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An Enhanced Approach for Solving Fuzzy Sequencing Problems with Trapezoidal Fuzzy Numbers

M. Ananthanarayanan[1] & R. Mahalakshmi[2]
[1]A. M. Jain College, University of Madras, Chennai-114, Tamil Nadu, India.
[2]Anna Adarsh College for Women, University of Madras, Chennai-40, Tamil Nadu, India.

Abstract
Discuss the situation of planning n occupations to 2 machines, the handling time as trapezoidal fuzzy nos. The fuzzy sequencing issue is changed over into crísp value by utilizing Python's program. Consequently the ideal succession of the positions is found by using Johnson's calculation, in which total elapsed time and inactive time for each machine is obtained. An illustrative mathematical model is given to show the execution of the fuzzy sequencing problems.

Keywords
Fuzzy number, Tríangular fuzzy number, Trapezoídal fuzzy number, Fuzzy arithmetic operations, Fuzzy Sequencing problems, Total elapsed time, Idle time, Robust's Ranking Technique, Johnson's Algorithm.

Introduction:

The term fuzzy logic was introduced in 1965 with the proposal of fuzzy set theory by Lotfi Zadeh. Fuzzy logic had, however, been studied since the 1920s, as infinite-valued logic—notably by Łukasiewicz and Tarski.

The main role of the classical sequencing problem is to find the Ideal succession of the jobs on machines so as to minimize the total amount of time required to complete the process of all the jobs.

Kripa. K, Govindarajan. R [1] prèsented the different methods to solve fuzzy sequencing problem using fuzzy technological values.

Fuyu Yuan, XinXuet al [3], proposed a novel fuzzy model for solving multi-objective permutation flow shop scheduling problem.

Dr. S.U. Malini and S. Kalaivani [5], introduced a method to solve fuzzy sequencing problem using octagonal fuzzy number.

M N Kanchana, K M Sangeetha, S SasiRekha [4] solved Fuzzy sequencing problem using Tríangular Fuzzy Numbers.

Nrimala.G and Anju.R [2], introduced fuzzy ranking method in "An application of fuzzy quantifiers" in sequencing problem. The essential ideas and meanings of fuzzy numbers and manages the proposed new calculation are analysed. The Fuzzy ideal arrangement is gotten utilizing Fuzzy Sequencing problem. Atlast, the problem is solved.

Definitions:

Fuzzy Number:

A fuzzynumber is a generalization of a regular real number and which does not refer to a single value but rather to a associated a bunch of possible values, where each possible value has its weight somewhere in the range of 0 and 1.

A fuzzynumber is a convex standardized fuzzyset on the genuine line R to such an extent that, there exist at least one (i) $x \in X$ with $\mu_{\underline{A}}(x) = 1$ (ii) $\mu_{\underline{A}}(x)$ is piece-wise continuous.

Tríangular Fuzzy Number:

A fuzzy number with membership func. $\underline{\mathring{A}} = (\lambda, \vartheta, \tau)$ in the form is called a Tríangular fuzzynumber.

$$\mu_A(x) = \begin{cases} \left[\dfrac{x-\lambda}{\vartheta-\lambda}\right] & \lambda \le x \le \vartheta \\ 1 & x = \vartheta \\ \left[\dfrac{\eta-x}{\eta-\tau}\right] & \tau \le x \le \eta \\ 0 & otherwise \end{cases}$$

Trapezoídal Fuzzy Number:

A fuzzy number with membership func. $\underline{\mathring{A}} = (\lambda, \vartheta, \tau, \eta)$ in the form

$$\mu_A(x) = \begin{cases} \left[\dfrac{x-\lambda}{\vartheta-\lambda}\right] & \lambda \le x \le \vartheta \\ 1 & \vartheta \le x \le \tau \\ \left[\dfrac{\eta-x}{\eta-\tau}\right] & \tau \le x \le \eta \\ 0 & otherwise \end{cases}$$

is called a Trapezoídal fuzzynumber.

Fuzzy arithmetic operations:

Summation: $(\lambda_1, \vartheta_1, \tau_1) + (\lambda_2, \vartheta_2, \tau_2) = (\lambda_1 + \lambda_2, \vartheta_1 + \vartheta_2, \tau_1 + \tau_2)$

Difference: $(\lambda_1, \vartheta_1, \tau_1) - (\lambda_2, \vartheta_2, \tau_2) = (\lambda_1 - \lambda_2, \vartheta_1 - \vartheta_2, \tau_1 - \tau_2)$

Fuzzy Sequencing Problem:

It is a choice of a fitting request where the quantity of positions can be relegated to a limited number of machines to enhance the result with regards to time, cost or benefit.

Total elapsed time:

It's the time from the beginning of the principal venture to culmination of the last undertaking.

Idle time:

Inactive time on a task is the time that venture stays inactive during the all out passed time.

Robust's Ranking Technique:

$$R(\underline{a}) = \int_0^1 0.5\,(a_l, a_u)\,d\alpha.$$

Method I:

Algorithm:

Discussing the issue of handling n-occupations, on 2 machines under the accompanying suppositions.

Step 1 : Convert Trapezoídal fuzzynumbers(TFN) in to crísp value using Robust Ranking Method.

Step 2: The ideal succession is tracked down utilizing the accompanying method.

- Deciding the base handling time among the undertakings A_i's for Machine A and undertaking B_i's for Machine B.
- Among the base handling time is A_r which is one of A_i's, where r^{th} undertaking is inserted at the initial of the sequence
- If the base handling time is B_s which is one of B_i's where s^{th} undertaking is inserted at the end of sequence (since the order A-B).
- If $A_r = B_s$ then allocate the r^{th} undertaking first then s^{th} undertaking next.
- If any undertaking with equal processing time among A_i for machine A, then any one undertaking can be inserted first.
- Similarly if any undertaking with equal processing time among B_i for machine B, then any one undertaking can be assigned first.
- Elíminating the undertaking assígned and repeating the above steps. Placíng the remaíning undertakíngs to the fírst job or before the last job. Thís wíll contínue untíl all the undertakíng assígnments are processed to complete.

Step 3: Work out the absolute slipped by time and inactive time for every one of the machines.

Description of the model:

In this model, Trapezoídal fuzzy numbers are defuzzified by using the Python language then find the optimum sequence evaluated by Johnson's Bellman algorithm and also find two different models for evaluation of total elapsed time and idle times for the given projects.

Numerical example 1:

Fíve undertakíngs A, B, C, D and E with two Undertakíngs 1 and 2. The fuzzy handlíng tímes for every one of the assígnments on two undertakíngs are gíven beneath:

Job	Task A	Task B	Task C	Task D	Task E
Project 1	(3,5,6,7)	(5,8,11,12)	(9,10,11,15)	(5,8,10,11)	(7,8,10,11)
Project 2	(6,8,10,12)	(5,8,9,10)	(2,4,5,6)	(5,7,10,11)	(8,11,13,15)

Decide a grouping for the positions that will limit the all out slipped by time and carve out the inactive opportunity for each projects.

Solution:

Given:

Job	Task A	Task B	Task C	Task D	Task E
Project 1	(3,5,6,7)	(5,8,11,12)	(9,10,11,15)	(5,8,10,11)	(7,8,10,11)
Project 2	(6,8,10,12)	(5,8,9,10)	(2,4,5,6)	(5,7,10,11)	(8,11,13,15)

Using Robust Ranking Method (3,5,6,7) is $(a_\alpha^l, a_\alpha^u) = (2\alpha + 3, 7 - \alpha)$

$$R(\underline{a}_{11}) = \int_0^1 \ 0.5((a_\alpha^l, a_\alpha^u))d\alpha = \int_0^1 \ 0.5((a_\alpha^l + a_\alpha^u))d\alpha = \int_0^1 \ 0.5(2\alpha + 3 + \ 7 - \alpha)d\alpha.$$

$$= \int_0^1 \ 0.5(\alpha + 10)d\alpha = 0.5[0.5 + 10] = 5.25.$$

Trapezoídal fuzzynumbers hasbeen transformed ín to crísp values by usíng Robust Rankíng Method whích ís ímplemented by followíng 'Python' program .

#Program to convert Trapezoídal Fuzzy numbers into crísp values.

```
from scipy.integrate import quad

rowss=int(input("Mention no. of rows:"))
colss=int(input("Mention no. of cols:"))
mat=[]
x=5
def f(x):
  l=(x*(n2-n1))+n1
  u=n4-(x*(n4-n3))
  return ((l+u)/2)
for i in range(rowss):
  tmat=[]
  for j in range(colss):
    n1=int(input("Mention number1:"))
    n2=int(input("Mention number2:"))
    n3=int(input("Mention number3:"))
    n4=int(input("Mention number4:"))
    a, b = quad(f, 0, 1)
    tmat.append(round(a,2))
  mat.append(tmat)
for i in mat:
  print(i,"     ")
```

```
[5.25, 9.0, 11.25, 8.5, 9.0]
[9.0, 8.0, 4.25, 8.25, 11.75]
```

Thus the handling times are as per the following: **Job**	Task A	Task B	Task C	Task D	Task E
Project 1	5.25	9.0	11.25	9.0	5.25
Project 2	9.0	8.0	4.25	8.25	11.75

Ideal succession is gotten by Johnson's Bellman Algorithm:

A	E	D	B	C

To find the minimum total elapsed time:

Job	Project 1		Project 2		Idle Time	
	Log In	Log Out	Log In	Log Out	P_1	P_2
A	(0,0,0,0)	(3,5,6,7)	(3,5,6,7)	(9,13,16,19)	–	(3,5,6,7)
E	(3,5,6,7)	(10,13,16,18)	(10,13,16,18)	(28,24,29,33)	–	(1,0,0,-1)
D	(10,13,16,18)	(15,21,26,29)	(28,24,29,33)	(33,31,39,44)	–	–
B	(15,21,26,29)	(20,29,37,41)	(33,31,39,44)	(38,39,48,54)	–	–
C	(20,29,37,41)	(29,39,48,56)	(38,39,48,54)	(40,43,53,60)	(11,4,5,4)	–
				Total:	(11,4,5,4)	(4,5,6,6)

The mínimum total elapsed tíme = (40,43,53,60) hrs.
Idle tíme on project P_1 = (11,4,5,4) hrs.
Idle tíme on project P_2 = (4,5,6,6) hrs.

Method II:

Let us consider the Trapezoídal Fuzzy Numbers in terms of crísp values and evaluate mínimum elapsed time, idle tíme.

To find the minimum total elapsed time:

Job	Project 1		Project 2		Idle Time	
	Log In	Log Out	Log In	Log In	Log Out	Log In
A	0	5.25	5.25	14.25	–	5.25
E	5.25	10.5	14.25	26.0	–	3.75
D	10.5	19.5	26.0	34.25	–	6.5
B	19.5	28.5	34.25	42.25	–	5.25
C	28.5	39.75	42.25	46.5	6.75	2.5
				Total:	6.75	23.75

The mínimum total elapsed tíme = 46.5 hrs.
Idle tíme on project P_1 = 6.75 hrs.
Idle tíme on project P_2 = 23.75 hrs.

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