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Application of a two-stage iterative technique to a path planning issue in an indoor setting

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Abstract— Mobile robots are frequently associated with their ability to discover a collision-free pathway from any departure point to a certain destination in their surroundings. In this study, our attempts are to iteratively unravel a path planning issue via numerical techniques. Its foundation is a potential field approach that is driven by Laplace's equation to stimulate the formation of a potential function over the designated regions. This potential field is naturally utilized as a guide in the global approach to robot path planning. In order to address the path planning challenges, this paper presented a two-stage iterative technique called Alternating Group Explicit (AGE) method. By utilizing a finite-difference technique, the analysis proves that a smooth path can be produced along the course. Additionally, the simulation outcomes demonstrate that the proposed numerical approach provides a faster solution than the earlier work.
Index Terms—*Collision-Free, Elliptic Partial Differential Equation, Finite Difference Approximation, Iterative Approach, Optimal Path, Robot Navigation.*

Introduction

In achieving an autonomous mobile robot, path planning or navigation issues are classified as crucial components. A fully autonomous mobile robot needs to have the capability to project a route effectively and consistently from any point of departure to its final location without running into any impediments along the way. Efficient algorithms in solving these problems have vital applications in industrial robotics, computer animation, drug design, and automated surveillance, among other fields. Therefore, it is as expected that over the last two decades, research activity on this subject has continuously increased.

This study illustrates the simulation implementation of a heat transfer-based point-robot path planning using numerical potential functions in the designated area. The environment produces with this heat transfer model is not just devoid of local minima but also conducive to robot navigation control. In this study, the heat transfer problem is modeled after Laplace's equation with a solution known as a harmonic function. This function represents the temperature values in the configuration space (config-s) that will be used to simulate path generation. Numerous methods have been employed to obtain harmonic functions, however, numerical techniques are the most popular way because of the accessibility of rapid processing machines and their elegance and competence in handling the issue. In this study, several experiments were carried out to assess the effectiveness of employing a two-stage iterative technique to generate mobile robot pathways for various-sized config-s.

Related Work

In their groundbreaking work, [1] demonstrated that harmonic functions possessed several characteristics that were advantageous for robotic applications as well as offering a complete path planning algorithm. The work also claimed that harmonic function computation can be utilized for control while the configuration area is being updated as the function responds well to observe any changes in the area. Beforehand, [2] employed potential field concepts for collision avoidance and mobile robot, where each obstacle provides a repelling force while the objectives exert an attractive force. In contrast, [3] came to the conclusion that there exist potential functions that, topologically at least in certain types of domains, can direct the effector from practically any location to a certain point. However, the fundamental issue with these potential fields was that they were prone to the creation of local minima. Following that, [4] presents an extended approach for motion planning that unifies kinematic path planning.

Using solutions to Laplace's equations, [5] and [6] independently developed a global method that generates a smooth path, where the potential fields were computed globally throughout the entire region. According to prior studies [7-12], block approaches outperform conventional Jacobi and Gauss-Seidel (GS) iterative methods. There have been several other approaches suggested for solving path planning problems. For instance, an algorithm developed by [13] uses the efficient dynamic-programming method to help generate routes completely and continually even when there are obstacles in between, by updating distance estimates at each grid to the nearest target neigbourhood. In contrast, [14] investigated utilizing a geometry maze routing algorithm that can easily be extended with multiple autonomous robots in dynamic collision free problem. The viability of a hybrid genetic and modified simulated annealing algorithm for mobile path planning was also confirmed in [15]. Whereas [16] developed a roadmap approach utilizing the Voronoi Diagram to obtain the shortest path planning with no collision.

Methodology

In place of an actual robot vehicle, we imitate the notion of robot vehicle movement by utilizing a simulator with a point traveling in a predetermined area. The robot's path planning issue can be expressed as a steady-state heat transfer problem. Through the heat transfer analogy, the target location is viewed as a sink absorbing heat. Meanwhile, the config-s border and all obstructions in the space are regarded as heat sources with fixed constant temperature. A temperature distribution forms because of the heat conduction process and the config-s is filled with the heat flux lines that are streaming to the sink, also known as the target point. The point robot, target, and obstructions are seen to be able to communicate with one another through such a field. Following the heat flux that will stream from high-temperature sources to the lowest temperature point in the config-s, the point robot can then utilize the temperature distribution in the field as a guide to proceed with the path from the departure to the target location. A harmonic function is used to simulate the environment setting and compute the temperature distribution of the config-s.

Mathematically, a harmonic function on a domain $\Omega \subset R^n$ is a function that meets Laplace's equation as.

$$\nabla^2\Phi = \sum_{i=1}^{n} \frac{\partial^2 \Phi}{\partial x_i^2} = 0 \qquad (1)$$

where x_i stands for the *i*-th Cartesian coordinates and *n* represents the dimension. The inner walls, every obstruction in config-s, and every start and target points make up the domain Ω in the context of creating a robot path. Laplace's equation can be solved effectively through numerical methods. Jacobi and GS are two conventional used approaches for solving the problem; however, in this paper, (1) was resolved using a two-stage iterative technique for quicker computing.

In this model, a point in the config-s, designed in a grid-like structure, serves as the representation of the robot. Using a numerical approach, the function values related with each node are computed iteratively to meet (1). The departure point is given the highest temperature, while the target point is given the lowest. The boundaries and obstructions are given varied initial temperature values. With Dirichlet boundary conditions, $\Phi \mid \partial\Omega = c$, where *c* is constant, the solutions to Laplace's equation were observed. Following the temperature distribution using the steepest descent method, in which the algorithm descends along a negative gradient from the starting point through successively lower temperatures points up to the lowest temperature target point, allows for the creation of a smooth pathway once the environment's potential values have been established.

I. Formulation Of Alternating Group Explicit Iterative Method

Any linear system can be solved using Jacobi and GS iterative methods as documented in [5,17]. Not long ago, [18] used analytical solutions for obstacles with arbitrarily shaped. In order to solve Laplace's equation more quickly, this study projected a faster numerical solver using a two-stage iteration technique namely Alternating Group Explicit (AGE) method. Consider the following definition of the two-dimensional Laplace equation in (1):

$$\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}=0. \quad (2)$$

The straightforward finite difference scheme to approximate (2) is the 5-point finite difference approximation, commonly expressed as

$$U_{i-1,j}+U_{i+1,j}+U_{i,j-1}+U_{i,j+1}-4U_{i,j}=0. \quad (3)$$

Referring to the grid structure of the config-s, suppose that a rectangular grid in the *xy*-plane with both directions having the same grid spacing *h*, i.e. $x_i=ih$ and $y_j=jh$, is applied, then $u_{i,j}=u\left(x_i,y_j\right)$ where $i,j=0,1,2,\ldots,n$. Equation (3) can then be generalized and expressed in matrix form as

$$A\underset{\sim}{U}=\underset{\sim}{F}. \quad (4)$$

To demonstrate the AGE iterative method formulation, consider a group of techniques described by [10] that are based on dividing the matrix *A* into the sum of its constituent symmetric and positive definite matrices, as

$$A=G_1+G_2+G_3+G_4. \quad (5)$$

The element of G_1 and G_2 are the respective forward and backward differences in the *x*-plane, whereas G_3 and G_4 are alike differences towards the *y*-plane, wherein $diag\left(G_1\right)=diag\left(G_2\right)=\frac{1}{4}diag\left(A\right)$. The structures of G_3 and G_4 literally are identical to those of G_1 and G_2, respectively, by rearranging the points column-wise along the *y*-direction. Equation (5) then be rewritten as follows

$$\left(G_1+G_2+G_3+G_4\right)\underset{\sim}{U}=\underset{\sim}{F}. \quad (6)$$

Therefore, the explicit form of the AGE scheme [10] appears as

$$\underset{\sim}{U}^{\left(k+\frac{1}{4}\right)}=\left(r_1I+G_1\right)^{-1}\left[2\underset{\sim}{f}+\left(r_1I+G_1-2A\right)\right], \quad (7)$$

$$\underset{\sim}{U}^{\left(k+\frac{1}{2}\right)}=\left(r_1I+G_2\right)^{-1}\left[G_2\underset{\sim}{U}^{(k)}+r_1\underset{\sim}{U}^{\left(k+\frac{1}{4}\right)}\right], \quad (8)$$

$$\underset{\sim}{U}^{\left(k+\frac{3}{4}\right)}=\left(r_2I+G_3\right)^{-1}\left[G_3\underset{\sim}{U}^{(k)}+r_2\underset{\sim}{U}^{\left(k+\frac{1}{2}\right)}\right], \quad (9)$$

$$\underset{\sim}{U}^{(k+1)}=\left(r_2I+G_4\right)^{-1}\left[G_4\underset{\sim}{U}^{(k)}+r_2\underset{\sim}{U}^{\left(k+\frac{3}{4}\right)}\right], \quad (10)$$

Four-node points are computed concurrently in a parallel iteration from (7) to (10). Consequently, the computation of every nodal point in the config-s is substantially accelerated.

The minimal iterations number is a result of the uncertainty in the relaxation parameter values, in this case i.e., r_1 and r_2. According to earlier studies [19,20], the optimal values are typically selected so as to remain close to the SOR weighted parameter value, which is $1\le\omega<2$. The computation is then continual within the aforementioned range. Additionally, since the values of each parameter are predetermined prior to execution, the complexity of determining the values of parameters has no effect on the outcome of the entire computation. If the computing algorithm is configured with small ranges of parameter values in advance, it will undoubtedly change. Algorithm 1 describes the application of the AGE method to resolve (2).

Algorithm 1. AGE iterative scheme

i. Set up the designated config-s with the start and target points.

ii. Initializing starting point $U,\ \varepsilon\leftarrow1.0^{-16},\ iteration\leftarrow0$.

iii. Calculate in order

$$\underset{\sim}{U}^{\left(k+\frac{1}{4}\right)}=\left(r_1I+G_1\right)^{-1}\left[2\underset{\sim}{f}+\left(r_1I+G_1-2A\right)\right],$$

$$\underset{\sim}{U}^{\left(k+\frac{1}{2}\right)}=\left(r_1I+G_2\right)^{-1}\left[G_2\underset{\sim}{U}^{(k)}+r_1\underset{\sim}{U}^{\left(k+\frac{1}{4}\right)}\right],$$

$$\underset{\sim}{U}^{\left(k+\frac{3}{4}\right)}=\left(r_2I+G_3\right)^{-1}\left[G_3\underset{\sim}{U}^{(k)}+r_2\underset{\sim}{U}^{\left(k+\frac{1}{2}\right)}\right],$$

$$\underset{\sim}{U}^{(k+1)}=\left(r_2I+G_4\right)^{-1}\left[G_4\underset{\sim}{U}^{(k)}+r_2\underset{\sim}{U}^{\left(k+\frac{3}{4}\right)}\right].$$

iv. Verify the convergence test for $\varepsilon\leftarrow10^{-16}$, execute GDS creating a path to the target. Else, back to step (iii).

II. Experiments and Results

Three different sizes of designated config-s: 300x300, 600x600, and 900x900 have been tested. All starting locations were allocated randomly with no specific temperature values; however, the goal destination was allotted with fixed and lowest temperature values. Different numbers of obstructions in the form of Box, Line, Circle, L, and Z, were positioned in the config-s. The Dirichlet boundary condition was employed in the initial setup, and high-temperature values were fixed used to the walls and obstructions. Except for the target destination, which had the lowest temperature values, all other points were set to zero temperature values.

On an AMD A10-7400P Radeon R6 computer with 10 Compute Cores 4C+6G running at 2.50GHz and 8GB of RAM, the computation process was carried out. The iterative process of numerically computing temperature values at all points continues until the stopping condition is satisfied. The loop is ended when there are no more changes in temperature values and the difference between the calculation values is very small, i.e., 1.0^{-16} . This extremely high precision was required to prevent flat areas, also referred to as saddle points, in the solutions, which would lead to the path creation failing.

Tables I, II, and III indicate the iterations number, maximum error, and execution time (in seconds), respectively. Evidently, the AGE iterative method outperformed the GS method. Once the temperature values had been gathered, the path was formed through the steepest descent search upon the initial location to the target destination. The process of creating the pathways was incredibly fast, starting from the initial location, the algorithm simply chose its neighboring points with the lowest temperature value. This process continues until the desired target destination is reached. The pathways in the obstruction config-s were successfully generated, as shown in Fig. 1, based on the temperature distribution profile obtained via numerical computation. Every starting location (green square point) successfully ended at the specified destination point (red round point), escaping diverse obstructions set in place.

Table I. Iterations number against iterative methods

Methods	NxN		
	300	600	900
GS	47588	159652	342414
AGE	21823	50656	116470

Table II. Maximum error against iterative methods

Methods	NxN		
	300	600	900
GS	9.9920E-16	9.9920E-16	9.9920E-16
AGE	9.9920E-16	9.9920E-16	9.9920E-16

Table III. Execution time of various iterative methods

Methods	NxN		
	300	600	900
GS	179.56	4497.97	21691.57
AGE	233.08	2815.31	13046.90

III. Conclusion

The experiment in this study demonstrates that addressing robot path planning problems using numerical techniques is highly interesting and achievable by virtue of recently advanced and sophisticated approaches, as well as the accessibility of fast machine at the present time. The AGE iterative method outperformed the conventional GS method, as indicated in the results table. The finding of this study also shows that the AGE iterative method is hypothetically applicable in path planning problems and is better suited for larger scales. The performance is not negatively impacted by increasing number of obstructions; in fact, the computation progresses more quickly due to the disregard of the obstacle-occupied areas. The outcomes of this study are basically classified as a family of full-sweep iterations. To speed up the convergence rate of the standard proposed iterative approaches, further investigation on half-sweep [9,11,12,21,22] and quarter-sweep [19,23-26] iterations can also be taken into consideration.

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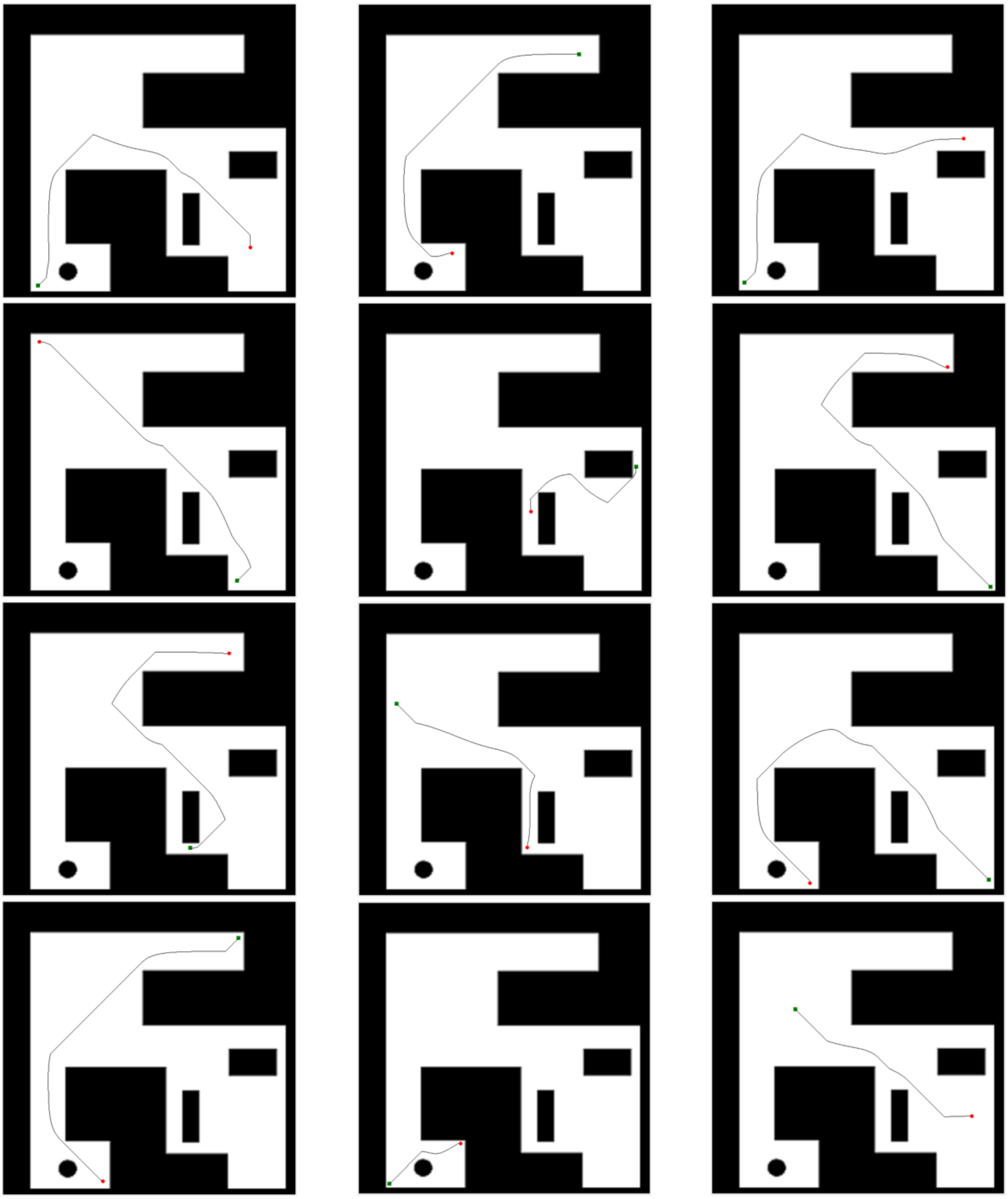

Fig.1. The paths generated with AGE iterative method in configuration space with obstructions.