# Modeling Of Communication Network with Bulk Arrival Queuing Model Under Intuitionistic Fuzzy Environment Using Robust Ranking Method

**Lakshmana Rao Nambaru[1], B. Satyanarayana[2], K. Chandan[3], P. Sudhakara Babu[4], Ganapathi Swamy Chintada[5]**

[1]Research Scholar, Department. of Statistics, Acharya Nagarjuna University, Guntur, Andhra Pradesh,
[2]Professor, Department. of Mathematics, Acharyathia Nagarjuna University, Guntur, Andhra Pradesh
[3]Professor (Rtd.), Department. of Statistics, Acharya Nagarjuna University, Guntur, Andhra Pradesh.
[4]Research Scholar, Department. of Statistics, Acharya Nagarjuna University, Guntur, Andhra Pradesh,
[5*]Associate Professor, Department. of Community Medicine, GSL Medical College, Rajahmundry, Andhra Pradesh, ganesh051981@gmail.com

***Corresponding Author:** Dr Ganapathi Swamy Chintada
*Associate professor in Statistics, Department of Community Medicine, GSL Medical College, Rajahmundry, Andhra Pradesh, ganesh051981@gmail.com

**Abstract:**

This work has been studied by dynamic system of communication flow in the presence of bulk arrival queueing theory under fuzzy and intuitionistic fuzzy environment by using Robust ranking technique. Also, we studied the application of bulk arrival queue model to communication network under ambiguous data, where the arrival rate and service rate are IFS in nature. We construct the membership and non-membership functions of triangular and trapezoidal fuzzy numbers through IFS α-cut approach and it is illustrated with the numerical example.

***Keywords:*** *Bulk arrival, Membership functions, fuzzy queue, α-cut approach, Intuitionistic fuzzy***,** *Robust ranking.*

## 1. Introduction

Queueing theory is the mathematical study of waiting lines. This is an important branch of mathematics that includes applied probability, statistical distributions, calculus, matrix theory, and complex analysis. This also applies to the field of decision science.
The main objective of queuing theory is to reduce customer waiting times, to make sure the customer receives the most efficient service possible, by reducing the length of the queue and balancing customer waiting time with server downtime, designing the right system by providing appropriate services for a specific cost structure.
In ordinary queueing problems, it is assume that customers arrive singly at a service complex. However, this assumption is ignored in many real-life queueing situations. Patients arriving at health centre, ships arriving at a port, people going to theatres, shopping malls, and so on, are some of the examples of queueing situations in which customers do not arrive singly, but in bulk or batches. Also, the size of an arriving group may be a random variable or a fixed number. Mathematically and also from practical point of view, the cases when the size of an arriving group is a random variable, are more frequent, and also more arduous to handle.

A bulk queueing model is described in term of inter arrival times of batches of customers, batch sizes, service times of customer groups and group sizes. Such types of queue system produce practical models for evaluation of performance in communication network and computer systems.

Probability queueing models have assumed to Poisson process and having exponential service times but the fuzzy queues are realistic than the normally used in crisp queues. In reality the arrival pattern, service pattern are normally determined by linguistic terms such as large, small or moderate which can be finest determined by the fuzzy sets. Many of them [1-5] have been analyzed by fuzzy queuing theory; they have applied zadeh's extension law. Chen [6] developed by parametric non-linear programming approach to fuzzy queues with bulk service and derives the membership function of steady state performance measure in bulk arrival system.Reeta Bhardwaj [8] analyzed membership functions of triangular fuzzy numbers for bulk arrival queuing model.Palpandi and Geetharamani [7] proposed a model a various performance for bulk arrival queueing systems with varying batch sizes of triangular and trapezoidal fuzzy numbers using Robust ranking technique.In this object we have made a focus on queue models to describe steady state behavior of system and end delay in packet networks which provide insight into some aspects of networking, as congestion information loss and delay variation for a packet flow. We analyzed for the average length of queue in the system view of that the arrival rate and service rate are fuzzified based on zadeh's extension principle the membership and non-membership functions are constructed for the fuzzfied arrivals and service by using robust ranking technique.

## 2. Model Description:

The jobs of typical arrival were considered in bulk of packets or clumps, as there is either nothing or a large number of jobs waiting in a queue for their execution. Every job is being performed in a sequential order, i.e., no priority is considered. Though each task on arrival can generate an 'interrupt' of the processor, which we have not counted in our study, it is generated in the model by using a flow control mechanism that drops bits, inducing a dependency between messages arriving and service transmission completion. The following symbols and notations are used:

$\lambda_x$ = Mean arrival rate of messages containg x packets or mean external rate of port x packets

$\lambda = \sum_x \lambda_x$ Composite mean arrival of packets of size x or aggregate total network through put rate

μ = Mean transmission rate

$\delta_x$ = Covariance between arriving and transmitted pakets

$\delta = \sum_x \delta_x$ Covariance between Composite arriving and transmitted pakets

$C_x$ = Probability of a batch of size x packets arriving in buffer=$\frac{\lambda_x}{\lambda}$

$L_s$ = Mean queue length of system

Var = Variance in system

The arrival rate and service rate are as follows Poisson arrivals, exponential service time distribution. The performance measures of this queueing system (crisp) are taken [9] as follows

1. The mean queue length of system is

$L_s = \frac{\rho}{(1-\beta)(1-\rho)}$ where $\rho = \frac{\lambda-\delta}{(1-\beta)(\mu-\delta)}$ ........ (1)

2. The variance of number of packets in the system is

Variance = $\frac{\beta\rho(1-\rho)+\rho}{(1-\beta)^2(1-\rho)^2}$ ........ (2)

## 3.Model with fuzzy parameter

### 3.1.Definitions

**Fuzzy Set**

Let A be a conventional set, $\mu_A(x)$ be a task from A to [0,1]. A fuzzy set A with the membership function $\mu_A(x)$ is explained as A={x,$\mu_A(x)$;x ∈ X} where X is a existing set and $\mu_A(x) \in [0,1]$.

**Intutionistic Fuzzy (IF) Set**

Let X be a non-empty set. An intuitionistic fuzzy set C in X is an object having the form $C = \left\{(x, \mu_C^{(x)}, \phi_C^{(x)}) : x \in X\right\}$, here $\mu_C^{(x)}, \phi_C^{(x)} : X \to [0,1]$, are the degree of membership ( $\mu_C^{(x)}$ ) and non-membership ( $\phi_C^{(x)}$ ) of elements $x \in X$ to the set C, which is subset of X and $0 \le \mu_C^{(x)} + \phi_C^{(x)} \le 1$, $\forall x \in X$ . $\psi_C^{(x)} = 1 - \mu_C^{(x)} - \phi_C^{(x)}$ is the degree of hesitation.

### 3.2. Membership and Non-membership Functions

**Triangular Membership function**

A triangular fuzzy number $\tilde{A}$ is defined by (d1, d2, d3), where di∈R and d1≤d2≤d3.

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-d_1}{d_2-d_1} & for d_1 \le x \le d_2 \\ \frac{d_3-x}{d_3-d_2} & for d_2 \le x \le d_3 \\ 0 & Otherwise. \end{cases}$$

**Trapezoidal Membership function**

A trapezoidal fuzzy number $\tilde{A}$ is defined by (d1, d2, d3, d4) where di∈R and d1≤d2≤d3≤d4.

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-d_1}{d_2-d_1} & for d_1 \le x \le d_2 \\ 1 & for d_2 \le x \le d_3 \\ \frac{d_4-x}{d_4-d_3} & for d_3 \le x \le d_4 \\ 0 & Otherwise. \end{cases}$$

**Triangular Non-membership function**

A triangular intuitionistic fuzzy number $\tilde{A}$ is defined by (d1, d2, d3), where di∈R and d1≤d2≤d3.

$$\Omega_{\bar{A}}(x) = \begin{cases} \frac{d_2 - x}{d_2 - d_1} & for d_1 \leq x \leq d_2 \\ \frac{x - d_3}{d_3 - d_2} & for d_2 \leq x \leq d_3 \\ 0 & Otherwise. \end{cases}$$

**Trapezoidal Non- membership function**

A trapezoidal intuitionistic fuzzy number $\bar{A}$ is defined by (d1, d2, d3, d4) where $d_i \in R$ and d1≤d2≤d3≤d4.

$$\Omega_{\bar{A}}(x) = \begin{cases} \frac{d_2 - x}{d_2 - d_1} & for d_1 \leq x \leq d_2 \\ 1 & for d_2 \leq x \leq d_3 \\ \frac{x - d_3}{d_4 - d_3} & for d_3 \leq x \leq d_4 \\ 0 & Otherwise. \end{cases}$$

We considered a bulk arrival queueing system which the customer arrives at a fuzzy and intuitionistic arrival rate$\tilde{\lambda}$, fuzzy service rate $\tilde{\mu}$and vacation time $\tilde{\delta}$ are approximately known. We construct the membership functions of the mean queuesystem length Ls and variance in the systemfor bulk arrival queueing system by using Robust ranking technique and are given as follows

**Robust Ranking Technique**

Robust ranking fuzzy method play major role at the time of decision making by transforming fuzzy values to crisp values.

The robust ranking technique is given by R($\tilde{a}$) = $\int_0^1 0.5(a_\alpha^L + a_\alpha^U)d\alpha$

Where $(a_\alpha^L + a_\alpha^U)$ is the α-cut level of the fuzzy number$\tilde{a}$.

The robust ranking index gives the representative value of the fuzzy number$\tilde{a}$.

**4. Numerical example**

Consider the bulk arrival queueing system where the parameters are taken in triangular and trapezoidalfuzzy and intuitionistic fuzzy numbers. α -cut is applied to convert the fuzzy into crisp values.

**Membership of Triangular Fuzzy Number**

Bulk arrival rate λ = [3, 4, 5], the service rate μ = [7, 8, 9] and fuzzy covariance rate δ = [0.03, 0.05, 0.07] while geometric distribution parameter β= 0.2.

Applying arithmetic operations on interval and α-cut we get

$\tilde{\lambda}(\alpha) = [\lambda_\alpha^L, \lambda_\alpha^U]$ = [α+3, 5-α],

$\tilde{\mu}(\alpha) = [\mu_\alpha^L, \mu_\alpha^U]$ = [α+7, 9-α],

$\tilde{\delta}(\alpha) = [\delta_\alpha^L, \delta_\alpha^U]$ = [0.02α+0.03, 0.07-0.02α]

We can proceed triangular fuzzy rates according to the Robust ranking technique

$R(\tilde{\lambda})$ = R(3,4,5) = $\int_0^1 0.5(a_\alpha^L + a_\alpha^U)d\alpha = \int_0^1 0.5\,(8)\,d\alpha$ = 4

Similarly

$R(\tilde{\mu})$ = R(7,8,9) = 8, $R(\tilde{\delta})$ = R(0.03,0.05,0.07) = 0.05.

Let it put in the values in equation (1) we get

Ls = 2.048

Variance = 4.6518

**Figure 1.1: Triangular Membership Function of Ls**

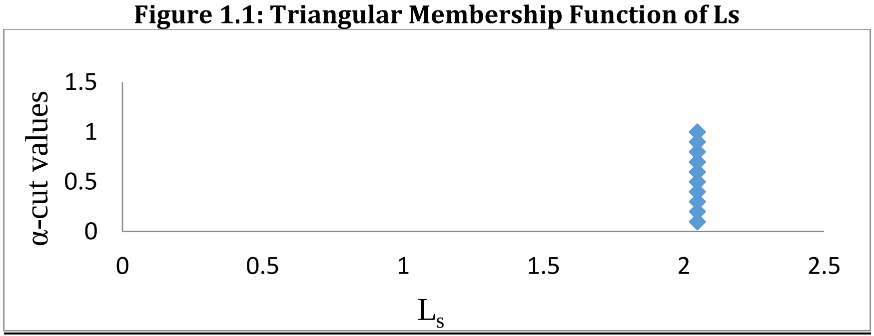
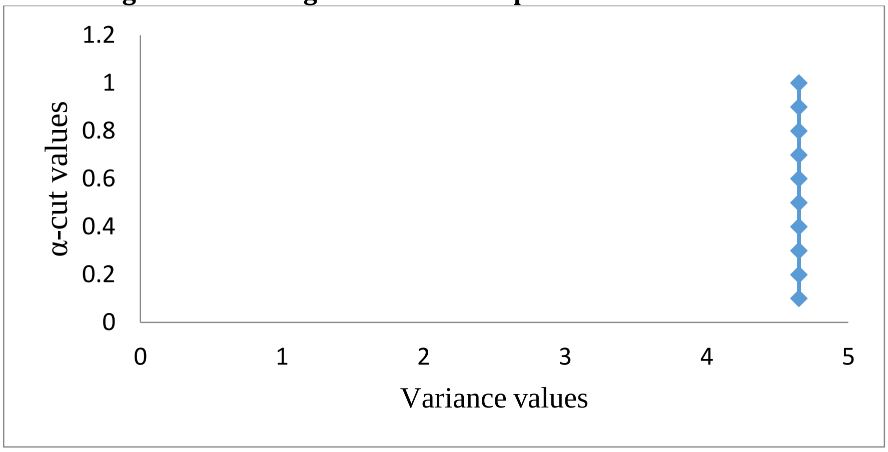


**Figure 1.2: Triangular Membership Function of Variance**




**Membership of Trapezoidal Fuzzy Number**

Bulk arrival rate λ = [3,4,5,6], the service rate μ = [7,8,9,10] and fuzzy covariance rate δ = [0.03,0.05,0.07,0.09] while geometric distribution parameter β= 0.2.

Applying arithmetic operations on interval and α- cut we get

$\tilde{\lambda}(\alpha) = [\lambda_\alpha^L, \lambda_\alpha^U] = [\alpha+3, 6-\alpha]$,

$\tilde{\mu}(\alpha) = [\mu_\alpha^L, \mu_\alpha^U] = [\alpha+7, 10-\alpha]$,

$\tilde{\delta}(\alpha) = [\delta_\alpha^L, \delta_\alpha^U] = [0.02\alpha+0.03, 0.09-0.02\alpha]$

We can proceed trapezoidal fuzzy rates according to the Robust ranking technique

$R(\tilde{\lambda}) = R(3,4,5,6) = \int_0^1 0.5(a_\alpha^L + a_\alpha^U)d\alpha = \int_0^1 0.5\ (8)\ d\alpha = 4.5$

Similarly

$R(\widetilde{\mu}) = R(7,8,9) = 8.5$, $R(\tilde{\delta}) = R(0.03,0.05,0.07) = 0.06$.

Let it put in the values in equation (1) we get

Ls = 2.3996

Variance = 9.3666

**Figure 1.3: Trapezoidal Membership Function of Ls**

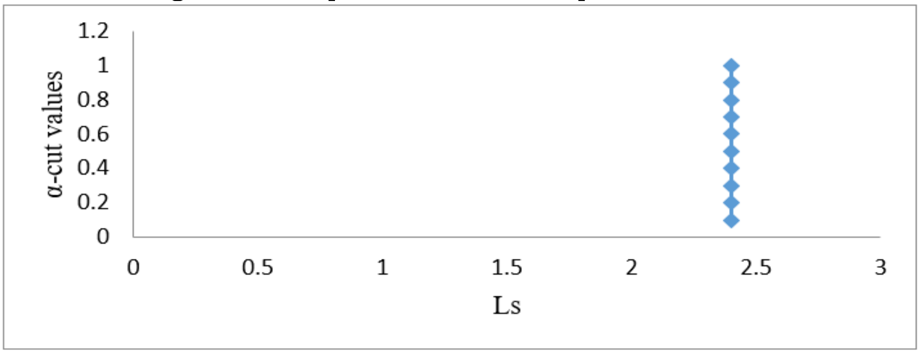
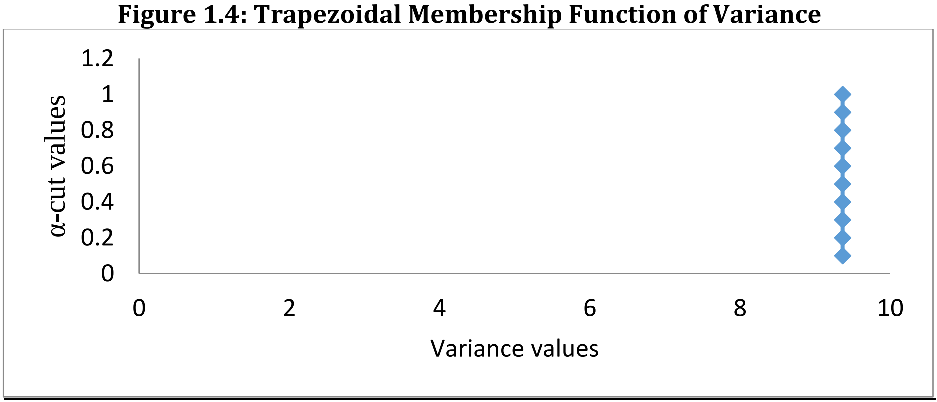



**Figure 1.4: Trapezoidal Membership Function of Variance**



**Non-Membership of Triangular Fuzzy Number**

Bulk arrival rate λ = [3, 4, 5], the service rate μ = [7, 8, 9] and fuzzy covariance rate δ = [0.03, 0.05, 0.07] while geometric distribution parameter β= 0.2.

Applying arithmetic operations on interval and α- cut we get

$\tilde{\lambda}(\alpha) = [\lambda_\alpha^L, \lambda_\alpha^U] = [4- \alpha, 4+\alpha]$,

$\tilde{\mu}(\alpha) = [\mu_\alpha^L, \mu_\alpha^U] = [8- \alpha, 8+\alpha]$,

$\tilde{\delta}(\alpha) = [\delta_\alpha^L, \delta_\alpha^U] = [0.05-0.02\alpha, 0.05+0.02\alpha]$

We can proceed IF triangular rates according to the Robust ranking technique

$R(\tilde{\lambda}) = R(3,4,5) = \int_0^1 0.5(a_\alpha^L + a_\alpha^U)d\alpha = \int_0^1 0.5\ (8)\ d\alpha = 4$

Similarly

$R(\widetilde{\mu}) = R(7,8,9) = 8$, $R(\tilde{\delta}) = R(0.03,0.05,0.07) = 0.05$.

Let it put in the values in equation (1) we get

Ls = 2.048

Variance = 4.6518

**Figure 1.5: Triangular Non-Membership Function of Ls**

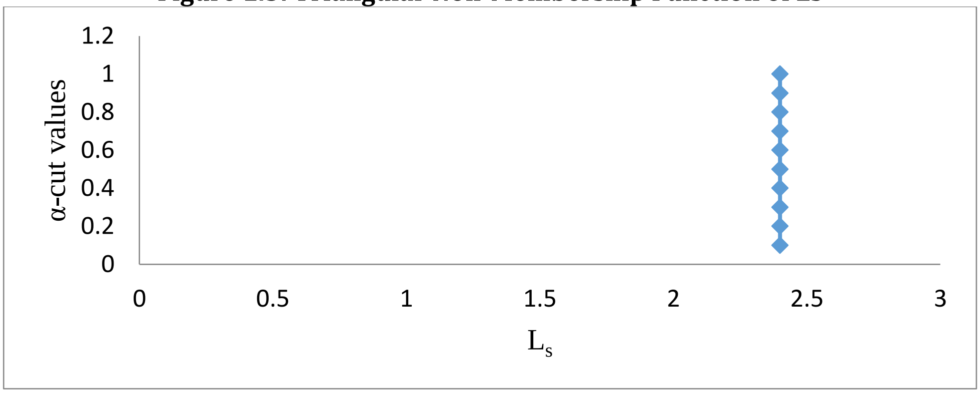
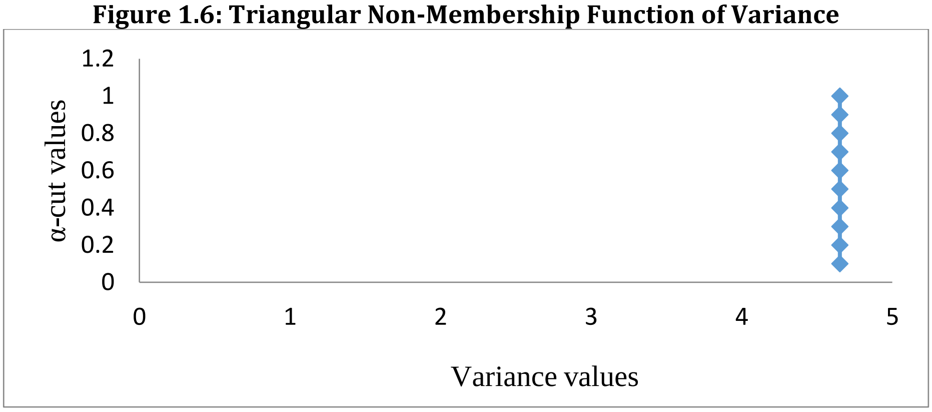



**Figure 1.6: Triangular Non-Membership Function of Variance**




**Non-membership of Trapezoidal Fuzzy Number**

Bulk arrival rate λ = [3,4,5,6], the service rate μ = [7,8,9,10] and fuzzy covariance rate δ = [0.03,0.05,0.07,0.09] while geometric distribution parameter β= 0.2.

Applying arithmetic operations on interval and α- cut we get

$\tilde{\lambda}(\alpha) = [\lambda_\alpha^L, \lambda_\alpha^U] = [4- \alpha, 5+\alpha]$,

$\tilde{\mu}(\alpha) = [\mu_\alpha^L, \mu_\alpha^U] = [8- \alpha, 9+\alpha]$

$\tilde{\delta}(\alpha) = [\delta_\alpha^L, \delta_\alpha^U] = [0.05-0.02\alpha, 0.07+0.02\alpha]$

We can proceed IF trapezoidal rates according to the Robust ranking technique

$R(\tilde{\lambda}) = R(3,4,5,6) = \int_0^1 0.5(a_\alpha^L + a_\alpha^U)d\alpha = \int_0^1 0.5\ (8)\ d\alpha = 4.5$

Similarly

$R(\widetilde{\mu}) = R(7,8,9) = 8.5$, $R(\tilde{\delta}) = R(0.03,0.05,0.07) = 0.06$.

Let it put in the values in equation (1) we get

Ls = 2.3996

Variance = 9.3666

**Figure 1.7: Trapezoidal Non-Membership Function of Ls**

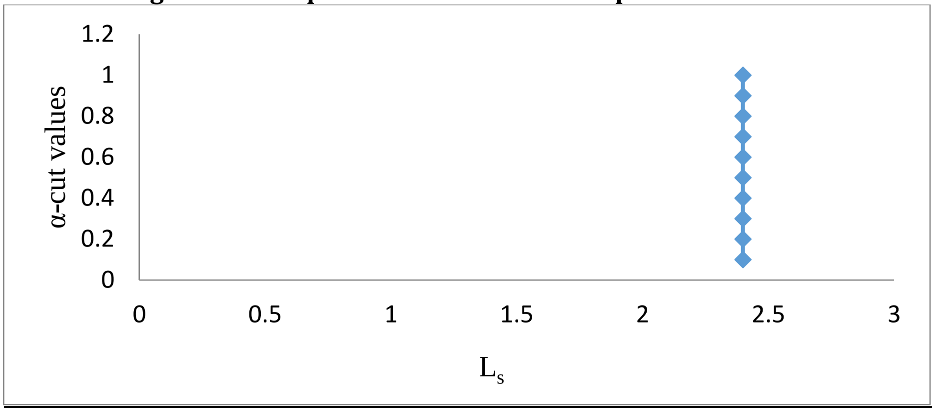
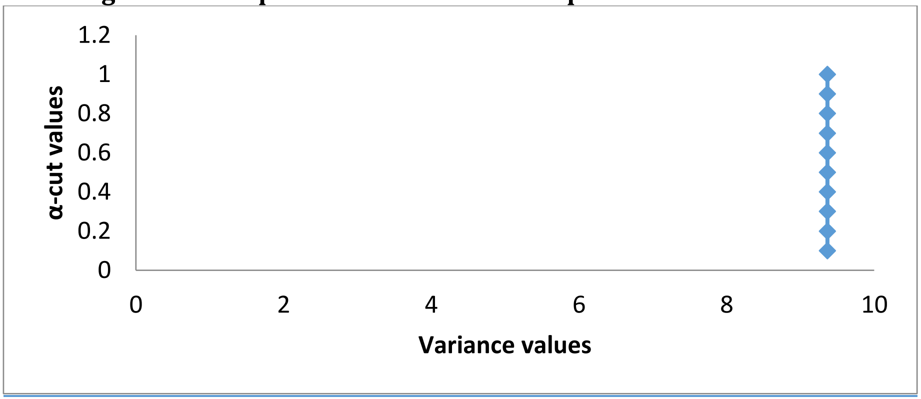



**Figure 1.8: Trapezoidal Non-Membership Function of Variance**




**Results**

For membership and non-membership functions of triangular
Ls = 2.048
Variance = 4.6518
For membership and non-membership functions of trapezoidal
Ls = 2.3996
Variance = 9.3666
Both are same length and variance

**5. Conclusion**

We construct the membership functions of triangular and trapezoidal fuzzyunder intuitionistic fuzzy numbers by using Robust ranking method. Also analyzed mean queue length and variance through IFS α-cut approach.

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