

# Effects Of Rotation Modulation on Double-Diffusive Convection in Oldroyd-B Liquids

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## ABSTRACT

In a variety of scientific and technical fields, having a solid understanding of the dynamics of fluid flow and the transport of heat is very necessary. The main aim of the study is Effects of Rotation Modulation on Double-Diffusive Convection in Oldroyd-B Liquids. A horizontally oriented system consists of two walls that are separated by an infinitely thin layer of Oldroyd-B liquid. The presence of a stress relaxation parameter in the system leads to instability when synchronous temperature modulation is applied.

**Keywords:** Transport, Horizontally, synchronous, Temperature, Modulation

## 1. INTRODUCTION

In a variety of scientific and technical fields, having a solid understanding of the dynamics of fluid flow and the transport of heat is very necessary. These phenomena are largely influenced by convection, which is defined as the process of fluid motion that is driven by density gradients. There is a phenomenon known as double diffusive convection, which occurs when two separate diffusion processes, which are often connected to temperature and concentration gradients, coexist, and interact to cause fluid motion. This phenomenon is one of the most fascinating aspects of convection. This phenomenon may be found everywhere in nature and plays an important role in several scientific disciplines, including oceanography and materials research.

Researchers have investigated a wide variety of modulation strategies in order to get a more in-depth understanding of double diffusive convection and its regulation. In the context of this discussion, the term "modulation" refers to the intentional change of system characteristics in order to analyse and impact the convection patterns. Because of these modulations, scientists and engineers can explore the behavior of double diffusive convection under various situations. As a result, our knowledge of the processes that are at play has advanced, and new opportunities for practical applications have been made available.

During this in-depth investigation, we are going to dig into the realm of double diffusive convection and its many modulations. First, we will have a conversation about the underlying principles of

double diffusive convection. Specifically, we will explain how this kind of convection is distinct from single diffusive convection and why it is important in both natural and engineered systems. After that, we will discuss the idea of modulation, as well as the relevance of this notion in the study of fluid dynamics. In the following sections, we will investigate several sorts of modulation strategies that are used in the context of double diffusive convection. These approaches include using external fields and changing the geometry of the system, as well as managing temperature and concentration gradients. We are going to talk about the mechanics behind each sort of modulation, as well as its applications and the insights that it offers. As we go through this discussion, we will show the practical importance of various modulation approaches by drawing examples from relevant research and applications that take place in the real world. When we are through with this adventure, we will have a complete comprehension of the adaptability and potential of modulation in deciphering the secrets of double diffusive convection.

## 2. LITERATURE REVIEW

**Athar, Maria & Khan, Yasir & Akram, Safia (2022)** Double-diffusive convection is a notable physical phenomenon that emerges within the field of fluid mechanics. The phenomenon is mostly linked to a convection mechanism that involves the interaction of two density gradients of different magnitudes and diffusion speeds. The main objective of this work is to examine the impact of double-diffusivity convection and partial slip, in the presence of an inclined magnetic field, on peristaltic propulsion inside an asymmetric channel. This investigation

specifically focuses on nanofluids with Oldroyd-4 constants. The mathematical modeling of the flow of an Oldroyd-4 constant nanofluid is considered in the presence of double-diffusivity convection and a tilted magnetic field. The use of lubrication approach helps to streamline the complex system of partial differential equations (PDEs) that exhibit nonlinearity. The numerical method is used to compute the solution of a system of linked nonlinear partial differential equations (PDEs). Moreover, this research investigates the impact of modifying the parameters related to slip, thermophoresis, Brownian motion, Grashof number of nanoparticles, Hartmann number, pumping, and trapping. It has been observed that an increase in both Brownian motion and thermophoresis limitations leads to a rise in temperature. The observed correlation between the increase in the Brownian motion parameter and the rise in kinetic energy of nanoparticles leads to the consequent heating of the nanofluid. Moreover, the level of concentration decreases in response to the escalation of both Brownian motion and thermophoresis limitations.

**Riaz, Muhammad & Rehman, Aziz (2022)** The objective of this study is to examine the mathematical equations governing double diffusive magneto-free convection in an Oldroyd-B fluid flow. These equations are derived from fundamental symmetries and are expressed in non-dimensional form. The analysis focuses on a boundary layer flow along a vertically heated plate, where the flow moves upwards. Additionally, the study considers the influence of an external magnetic field, which can either be stationary or in motion, in accordance with the plate. This study examines the thermal transport phenomena in the presence of a constant concentration, linked with a first-order chemical process, under the exponential heating of the symmetry of fluid flow. The use of the Laplace transform technique is employed in a symmetrical manner to address the non-dimensionalized partial differential equations governing velocity, mass, and energy. This study presents and discusses the individual contributions of mass, temperature, and mechanical components to the dynamics of fluid. A noteworthy characteristic pertaining to the dynamics of fluid velocity arises when the motion is examined in conjunction with the magnetic field strength in the presence of a plate. In the scenario, the fluid velocity does not exhibit a value of zero when it is quite far from the plate. Furthermore, the heat transfer phenomena, fluid flow dynamics, and their dependence on various factors are elucidated via the use of graphical representations. In addition, the study also examines some exceptional scenarios pertaining to plate movement.

**Manjula, S & Suresh, P. & Rao, M.G. (2021)** The study examines the impact of thermal modulation on double-diffusive stationary convection when an applied magnetic field and internal heating are present. The finite-amplitude Ginzburg-Landau model was used to conduct a stability study with weak nonlinearity. The finite amplitude of convection is achieved when the system is analyzed up to the third order. The research examines three distinct variations of temperature modulations. Out of Phase Modulation (OPM), Lower Boundary Modulation (LBMO), and In Phase Modulation (IPM) are three modulation techniques often used in communication systems. The finite amplitude is a mathematical function that depends on the amplitude  $\delta T$ , frequency  $\omega$ , and phase difference  $\theta$ . The impact of  $\delta T$  and  $\omega$  on heat and mass transfers has been examined and visually shown. The research findings indicate that thermal modulation may be a very effective method for controlling heat and mass transfers. Moreover, it has been shown that the internal Rayleigh number (Ri) has a positive effect on heat transmission while simultaneously decreasing mass transfer inside the system.

**Altawallbeh, Anas (2021)** The analytical investigation focuses on the study of double diffusive convection in a porous layer saturated with a binary viscoelastic fluid. The research considers the existence of a cross diffusion effect and an internal heat source. Both linear and nonlinear stability analysis methods are used in this study. The linear stability theory relies on the use of the normal mode approach, while the nonlinear theory is based upon a minimum representation of truncated double Fourier series. The modified Darcy law is used to represent the momentum equation for the viscoelastic fluid of the Oldroyd type. Analytical methods have been used to determine the onset criterion for stationary and oscillatory convection, as well as for steady heat and mass transport. The linear and nonlinear theories have been utilized to get these criteria. The present study aims to examine the collective impact of an internal heat source and cross diffusion. The graphical representation illustrates the impact of several factors, such as Dufour and Soret effects, internal heat, relaxation and retardation time, Lewis number, and concentration Rayleigh number, on the phenomena of stationary, oscillatory, and heat and mass transfer. The graphical representation of heat transmission is often expressed in terms of the Nusselt number, whereas the graphical representation of mass transfer is expressed in terms of the Sherwood number. The effect of Soret and Dufour factors on both stationary and oscillatory convection has been observed to be substantial. An increase in the internal heat parameter has both a destabilizing impact and enhances the heat transmission process.

Conversely, the variation in the internal heat parameter yields a varying impact on the process of mass transfer. It has been observed that there exists a critical value for the thermal Rayleigh number, whereby an increase in internal heat leads to a fall in the Sherwood number below this threshold, while an increase in internal heat results in a rise in the Sherwood number above this threshold.

**Raghunatha, K. R. & Shivakumara, I. S. (2021)** This study examines the stability of a viscoelastic fluid layer with triple diffusion, where the density of the fluid is influenced by three stratifying factors with varying diffusivities. The Oldroyd-B constitutive equation is used to represent the behavior of the viscoelastic fluid. The linear instability analysis is conducted to provide analytical formulations for both steady and oscillatory start. The demarcation of the crossover border between these two states is achieved by selecting a codimension-two point inside the viscoelastic parameters plane. The presence of disjointed closed oscillatory neutral curves, which are situated far below the stationary neutral curve, has been seen for certain combinations of governing parameters. This observation suggests that three crucial values of the thermal Rayleigh number are necessary to accurately determine the criterion for linear instability. Nevertheless, the occurrence of quasiperiodic bifurcation from the stationary equilibrium state is not seen, which contradicts the behavior observed in the context of inelastic couple stress and Newtonian fluids. A perturbation approach was used to investigate the weakly nonlinear stability of both stationary and oscillatory modes. The derivation of the cubic Landau equations is presented, followed by a discussion on the stability of the bifurcating solution. The stability of stationary bifurcation is influenced by the viscoelastic parameters, but their impact is not seen during the stationary start. The bifurcation of the stationary and oscillatory finite amplitude solution is determined by the selection of governing parameters, resulting in either subcritical or supercritical bifurcation. This study examines the impact of Prandtl number and viscoelastic factors on the characteristics of stationary and oscillatory convection modes in the context of heat and mass transport.

### 3. EFFECTS OF ROTATION MODULATION ON DOUBLE-DIFFUSIVE CONVECTION IN OLDROYD-B LIQUIDS

#### 3.1 Mathematical Formulation

A horizontally oriented system consists of two walls that are separated by an infinitely thin layer of Oldroyd-B liquid. The spatial separation between the two entities is denoted as  $d$ . The two planes are

positioned at the  $z$ -coordinate of zero and  $d$ , respectively. The two plates exhibit disparate thermal and compositional conditions. This phenomenon results in the formation of gradients in density and temperature, denoted as  $\Delta T$  and  $\Delta S$ , respectively. The density of the fluid exhibits a linear truncation with respect to both solute concentrations, denoted as  $S$ , and temperature, denoted as  $T$ . The two limits exhibit a free-free condition. The system undergoes rotational motion around the  $z$ -axis. The setup is shown in Figure 4.1. Hence, the governing equations pertaining to the issue are presented as follows.

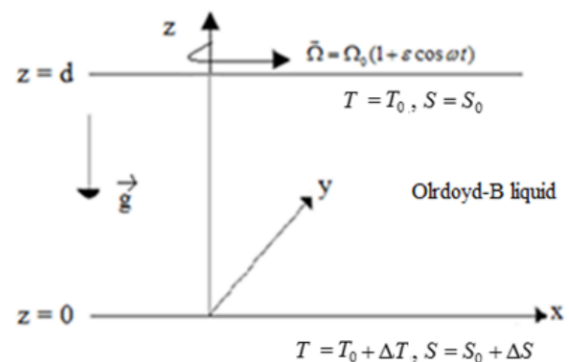


Fig 3.1 physical configuration

Nomenclature	
$d$	thickness of the liquid
$k$	dimensionless wave number
$pr$	Prandtl number
$q$	velocity
$Ra$	thermal Rayleigh number
$Rs$	solutal Rayleigh number
$t$	time
$T$	temperature
$T_0$	constant temperature of the upper boundary
$T_R$	reference temperature
$Le$	Lewis number
$Ta$	Taylor number
Greek symbols	
$\alpha$	thermal expansion coefficient
$\epsilon$	amplitude of modulation
$\kappa$	thermal diffusivity
$\kappa_s$	solutal diffusivity
$\lambda_1$	stress relaxation coefficient
$\lambda_2$	strain retardation coefficient
$\Lambda$	elasticity ratio ( $\lambda_2 / \lambda_1$ )
$\mu$	viscosity
$\omega$	frequency of modulation
$\rho$	density
$\rho_0$	reference density

**Continuity Equation:**

$$\nabla \cdot \vec{q} = 0,$$

**Conservation of momentum:**

$$\rho_0 \left[ \frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla) \vec{q} + 2(\vec{\Omega} \times \vec{q}) \right] = -\nabla p + \rho \vec{g} + \nabla \tau'$$

Where

$$\vec{\Omega} = \Omega_0(1 + \varepsilon \cos \omega t)$$

**Rheological Equation:**

$$\left( 1 + \lambda_1 \frac{\partial}{\partial t} \right) \tau' = \left( 1 + \lambda_2 \frac{\partial}{\partial t} \right) (\nabla_{\vec{q}} + \nabla_{\vec{q}tr})$$

**Conservation of Energy:**

$$\frac{\partial T}{\partial t} + (\vec{q} \cdot \nabla) T = k \nabla^2 T,$$

**Conservation of Species:**

$$\frac{\partial S}{\partial t} + (\vec{q} \cdot \nabla) S = k_s \nabla^2 S,$$

**Equation of State:**

$$\rho = \rho_0(1 - \alpha_t(T - T_0) + \alpha_s(S - S_0))$$

**3.2 Basic State**

At the basic state, the liquid is at rest. Therefore,

$$\vec{q} = \vec{q}_b = 0, p = p_b(z), \rho = \rho_b(z), S = S_b(z), T = T_b(z)$$

The variables of density ( $\rho_b$ ), pressure ( $p_b$ ), temperature ( $T_b$ ), and concentration ( $S_b$ ) are in accordance with one another.

$$\frac{dp_b}{dz} + \rho_b \vec{g} = 0$$

$$\frac{\partial T_b}{\partial t} = k \frac{\partial^2 T_b}{\partial z^2}$$

$$\frac{\partial S_b}{\partial t} = k_s \frac{\partial^2 S_b}{\partial z^2},$$

$$\rho = \rho_0(1 - \alpha_t(T - T_0) + \alpha_s(S - S_0))$$

Operating  $\left( 1 + \lambda_1 \frac{\partial}{\partial t} + \lambda_1 (\vec{q} \cdot \nabla) \right)$  on the rheological equation becomes,

$$\left( 1 + \lambda_1 \frac{\partial}{\partial t} \right) \left[ \rho_0 \frac{\partial \vec{q}}{\partial t} + \nabla p - \rho \vec{g} \right] = \mu \left( 1 + \lambda_2 \frac{\partial}{\partial t} \right) \nabla^2 \cdot \vec{q}$$

The fundamental state solutions for temperature and concentration are.

$$T_b = T_0 \left( 1 - \frac{z}{d} \right), S_b = S_0 \left( 1 - \frac{z}{d} \right)$$

**3.3 Stability Analysis**

The examination of the system's stability is conducted by the application of a minute perturbation. The perturbations may be described as follows:

$$\vec{q} = \vec{q}', p = p_b + p', \rho = \rho_b + \rho', S = S_b + S', T = T_b + T'$$

The prime symbols are used to represent the perturbations that have been applied. By using the equation, in conjunction with the fundamental state solutions, we get the following system of equations.

$$\frac{\partial T'}{\partial t} + w' \frac{\partial T'}{\partial z} + w' \frac{\partial T_b}{\partial z} = k \nabla^2 T'$$

$$\frac{\partial S'}{\partial t} + w' \frac{\partial S'}{\partial z} + w' \frac{\partial S_b}{\partial z} = k_s \nabla^2 S',$$

$$\left( 1 + \lambda_1 \frac{\partial}{\partial t} \right) \left[ \rho_0 \frac{\partial (\nabla^2 \psi)}{\partial t} - 2\rho_0 \vec{\Omega}(t) \frac{\partial V'}{\partial z} - \alpha_t \rho_0 g \nabla_1^2 T' + \alpha_s \rho_0 g \nabla_1^2 S' \right] = \mu \left( 1 + \lambda_2 \frac{\partial}{\partial t} \right) \nabla^4 \psi$$

By considering the equation in which all the derivatives with respect to y become zero, we may determine the y-component.

$$\left( 1 + \lambda_1 \frac{\partial}{\partial t} \right) \left[ \rho_0 \frac{\partial V}{\partial t} + 2\rho_0 \vec{\Omega}(t) \frac{\partial \Psi}{\partial z} \right] = \mu \left( 1 + \lambda_2 \frac{\partial}{\partial t} \right) \nabla^2 V$$

Where  $\nabla_1^2 = \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right), V = \nabla \times \vec{q}$

The scales for non-dimensionalizing eqs are

$$w^* = \frac{w'}{k/d}, t^* = \frac{t}{d^2/k}, T^* = \frac{T'}{\Delta T}, S^* = \frac{S}{\Delta S}, \nabla^* = d \nabla, (x^*, y^*, z^*) = \left( \frac{x}{d}, \frac{y}{d}, \frac{z}{d} \right)$$

**3.4 NON-LINEAR THEORY**

In order to facilitate mathematical analysis, we restrict our consideration to two-dimensional rolls, so rendering the physical parameters in the

equations independent of the y-coordinate. The stream function is introduced in this context  $\psi$ , so that  $u = \frac{\partial \psi}{\partial z}$  and  $w = -\frac{\partial \psi}{\partial x}$  which satisfy equation of continuity. This gives eqs. that are dimensionless.

$$\left(1 + \Lambda_1 \frac{\partial}{\partial t}\right) \left[ \frac{1}{Pr} \frac{\partial(\nabla^2 \Psi)}{\partial t} - Ta^{1/2} (1 + \varepsilon \cos \omega t) \frac{\partial V}{\partial z} + Ra \nabla_1^2 T - Rs \nabla_1^2 S \right] = \left(1 + \Lambda_2 \frac{\partial}{\partial t}\right) \nabla^4 \psi$$

$$\left(1 + \Lambda_1 \frac{\partial}{\partial t}\right) \left[ \frac{1}{Pr} \frac{\partial V}{\partial t} - Ta^{1/2} (1 + \varepsilon \cos \omega t) \frac{\partial \psi}{\partial z} \right] = \left(1 + \Lambda_2 \frac{\partial}{\partial t}\right) \nabla^2 V$$

$$\left(\frac{\partial}{\partial t} - \nabla^2\right) T + (\vec{q} \cdot \nabla) T = -\frac{\partial \psi}{\partial x},$$

$$\left(\frac{\partial}{\partial t} - \frac{1}{Le} \nabla^2\right) S + (\vec{q} \cdot \nabla) S = -\frac{\partial \psi}{\partial x}$$

#### 4. RESULTS

The issue at hand pertains to the examination of the consequences resulting from the modification of gravity on the convection of two components inside Oldroyd-B liquids. The linear problem is solved using the approach proposed by Venezian in 1969. The characteristics that most influence the transport of heat and mass are  $Le$ ,  $Ra$ ,  $Rs$ ,  $Pr$ ,  $Ta$ ,  $\Lambda_1$ ,  $\Lambda_2$ , and  $\omega$ . The features of the fluid, including  $Le$ ,  $Ra$ ,  $Rs$ ,  $Pr$ ,  $Ta$ ,  $\Lambda_1$ ,  $\Lambda_2$ , play a significant role in the convection process. Additionally,  $\omega$  serves as an external mechanism for regulating convection. The thermal modulation amplitude, denoted as  $\varepsilon$ , is of a diminutive magnitude. The outcomes are mostly contingent upon the modulation frequency, denoted as  $\omega$ . If the numerical value of this parameter is less than one, it indicates that the modulation period is quite long. This phenomenon leads to the amplification of disturbances, resulting in the increased significance of finite amplitude effects. When the angular frequency approaches infinity, the impact of modulation on  $Ra_{2c}$  tends to zero, resulting in few perceptible effects. Therefore, the research focuses on moderate values of  $\omega$ .

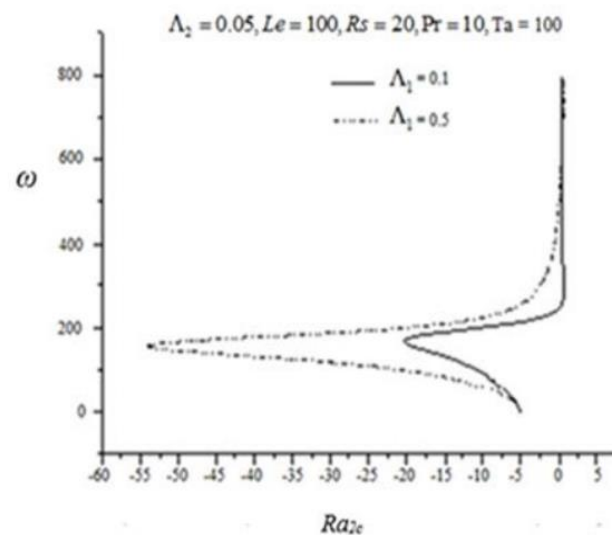
**Table 4.1 Values of correction Rayleigh number,  $Ra_{2c}$ , Nusselt number,  $Nu$ , and Sherwood number,  $Sh$ , for  $Le = 100$ ,  $Pr = 10$ ,  $Rs = 20$ ,  $\omega = 10$ ,  $\varepsilon = 0.1$ .**

Newtonian fluid	$\Lambda_1$	0.1	0.5	0.8
$\Lambda_1 = \Lambda_2$	$Ra_{2c}$	657	643.25	630.59

	$Nu$	1.3114	1.6531	1.9672
	$Sh$	1.7754	2.1342	2.3641
Maxwell fluid	$\Lambda_1$	0.1	0.5	0.8
$\Lambda_2 = 0$	$Ra_{2c}$	148.4	145.32	137.31
	$Nu$	1.8234	2.2763	2.6874
	$Sh$	2.596	3.0015	3.3419
Oldroyd-B fluid	$\Lambda_1$	0.1	0.1	0.1
$\Lambda_1 \neq \Lambda_2$	$\Lambda_2$	0.05	0.08	0.09
	$Ra_{2c}$	180.39	172.53	168.38
	$Nu$	1.6218	1.8534	2.2116
	$Sh$	1.9963	2.5531	3.0167

We conclude that

1.  $Ra_{2c}^{Maxwell\ fluid} < Ra_{2c}^{Oldroyd-B\ fluid} < Ra_{2c}^{Newtonian\ fluid}$ ,
2.  $Nu^{Maxwell\ fluid} > Nu^{Oldroyd-B\ fluid} > Nu^{Newtonian\ fluid}$ ,
3.  $Sh^{Maxwell\ fluid} > Sh^{Oldroyd-B\ fluid} > Sh^{Newtonian\ fluid}$ .



**Fig 4.1 plot of  $Ra_{2c}$  versus  $\omega$  for different values of  $\Lambda_1$ .**

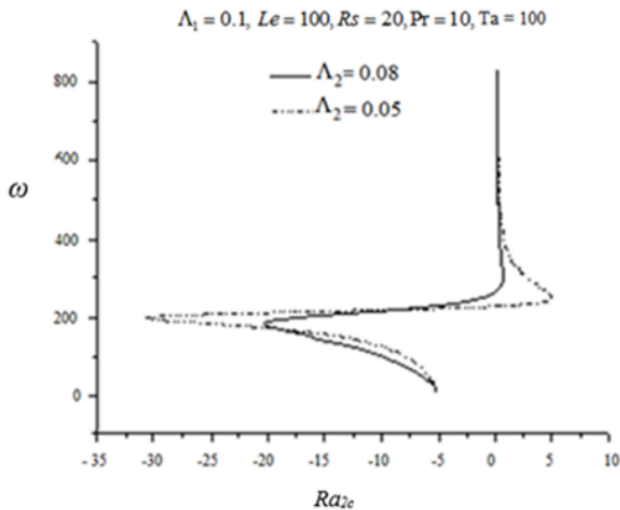


Fig 4.2 plot of  $Ra_{2c}$  versus  $\omega$  for different values of  $\Lambda_2$

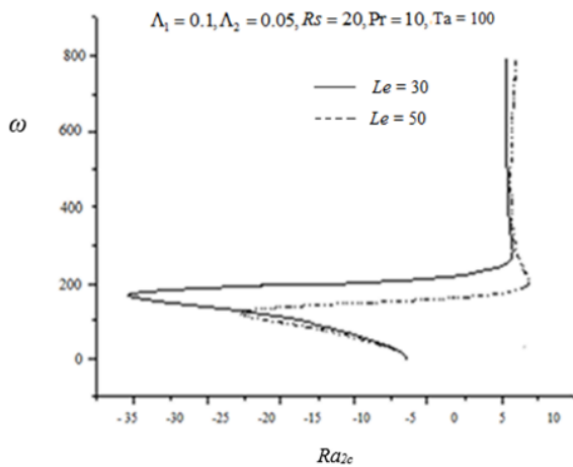


Fig 4.3 plot of  $Ra_{2c}$  versus  $\omega$  for different values of  $Le$ .

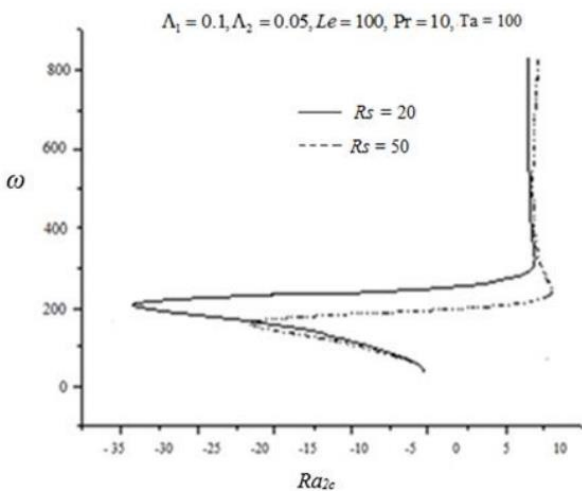


Fig 4.4 plot of  $Ra_{2c}$  versus  $\omega$  for different values of  $R_s$

## 5. CONCLUSION

The presence of a stress relaxation parameter in the system leads to instability when synchronous temperature modulation is applied. The strain retardation parameter plays a crucial role in ensuring the stability of the system while using synchronous temperature modulation. The values of stress relaxation parameter positively influence the rates of heat and mass transfers, whereas the rates decrease with an increase in the Lewis number, solutal Rayleigh number, and strain retardation parameter.

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