\frac{1}{M_0 \lambda} = 0.055 \times 10^{-24}$
1.	50	1.50	19.25	9.62	6.47	4.81
2.	55	1.65	18.67	9.33	6.28	4.66
3.	60	1.80	18.04	9.02	6.07	4.51
4.	65	1.95	17.35	8.67	5.84	4.33
5.	70	2.10	16.61	8.30	5.60	4.15
6.	75	2.25	15.81	7.92	5.31	3.95
7.	80	2.40	14.96	7.48	5.08	3.74
8.	85	2.55	14,05	7.02	4.75	3.51
9.	90	2.70	13.09	6.54	4.44	3.27
10.	95	2.85	12.07	6.03	4.09	3.01
11.	99	2.97	11.21	5.60	3.81	2.80

3.4. Table:

Relativistic F <sub>rel</sub> of SBHs of wavelength (5.00 x 10 <sup>-7</sup> ) of Cyan colour in XRBs						
S. N.	(% speed of black holes)	Speed of black holes (x 10 <sup>8</sup> m/s)	$F_{rel} = \frac{1}{M_0 \lambda} \left(1 - \frac{v^2}{2c^2}\right) \times 10^{-26}$			
			For M=5 M <sub>0</sub> $\frac{1}{M_0 \lambda} = 0.20 \times 10^{-24}$	For M=10 M <sub>0</sub> $\frac{1}{M_0 \lambda} = 0.10 \times 10^{-24}$	For M=15 M <sub>0</sub> $\frac{1}{M_0 \lambda} = 0.07 \times 10^{-24}$	For M=20M <sub>0</sub> $\frac{1}{M_0 \lambda} = 0.05 \times 10^{-24}$
1.	50	1.50	17.50	8.75	6.12	4.37
2.	55	1.65	16.97	8.48	5.94	4.24
3.	60	1.80	16.40	8.20	5.74	4.10
4.	65	1.95	15.77	7.88	5.52	3.94
5.	70	2.10	15,10	7.55	5.28	3.77
6.	75	2.25	14.37	7.18	5.03	3.59
7.	80	2.40	13.60	6.80	4.76	3.40
8.	85	2.55	12.77	6.38	4.47	3.19
9.	90	2.70	11.90	5.95	4.16	2.97
10.	95	2.85	10.97	5.47	3.84	2.73
11.	99	2.97	10.19	5.10	3.56	2.55

3.5. Table:

Relativistic F <sub>rel</sub> of SBHs of wavelength of wavelength (5.50 x 10 <sup>-7</sup> ) of Green colour in XRBs						
S. N.	(% speed of black holes)	Speed of black holes (x 10 <sup>8</sup> m/s)	$F_{rel} = \frac{1}{M_0 \lambda} \left(1 - \frac{v^2}{2c^2}\right) \times 10^{-26}$			
			For M=5 M <sub>0</sub> $\frac{1}{M_0 \lambda} = 0.18 \times 10^{-24}$	For M=10 M <sub>0</sub> $\frac{1}{M_0 \lambda} = 0.09 \times 10^{-24}$	For M=15 M <sub>0</sub> $\frac{1}{M_0 \lambda} = 0.06 \times 10^{-24}$	For M=20M <sub>0</sub> $\frac{1}{M_0 \lambda} = 0.04 \times 10^{-24}$
1.	50	1.50	15.75	7.87	5.25	3.50

2.	55	1.65	15.27	7.63	5.09	5.39
3.	60	1.80	14.76	7.38	4.92	3.28
4.	65	1.95	14.19	7.09	4.73	3.15
5.	70	2.10	13.59	6.79	4.53	3.02
6.	75	2.25	12.93	6.46	4.31	2.87
7.	80	2.40	12.24	6.12	4.08	2.72
8.	85	2.55	11.49	5.74	3.83	2.55
9.	90	2.70	10.71	5.35	3.57	2.38
10.	95	2.85	9.47	4.93	3.29	2.19
11.	99	2.97	9.17	4.58	3.05	2.03

3.6. Table:

Relativistic $F_{rel}$ of SBHs of wavelength of wavelength ( $5.80 \times 10^{-7}$ ) of Yellow colour in XRBs						
S. N.	(% speed of black holes)	Speed of black holes ( $\times 10^8$ m/s)	$F_{rel} = \frac{1}{M_0 \lambda} \left(1 - \frac{v^2}{2c^2}\right) \times 10^{-26}$			
			For M=5 $M_\odot$ $\frac{1}{M_0 \lambda} = 0.172 \times 10^{-24}$	For M=10 $M_\odot$ $\frac{1}{M_0 \lambda} = 0.086 \times 10^{-24}$	For M=15 $M_\odot$ $\frac{1}{M_0 \lambda} = 0.107 \times 10^{-24}$	For M=20 $M_\odot$ $\frac{1}{M_0 \lambda} = 0.043 \times 10^{-24}$
1.	50	1.50	15.05	7.52	9.36	3.76
2.	55	1.65	14.59	7.29	9.08	3.64
3.	60	1.80	14.10	7.05	8.77	3.52
4.	65	1.95	13.56	6.78	8.43	3.39
5.	70	2.10	12.98	6.49	8.07	3.24
6.	75	2.25	12.36	6.18	7.69	3.09
7.	80	2.40	11.69	5.84	7.27	2.92
8.	85	2.55	10.98	5.49	6.83	2.74
9.	90	2.70	10.23	5.11	6.36	2.55
10.	95	2.85	9.43	4.71	5.87	2.35
11.	99	2.97	8.77	4.38	5.45	2.19

3.7. Table:

Relativistic $F_{rel}$ of SBHs of wavelength ( $6.00 \times 10^{-7}$ ) of Orange colour in XRBs						
S. N.	(% speed of black holes)	Speed of black holes ( $\times 10^8$ m/s)	$F_{rel} = \frac{1}{M_0 \lambda} \left(1 - \frac{v^2}{2c^2}\right) \times 10^{-26}$			
			For M=5 $M_\odot$ $\frac{1}{M_0 \lambda} = 0.166 \times 10^{-24}$	For M=10 $M_\odot$ $\frac{1}{M_0 \lambda} = 0.083 \times 10^{-24}$	For M=15 $M_\odot$ $\frac{1}{M_0 \lambda} = 0.063 \times 10^{-24}$	For M=20 $M_\odot$ $\frac{1}{M_0 \lambda} = 0.041 \times 10^{-24}$
1.	50	1.50	14.52	7.26	5.51	3.63
2.	55	1.65	14.08	7.04	5.34	3.52
3.	60	1.80	13.61	6.80	5.16	3.40
4.	65	1.95	13.09	6.54	4.96	3.27
5.	70	2.10	12.53	6.27	4.75	3.13
6.	75	2.25	11.93	5.96	4.52	2.98
7.	80	2.40	11.28	5.64	4.28	2.82
8.	85	2.55	10.60	5.30	4.02	2.65

9.	90	2.70	9.87	4.93	3.74	2.46
10.	95	2.85	9.10	4.55	3.45	2.29
11.	99	2.97	8.46	4.23	3.41	2.12

3.8.Table:

Relativistic $F_{rel}$ of SBHs of wavelength ( $7.00 \times 10^{-7}$ ) of Red colour in XRBs						
S. N.	(%) speed of black holes	Speed of black holes ( $\times 10^8$ m/s)	$F_{rel} = \frac{1}{M_0 \lambda} \left( 1 - \frac{v^2}{2c^2} \right) \times 10^{-26}$			
			For $M=5 M_\odot$ $\frac{1}{M_0 \lambda} = 0.142 \times 10^{-24}$	For $M=10 M_\odot$ $\frac{1}{M_0 \lambda} = 0.071 \times 10^{-24}$	For $M=15 M_\odot$ $\frac{1}{M_0 \lambda} = 0.052 \times 10^{-24}$	For $M=20 M_\odot$ $\frac{1}{M_0 \lambda} = 0.036 \times 10^{-24}$
1.	50	1.50	12.42	6.21	4.55	3.15
2.	55	1.65	12.05	6.02	4.41	3.05
3.	60	1.80	11.64	5.82	4.26	2.95
4.	65	1.95	11.20	5.60	4.10	2.83
5.	70	2.10	10.72	5.36	3.92	2.71
6.	75	2.25	10.20	5.10	3.73	2.58
7.	80	2.40	9.65	4.82	3.53	2.44
8.	85	2.55	9.07	4.53	3.32	2.29
9.	90	2.70	8.44	4.22	3.09	2.14
10.	95	2.85	7.79	3.89	2.85	1.97
11.	99	2.97	7.24	3.62	2.65	1.83

4. Graph:

4.1. Graph (Violet colour):

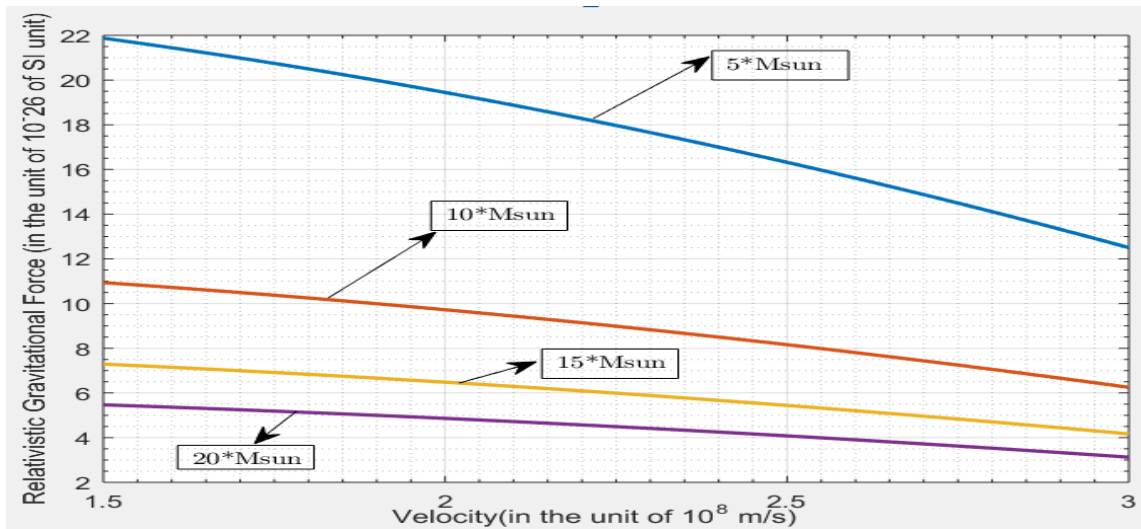


Figure.1: Relativistic gravitational force between black holes and violet colour.

4.2. Graph (blue colour):

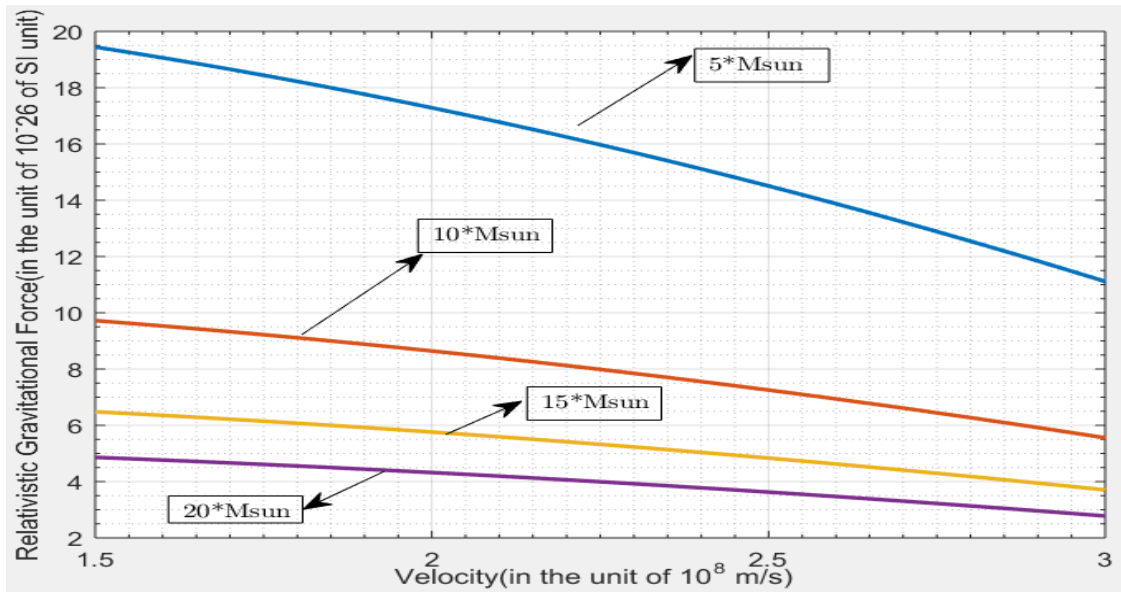


Figure.2: Relativistic gravitational force between black holes and blue colour.

4.3. Graph (cyan colour):

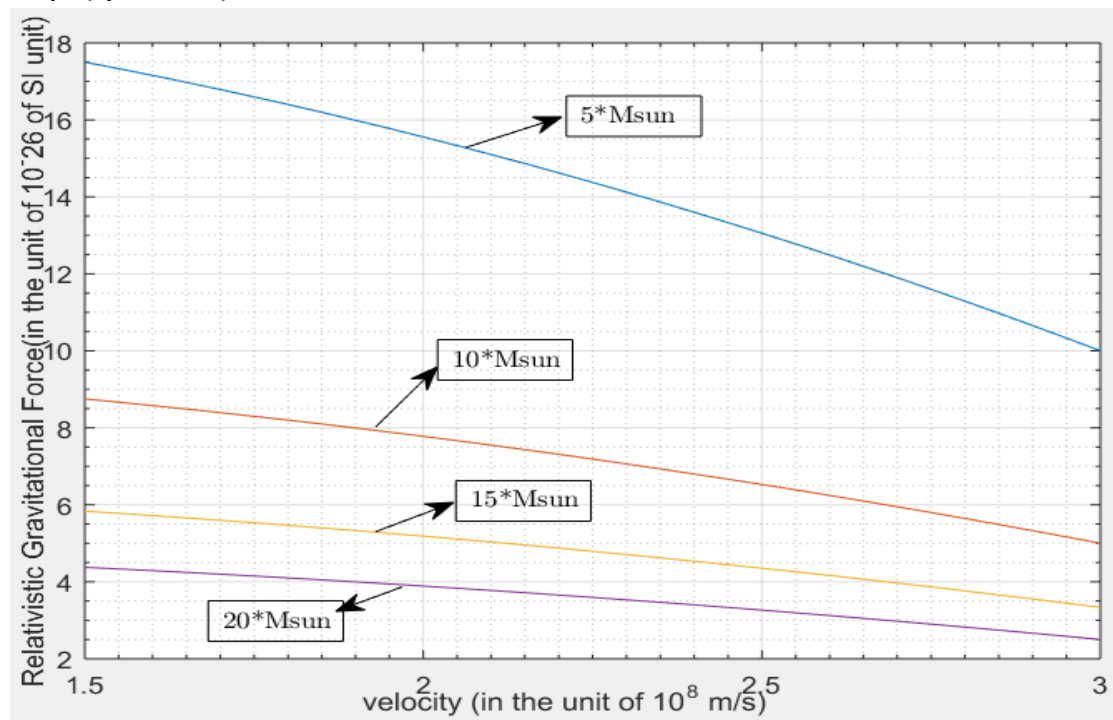


Figure.3: Relativistic gravitational force between black holes and cyan colour.

4.4. Graph (green colour):

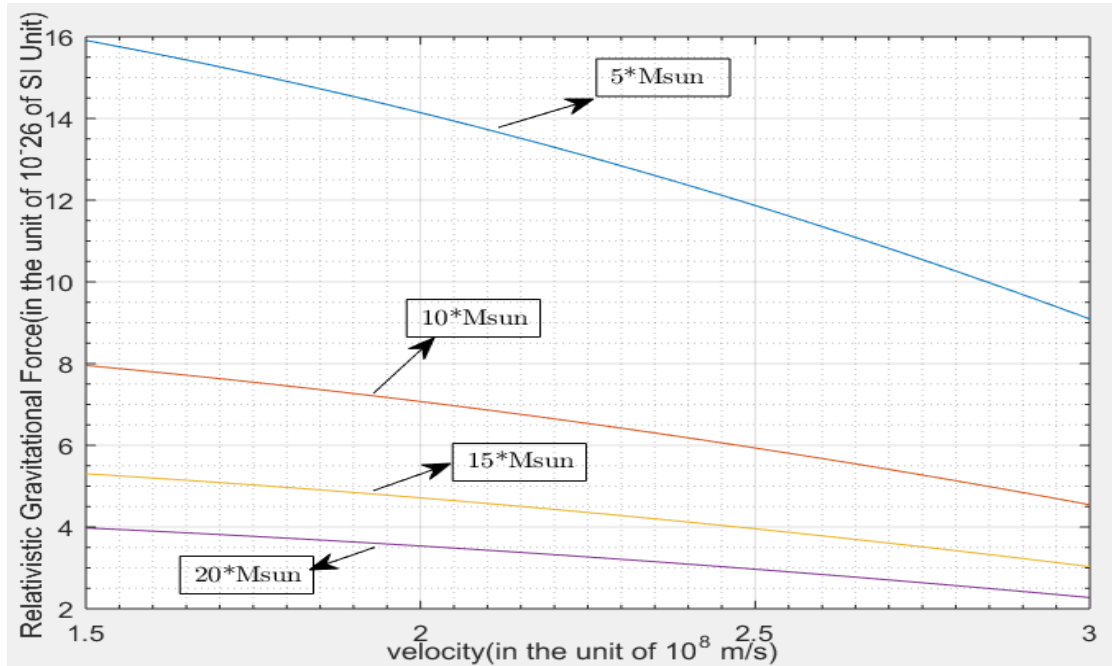


Figure.4: Relativistic gravitational force between black holes and green colour.

4.5. Graph (yellow colour):

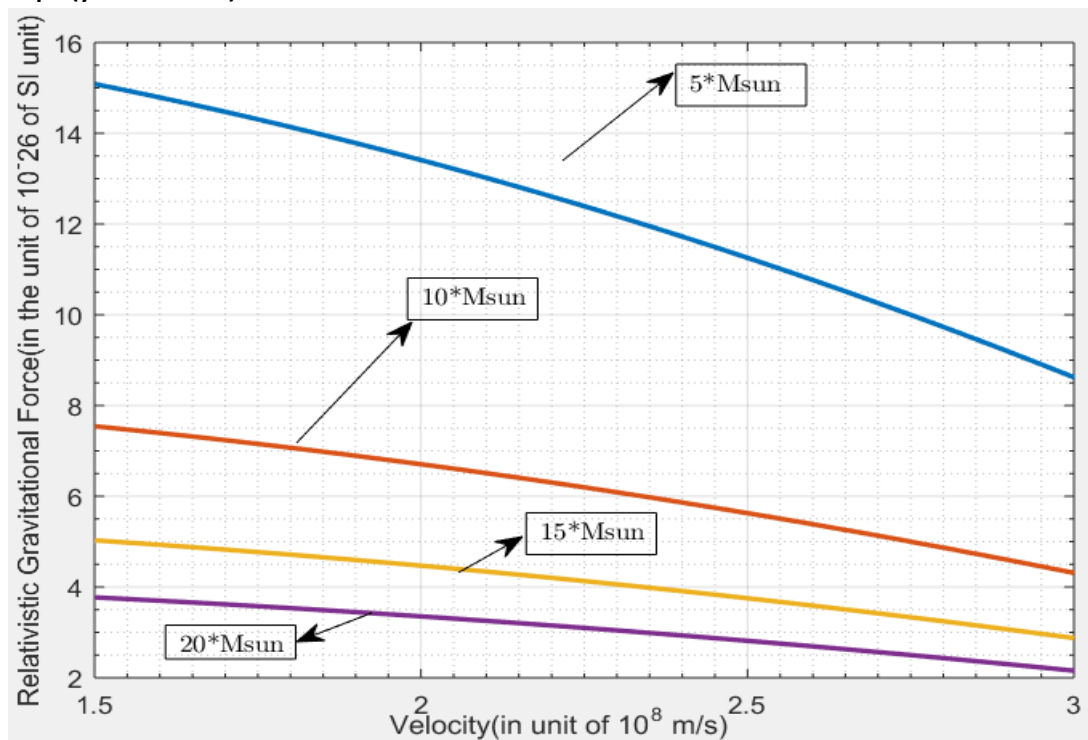


Figure.5: Relativistic gravitational force between black holes and yellow colour.

4.6. Graph (orange colour):

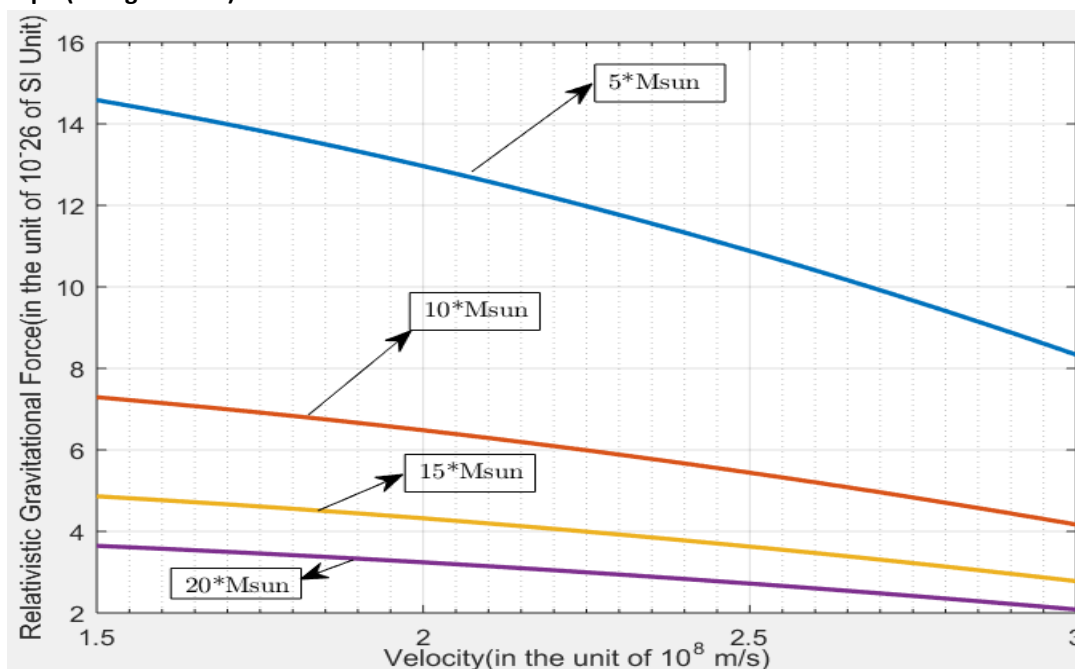


Figure.6: Relativistic gravitational force between black holes and orange colour.

4.7. Graph (red colour):

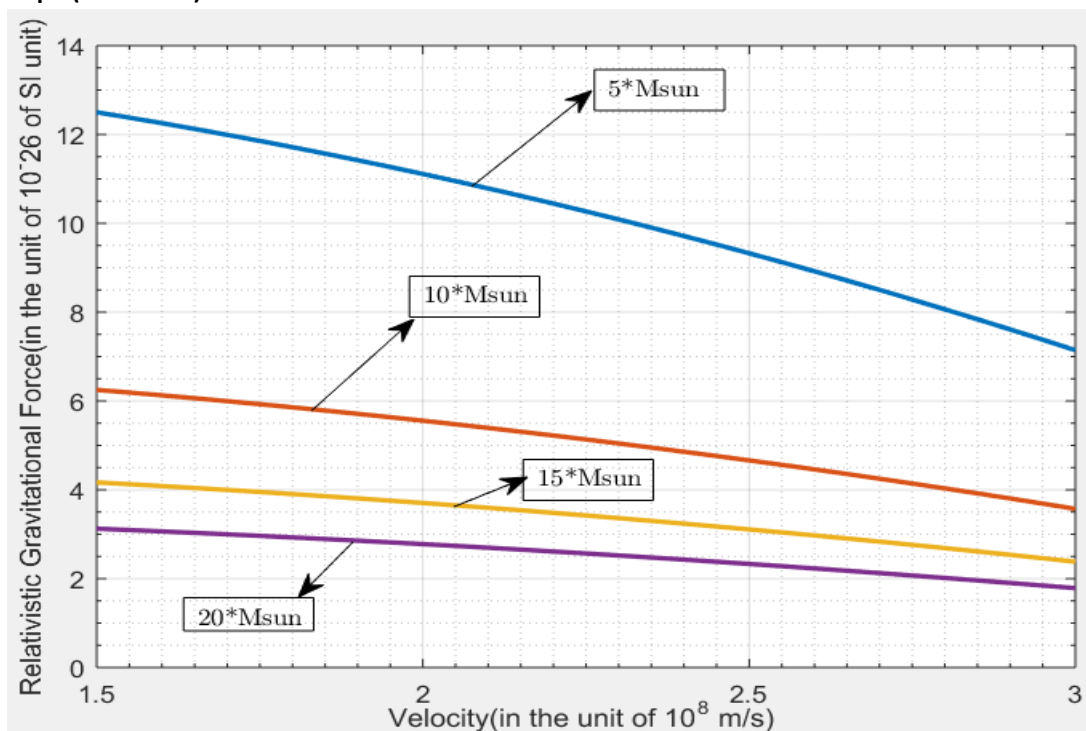


Figure.7: Relativistic gravitational force between black holes and red colour.

5. Result and Discussion:

The present work is concerned with the attraction/trapping of light particles passing nearby the event horizon of black holes. The work starts from the Newton's laws of gravitation applying between the mass of BHs and light

particles. The proper mathematical operation for this work gives a model for gravitational force of SBHs on light particles represented by the

equation 
$$F = \frac{hc^3}{GM\lambda}$$
. The black holes have

velocity from 50%-99.99% of speed of light. This causes the variation of mass of black holes. So the mass variation formula as proposed by Einstein is applied and with suitable mathematical operation, it gives a new model for gravitational force of spinning black holes acting on the light particles

$$F = \frac{hc^3}{GM_0\lambda} \left( 1 - \frac{v^2}{c^2} \right)$$

given by the equation

From the model for the gravitational force of spinning black holes, it is clear that this force depends on the rest mass and the wavelength of light particles. We know that the light is an electromagnetic radiation consisting of gamma radiation ( $\gamma$ ), x-ray radiation, ultraviolet radiation, visible ray radiation, infra-red ray radiation, microwave radiation and Hawking radiation. For the visibility of anything, there is no role of other radiations except visible radiations, because no sensation is given by other radiations at the retina of human eyes. Only visible part of electromagnetic spectrum gives the sensations of vision at the retina of human beings. So in the present work, only visible part of electromagnetic radiation is taken into considerations. We applied the model for the masses like  $5M_\odot$ ,  $10M_\odot$ ,  $15M_\odot$  and  $20 M_\odot$  existing in the XRBs with all the wavelengths of visible part of the electromagnetic spectrum. From the observation of tables from 3.2 to 3.8 and figures 1 to 7, it is clear that the red color is trapped by black holes less than to that of other components of visible part of electromagnetic spectrum.

Among all the radiation, Hawking radiation has the longest wavelength and gamma radiation ( $\gamma$ ) has the shortest wavelength. Due to this reason, the gravitational force is maximum for all the radiations rather than Hawking radiation. This is only reason due to which the escaping of Hawking radiation from black holes is possible. Hence the result obtained by our work justifies the emission of Hawking radiation from the black holes.

#### **6. Conclusions:**

The radiations of shorter wavelength are more attracted than the radiations of longer wavelength. Hence the gamma radiations ( $\gamma$ ) are attracted more than other radiations of

wavelength, while the Hawking radiations are less attracted than other radiations.

The red color is trapped by black holes less than to that of other components of visible part of electromagnetic spectrum.

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