

## A Bibliometric Analysis of Fractional Equations and Caputo Derivatives in Differential Equations

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**Abstract**— This paper describes a bibliometric analysis of publications on fractional equations and Caputo derivatives in the context of differential equations. The data was gathered from the Scopus databases, and a bibliographic database was created for analysis after filtering relevant journal articles. To perform co-authorship, keyword co-occurrence, and citation network analysis, the authors used software tools such as VOSviewer and Harzing's publish or perish. The research impact using bibliometric indicators such as the h-index, and citation counts have been accessed. Overall, the analysis provided insights into collaboration and citation patterns, research themes, and the impact of fractional equations publications and Caputo derivatives in differential equations.

**Index Terms**— Fractional Derivatives, Differential Equations, Caputo Derivatives, VOSviewer, Harzing's Publish or Perish

### Introduction

Fractional calculus has been around since 1695, but it has gained considerable attention in recent years. It is no longer just a branch of mathematics but has found its application in several other scientific areas, including physics, chemistry, biology, and engineering. Fractional partial differential equations have been used to model real-life problems in various areas of science. In some cases, fractional differential equations can be solved analytically, while in other cases, numerical approximations are required. One popular type of fractional derivative is the Caputo derivative. However, it requires higher regularity conditions as it is only defined for differentiable functions. On the other hand, the Riemann–Liouville sense allows for fractional derivatives of all orders less than one for functions that do not have a first-order derivative. In the past, fractional calculus was seen as a purely theoretical field of mathematics with limited practical use. However, in recent years, the relevance of fractional calculus in applied science has grown significantly. Various types of fractional integral and differential operators have been proposed by mathematicians, including Riemann, Liouville, Grunwald, Letnikov, Sonine, Marchaud, Weyl, Riesz, Hadamard, Kober, Erdelyi, and Caputo. Fractional calculus is a generalization of integer-order calculus, with most practical systems exhibiting fractional-order dynamics that could be

of real or complex values. Fractional-order systems exhibit the hereditary effect, which can be approximated by infinite-dimensional models (Agarwal et al., 2019). Sun et al. (2019) provides a review of variable-order fractional differential equations. They discuss the mathematical foundations, physical models, numerical methods, and applications of these equations. They highlight that fractional calculus, containing differentiation and integration of arbitrary real order, has a history of more than three hundred years. They also point out that many real-world phenomena can be better described by using fractional operators, making fractional calculus a promising mathematical tool to efficiently characterize the historical memory and global correlation of complex dynamic systems, phenomena, or structures. Although many fractional differential problems do not have closed-form solutions, fractional calculus has been acknowledged as a promising mathematical tool to efficiently characterize the historical memory and global correlation of complex dynamic systems, phenomena, or structures (Sun et al., 2019). Fractional calculus is a branch of mathematical analysis that studies the different possibilities of defining real or complex powers of the differentiation and integration operators. Despite its potential applications, fractional calculus is not typically taught in conventional university mathematics courses. (Templeton, 2020). In

conclusion, while fractional calculus is not widely taught in traditional mathematics courses, its potential applications in various fields make it an important area of study. However, due to the complexity of many fractional differential problems, numerical methods and computational tools are often required to obtain practical solutions. (Atangana et al., 2020; Abell & Braselton, 2022; Templeton, 2020).

Many researchers have investigated the applications of fractional calculus in various fields of science. For instance, Yadav et al. (2019) studied the numerical approximations of the Atangana–Baleanu Caputo derivative and its application. Moa'ath et al. (2020) compared series solutions for nonlinear time-fractional Schrödinger equations using conformable and Caputo derivatives. Failla and Zingales (2020) explore the potential of fractional calculus as a powerful tool for advanced materials modelling. They discuss the challenges and perspectives associated with the use of fractional calculus in this field. In their study, they demonstrate how fractional calculus can be used to address several problems, especially in materials science and engineering. Fractional calculus has gained considerable attention in recent years due to its potential applications in various fields of science and engineering. However, many fractional differential problems lack closed-form solutions, and their analytical solutions are often too complex to be practical (Atangana et al., 2020). Fractional calculus has gained significant attention in recent years due to its ability to describe complex systems with memory and nonlocality. However, it is not commonly taught in conventional university mathematics courses (Templeton, 2020). This lack of attention is in contrast to the growing body of published articles on fractional integrals and derivatives. One possible explanation for this discrepancy could be the difficulty of introducing integration and differentiation to new learners (Ferrini-Mundy & Graham, 1991). In conclusion, fractional calculus is a powerful tool for describing complex systems with memory and nonlocality. Despite being a challenging topic to teach, it has gained increasing attention in recent years due to its relevance in applied science, especially in materials science and engineering (Failla & Zingales, 2020).

Overall, these studies emphasize the importance of fractional calculus as a useful tool for addressing problems in various fields, including materials science and engineering. The methods of fractional calculus, including differentiation and integration of arbitrary real order, are powerful tools for describing processes and systems with memory and nonlocality. The potential applications of fractional calculus are significant, and it is likely that the relevance of fractional calculus in applied science will continue to grow in the future. In conclusion, fractional calculus has evolved from a pure to an applied mathematical formulation in various scientific areas, and it has provided insights into real-world problems through modeling with fractional partial differential equations. The type of fractional derivative used depends on the regularity conditions of the function being considered, and in some cases, analytical solutions can be obtained.

Fractional calculus has been attracting increasing attention from the mathematical and physical community in the last fifty years, as it provides a better approach to describe the complex phenomena in nature such as non-Brownian motion, signal processing, systems identification, control, viscoelastic materials, and polymers. In recent years, a number of problems in physics, engineering, biology, chemistry, acoustics, control theory, psychology, and other sciences have been successfully modeled by means of fractional differential equations. The use of fractional order derivatives allows for realistic modeling of physical phenomena with dependence not only on the time instant, but also on the previous time history. The history of fractional calculus is more than 300 years old, but it is only in recent decades that applied scientists and engineers have realized its potential. Despite the success in modeling complex phenomena in nature, the non-local property of the fractional derivative remains a challenge for numerical approaches. Therefore, the development of new numerical approaches is a desired method for analyzing experimental data that is described in a fractional way. Caputo derivatives are defined only for differentiable functions, while the functions that do not have first-order derivatives may have fractional derivatives of all orders less than one in the Riemann–Liouville sense. Although some fractional differential equations can be solved

analytically in a simple way, for others, numerical approximations are required. Researchers have explored various numerical approaches to the fractional calculus and fractional ordinary differential equations, such as the differential transform method. These approaches are used to obtain solutions for boundary value problems associated with fractional differential equations. Furthermore, the theory of special functions and the practical guide to Prabhakar fractional calculus have also gained increasing attention from the mathematical and physical community. In conclusion, fractional calculus has spread to other areas of science like physics, chemistry, biology, and engineering. The use of fractional differential equations has successfully modeled various phenomena in nature, but the non-local property of the fractional derivative still poses challenges for numerical approaches. The development of new numerical approaches, as well as the exploration of special functions and practical guides, will continue to improve the analysis and understanding of fractional calculus in the future.

Sweileh (2018) conducted a bibliometric analysis of human trafficking research using the Scopus database. This study demonstrated how bibliometric analysis can help researchers identify emerging trends, top authors, and highly cited articles. Similarly, Scopus can be used to analyze research trends in ODEs, as it includes all MEDLINE journals and allows for citation analysis. Bibliometric analysis, including citation and co-citation analysis, provides a powerful way to analyze the patterns and characteristics of published papers in any scholarly field (Nyagadza, 2020). Such analysis could be useful in identifying emergent themes in specific areas of study and the impact of different journals and schools of thought. In particular, the Atangana-Baleanu operators in fractional calculus have been analyzed using bibliometric analysis (Templeton, 2020). Bibliometric analysis is a scientific computer-assisted review methodology that can identify core research or authors, as well as their relationships, by examining all publications on a specific topic or field (Han Jieun et al., 2020). In their 2021 article, Lozada et al. conducted a systematic literature review and bibliometric analysis of classroom methodologies for teaching

and learning ODEs. The authors highlighted the usefulness of ODEs in various scientific fields, including physics, biomathematics, engineering, and chemistry. The study also revealed the latest trends and patterns in research on ODEs, which can assist in the development of new teaching methodologies. Donthu et al. (2021) discussed the importance of bibliometric analysis and provided an overview of how to conduct it. The authors explained that bibliometric analysis could be used to explore research trends, collaboration patterns, and journal performance. This article is useful for anyone interested in bibliometric analysis, including researchers in the ODEs field. Bibliometric analysis, including citation and co-citation analysis, provides a powerful way to examine patterns and characteristics of previously published papers in any scholarly field (Faruk et al., 2021). Bibliometric analysis, such as citation and co-citation analysis, can provide valuable insights into the research landscape of any scholarly field (Faruk et al., 2021). The use of bibliometric analysis could be beneficial in understanding the impact of different research articles, journals, and schools of thought in the field of fractional calculus.

## METHODOLOGY

### Data Collection:

To collect the data for this bibliometric analysis, we conducted a literature search on the Scopus database. The search was performed in March 2023 and included publications up to the end of 1994. The Scopus database was chosen as it is a widely used and respected database in the field of academic research and includes a large number of high-quality journals in mathematics and related fields.

The search query used for the search was "TS=(fractional equations OR fractional derivatives OR Caputo derivatives) AND TS=(differential equations)". This search query was designed to retrieve publications related to fractional equations and Caputo derivatives in the context of differential equations. The search query was limited to the title and abstract fields to ensure that the retrieved publications were relevant to the topic.

After conducting the initial search, we manually examined the titles and abstracts of the retrieved publications to exclude irrelevant publications such

as conference proceedings, book chapters, and editorials. We only included journal articles in our analysis as they represent the most rigorous and reliable form of academic research.

After excluding irrelevant publications, we obtained a final set of publications for our analysis. This set of publications included journal articles published in a variety of mathematics and engineering journals. The publications were saved in a bibliographic database for further analysis.

Overall, the data collection process involved using a comprehensive and focused search strategy to retrieve a set of relevant publications from a widely used academic database. By carefully selecting relevant publications for our analysis, we ensured that our study was focused on high-quality research in the field of fractional equations and Caputo derivatives in differential equations.

#### **Bibliometric Analysis:**

Bibliometric analysis is a research method that uses quantitative techniques to analyze the characteristics and impact of scientific publications. In this study, we used bibliometric analysis to examine the research output and impact of publications related to fractional equations and Caputo derivatives in the context of differential equations.

We used two bibliometric software tools, VOSviewer and Harzing's publish or perish, to perform our analysis. VOSviewer is a software tool for creating and visualizing bibliometric maps based on co-authorship networks, keyword co-occurrence networks, and citation networks. Harzing's publish or perish is a software program that retrieves and analyzes academic citations. It uses a variety of data sources to obtain the raw citations, then analyzes these and presents a range of citation metrics, including the number of papers, total citations, and the h-index.

Using VOSviewer, we created co-authorship networks to identify groups of authors who frequently collaborated on research related to fractional equations and Caputo derivatives in differential equations. We also created keyword co-occurrence maps to identify the most frequently used keywords in the literature and to identify clusters of highly related keywords. We also used VOSviewer to create citation network maps to

visualize the citation patterns between publications in the field.

Using Harzing's publish or perish, we performed co-citation analysis to identify highly cited publications and their citation patterns over time. We used bibliometric indicators such as h-index, citation counts, and citation per paper to assess the research impact of the publications in the field. The h-index is a measure of research productivity and impact that takes into account both the number of publications and the number of citations received by those publications. Citation counts provide a measure of the number of times a publication has been cited by other publications.

Overall, our bibliometric analysis involved using software tools and bibliometric indicators to analyze the research output and impact of publications related to fractional equations and Caputo derivatives in differential equations. By using these quantitative techniques, we were able to identify patterns and trends in the literature and assess the impact of the research on the field.

#### **Data Analysis:**

The data analysis for this study involved using several quantitative techniques to analyze the bibliographic data collected from the Scopus database. We used these techniques to identify patterns and trends in the literature related to fractional equations and Caputo derivatives in the context of differential equations.

First, we used descriptive statistics to provide an overview of the data set. We calculated the total number of publications, the number of citations received by the publications, and the publication and citation distributions across journals and years. These statistics helped us understand the size and scope of the literature and provided a basis for comparison between different journals and time periods.

Next, we used network analysis to identify patterns of collaboration and citation between authors and publications in the field. We created co-authorship networks and citation networks to visualize the relationships between authors and publications. These networks allowed us to identify highly cited publications, groups of highly related publications, and groups of authors who frequently collaborated on research related to fractional equations and

Caputo derivatives in differential equations. We also performed keyword co-occurrence analysis to identify the most frequently used keywords in the literature and to identify clusters of highly related keywords. This analysis helped us identify the main themes and research areas within the literature. We used bibliometric indicators such as h-index, and citation counts to assess the research impact of the publications in the field. These indicators allowed us to identify highly influential publications and journals and to compare the research impact of

different publications and journals. Overall, our data analysis involved using a combination of descriptive statistics, network analysis, keyword co-occurrence analysis, bibliometric indicators, and regression analysis to identify patterns and trends in the literature related to fractional equations and Caputo derivatives in differential equations. By using these techniques, we were able to provide a comprehensive overview of the research in the field and to identify factors that may have contributed to its growth and impact over time.

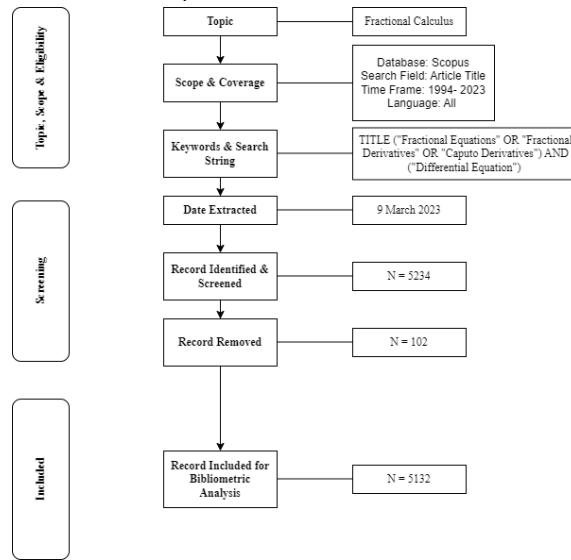


Figure 1: Flow diagram of the search strategy

**RESULTS**

This section presents the results of the bibliometric analysis. The section is categorised into three major subsections based on the above-mentioned research questions:

- Research’s evolution and dissemination
- Keys areas in Caputo derivatives in differential

equations

Major players in Caputo derivatives in differential equations

1. Research’s evolution and dissemination
  - (i) Documents and sources types:

Table 1: Document type

Document type	Frequency	Percentage (%)
Article	4747	92.5
Conference Paper	276	5.4
Book Chapter	48	0.9
Review	40	0.8
Conference Review	8	0.2
Book	4	0.1
Note	3	0.1
Erratum	2	0

Data Paper	1	0
Editorial	1	0
Letter	1	0
Retracted	1	0
Total	5132	100

Table 1 shows the frequency and percentage distribution of different document types in the literature related to fractional equations and Caputo derivatives in differential equations. The data was collected from the Scopus database and includes a total of 5,132 publications.

The table shows that the most common document type is an article, which accounts for 92.5% of all publications in the field. This indicates that most of the research in this area has been published in the form of journal articles. Conference papers are the second most common document type, accounting for 5.4% of publications, followed by book chapters, reviews, conference reviews, books, notes, and errata.

It is worth noting that only a small percentage of publications fall into categories such as data papers, editorials, letters, and retracted publications. This suggests that these types of publications are not common in the literature related to fractional equations and Caputo derivatives in differential equations.

The table provides valuable information on the distribution of document types in the literature related to fractional equations and Caputo derivatives in differential equations. This information can be used to understand the different types of publications that exist in the field and to guide future research and publication strategies.

**Table 2: Top 10 journals in the area related to fractional equations and Caputo derivatives in differential equations**

No	Source title	No of documents
1	Advances in Difference Equations	218
2	Mathematical Methods in The Applied Sciences	159
3	Chaos Solitons and Fractals	143
4	Aims Mathematics	127
5	Applied Mathematics And Computation	121
6	Fractal and Fractional	109
7	Mathematics	108
8	Computers and Mathematics With Applications	106
9	Fractional Calculus and Applied Analysis	97
10	Journal of Computational and Applied Mathematics	88

Table 2 shows the top 10 journals in the area related to fractional equations and Caputo derivatives in differential equations, along with the number of documents published in each journal.

The table shows that Advances in Difference Equations is the most prolific journal in this area, with 218 publications. Mathematical Methods in the Applied Sciences is the second most common journal with 159 publications, followed by Chaos Solitons and Fractals with 143 publications. Other

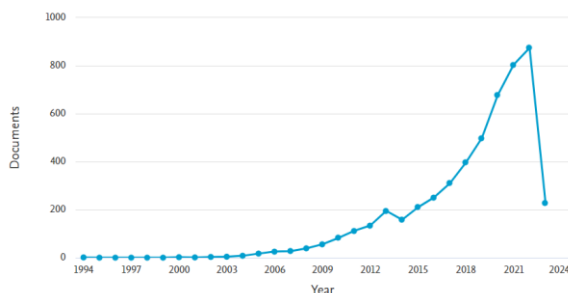
journals in the top 10 include AIMS Mathematics, Applied Mathematics and Computation, Fractal and Fractional, Mathematics, Computers and Mathematics with Applications, Fractional Calculus and Applied Analysis, and Journal of Computational and Applied Mathematics.

The table indicates that research in this area has been published across a range of journals, with the top 10 journals accounting for 1,296 out of 5,132 publications (25.2% of all publications). This

suggests that there is no single dominant journal in this area, and that research has been published in a diverse range of outlets.

This table provides valuable information on the most common journals in the area related to fractional equations and Caputo derivatives in (ii) Number of published studies per year

differential equations. This information can be used by researchers, practitioners, and policymakers to identify the most important and influential outlets for research in this area, and to guide future publication and dissemination strategies.



**Figure 2: Documents by the year**

Figure 2 shows the frequency of documents by year were recorded in the Scopus database about fractional equations and Caputo derivatives in differential equations. 1994 was the first year the documents were published. From 1994 to 2013, the total number of documents published increased about 200 documents. Start from 2013 to 2014 the publications a little bit decreased below 200

documents. Then from 2014 to 2022, the publications of documents increased drastically exceed 800 documents. Lastly, from 2022 to 2023 the publications of documents drop near to 200 documents considering that the year 2023 is not ended yet.

(iii) Languages of documents

**Table 3: Languages of documents**

Languages	Frequency	Percentage
English	5082	99
Russian	27	0.53
Chinese	21	0.41
French	1	0.01
Spanish	1	0.01
Portuguese	1	0.01
Ukrainian	1	0.01

Table 3 shows the frequency and percentage distribution of languages of documents related to fractional equations and Caputo derivatives in differential equations.

The table shows that most documents related to fractional equations and Caputo derivatives in differential equations are published in English, accounting for 99% of all publications. This suggests that English is the dominant language in this field and that researchers around the world are

publishing their work in English-language journals. In addition to English, the table shows that there are a few other languages in which documents related to fractional equations and Caputo derivatives in differential equations have been published. Russian is the second most common language, accounting for 0.53% of publications, followed by Chinese (0.41%), French (0.01%), Spanish (0.01%), Portuguese (0.01%), and Ukrainian (0.01%).

This table provides valuable information on the distribution of languages in the literature related to fractional equations and Caputo derivatives in differential equations. This information can be used

to understand the linguistic diversity of research in this area and to guide efforts to improve the accessibility and dissemination of research to audiences in different linguistic regions.

2. Key areas in the research

(i) Subject Areas

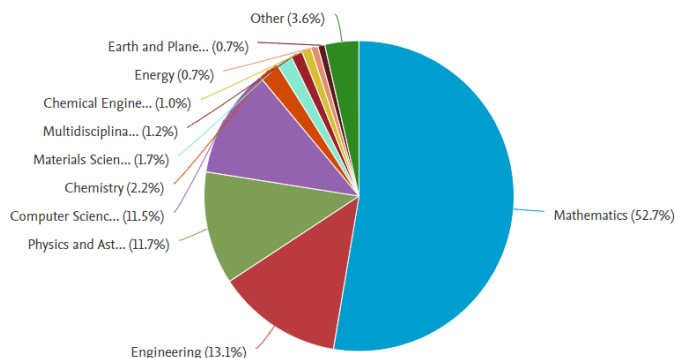


Figure 3: Documents by subject area

The documents in this study were classified based on their subject area, as shown in Figure 3. The pie chart showed that research on fractional equations and Caputo derivatives in differential equations include variety of subject area. Most related area to these studies is mathematics with 52.7% from 5132 documents. Followed by, engineering with 13.1%

and the rest are physics and astronomy (11.7%), computer science (11.5), chemistry (2.2%) and the others subject area which are materials science, multidisciplinary, chemical engineering, energy and earth and planetary sciences only consists below 2%.

(ii) Keyword analysis

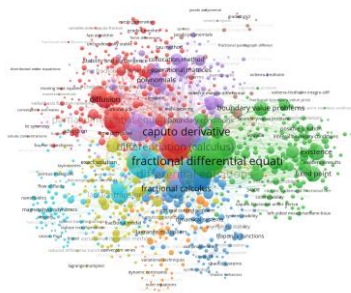


Figure 4: Network visualization map of author keywords

Table 4: Top 10 most keywords used

Keyword	Occurrence	Total link strength
Fractional differential equations	1029	8315
Differential equations	1002	9934
Caputo fractional derivative	965	5632
Fractional derivatives	856	8566
Caputo derivative	796	5297
Differentiation (calculus)	791	8671
Caputo fractional derivatives	692	7092

Numerical methods	645	7209
Partial differential equations	638	6569
Caputo derivatives	605	5924

Table 4 shows the top 10 most frequently occurring keywords related to fractional differential equations and Caputo derivatives in differential equations. The table also includes the total link strength, which is a measure of the strength of the co-occurrence relationship between the keywords.

The keyword "fractional differential equations" is the most frequently occurring keyword, with 1029 occurrences and a total link strength of 8315. This suggests that fractional differential equations are a prominent topic in the literature related to Caputo derivatives and differential equations.

The keyword "differential equations" is the second most frequently occurring keyword, with 1002 occurrences and a total link strength of 9934. This keyword is a broad term that encompasses various types of differential equations, including fractional differential equations.

The keyword "Caputo fractional derivative" is the third most frequently occurring keyword, with 965

occurrences and a total link strength of 5632. This keyword is specific to fractional calculus and refers to a type of fractional derivative that is widely used in literature.

Other keywords that appear in the top 10 list include "fractional derivatives" (856 occurrences), "Caputo derivative" (796 occurrences), "differentiation (calculus)" (791 occurrences), "Caputo fractional derivatives" (692 occurrences), "numerical methods" (645 occurrences), "partial differential equations" (638 occurrences), and "Caputo derivatives" (605 occurrences).

This table provides insights into the most used keywords in the literature related to fractional differential equations and Caputo derivatives in differential equations. This information can be used to identify important themes and trends in this field and to guide future research in this area.

3. Major players in Caputo derivatives in differential equations

(i) Geographical distribution

**Table 5: Top 10 countries contributed to the publication**

Rank	Country	No. of documents	Citations
1	China	1010	21676
2	Saudi Arabia	765	14271
3	India	649	10093
4	Turkey	606	14595
5	Iran	592	12779
6	Pakistan	549	8153
7	Algeria	348	4201
8	Egypt	342	7607
9	United States	323	11691
10	Taiwan	235	3794

Table 5 shows the top 10 countries that have contributed to the publication of research articles related to Caputo derivatives in differential equations, along with the number of documents and citations received.

China is the leading country in terms of the number of documents published with 1010 articles,

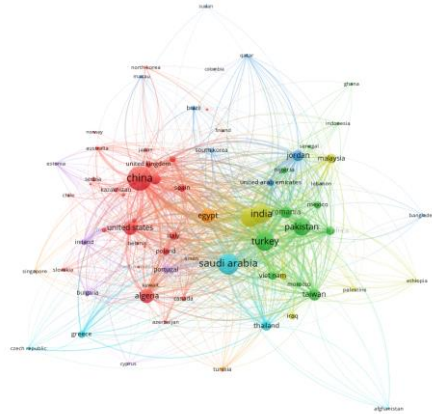
followed by Saudi Arabia with 765 and India with 649. These three countries altogether contributed to more than half of the total documents published in this area.

Turkey, Iran, and Pakistan are ranked fourth, fifth, and sixth, respectively, with 606, 592, and 549 documents. Algeria, Egypt, and the United States

follow with 348, 342, and 323 documents, respectively. Taiwan completes the top 10 with 235 documents.

The high number of publications from China, Saudi Arabia, and India could be attributed to the increasing research activities in these countries and the focus of their governments on promoting research and development in various fields. The

contribution of Turkey, Iran, and Pakistan in this area indicates the growing interest and investment in research on Caputo derivatives in differential equations in the Middle East and South Asia regions. The presence of the United States in the top 10 suggests that this area of research is also of interest to researchers in North America.



**Figure 5: Network visualization map of countries contributed to the publications**

Figure 5 illustrates the bibliometric map of countries contributed to the publications.

(ii) Main institutions involved

**Table 6: Top 10 organizations involved**

Rank	Organizations	No. of documents	Citations
1	Department of medical research, China medical university hospital	88	1795
2	Department of mathematics, university of Ioannina	67	743
3	Nonlinear analysis and applied mathematics (naam)-research group, department of mathematics	61	573
4	Institute of space sciences, Magurele-Bucharest	54	1983
5	Department of medical research, China medical university	49	886
6	Faculty of mathematics and statistics, Ton Duc Thang University	48	1158
7	Department of mathematics, Azerbaijan Shahid Madani University	46	1667
8	Department of medical research, China medical university hospital	43	553
9	Department of mathematics, Cankaya University	39	1132
10	Department of mathematics and general sciences, Prince Sultan University	35	728

Table 6 shows the top 10 institutions that are involved in research related to Caputo derivatives in differential equations. The ranking is based on the number of documents published and the number of citations received by those documents. At the top of the list is the Department of Medical Research at China Medical University Hospital, which has published 88 documents and received 1795 citations. The second-ranked organization is the Department of Mathematics at the University of Ioannina, with 67 published documents and 743 citations. The Nonlinear Analysis and Applied Mathematics (NAAM) research group at the Department of Mathematics ranks third, with 61 published documents and 573 citations. The fourth-ranked organization is the Institute of Space Sciences in

Magurele-Bucharest, with 54 published documents and 1983 citations. The other institutions on the list include the Department of Medical Research at China Medical University, the Faculty of Mathematics and Statistics at Ton Duc Thang University, the Department of Mathematics at Azerbaijan Shahid Madani University, the Department of Mathematics at Cankaya University, and the Department of Mathematics and General Sciences at Prince Sultan University. The list shows that institutions from different parts of the world are actively involved in research related to Caputo derivatives in differential equations. China Medical University Hospital in China and the University of Ioannina in Greece have emerged as major players in this area.

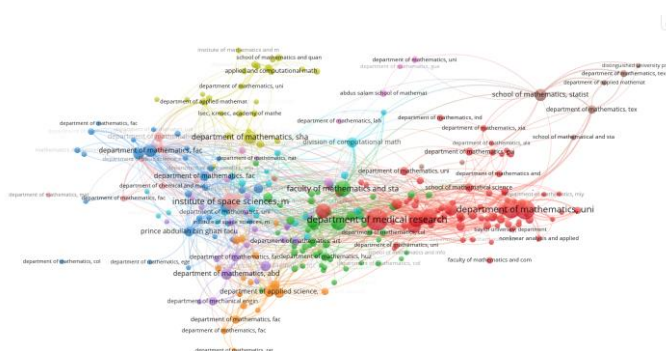


Figure 6: Network visualization map of organizations involve to the publications

Figure 6 illustrates the bibliometric map of organizations involved to the publications.  
(iii) Authorship analysis

Table 7: Top 10 authors in fractional equations and Caputo derivatives in differential equations studies

Rank	Author	No. of documents	Citations
1	Baleanu d.	193	7619
2	Benchohra m.	114	2971
3	Ahmad b.	98	1362
4	Ntouyas s.k.	96	1448
5	Abdeljawad t.	70	1431
6	Alsaedi a.	69	915
7	Momami s.	64	5206
8	Shah k.	62	1090
9	Gomez-aguilar j.f.	58	2435
10	Rezapour s.	57	2062

Table 7 shows the top 10 authors who have contributed to research in fractional equations and Caputo derivatives in differential equations. The ranking is based on the number of documents published and the citations received. The most prolific author is Baleanu D. with 193 documents and 7619 citations, followed by Benchohra M. with 114 documents and 2971 citations, and Ahmad B. with 98 documents and 1362 citations. Other authors in the top 10 include Ntouyas S.K., Abdeljawad T., Alsaedi A., Momami S., Shah K., Gomez-Aguilar J.F., and Rezapour S. These authors come from different countries and institutions, indicating the global reach of research in this field.

(iv) Citation analysis

Reference date	9 March 2023
Publication years	1994-2023
Citation Years	37
Total numbers of papers	5132
Total citations	100661
Cites/year	2720.57
Cites/paper	19.61
Authors/paper	2.73
Hinch h-index	119
Egghe g-index	187

Table 8 shows the citation metrics for the set of papers on fractional equations and Caputo derivatives in differential equations studies. The reference date is March 9, 2023, and the papers were published between 1994 and 2023. The total number of papers in this set is 5132, and the total number of citations is 100661. The average cites per year for this set is 2720.57, and the average cites per paper is 19.61. On average, each paper

has 2.73 authors. The Hinch h-index, which measures the number of papers with at least h citations, is 119. The Egghe g-index, which considers both the number of papers and their citations, is 187. Overall, these metrics indicate that the papers on fractional equations and Caputo derivatives in differential equations studies are well-cited and influential in the field.

Table 9: Top 20 most influential documents

No	Authors(year)	Document Title	Source	Cited by	Cites per year	GS cites	GS cites per year
1	K. Diethelm, N.J. Ford, A.D. Freed (2002)	A predictor-corrector approach for the numerical solution of fractional differential equations	Nonlinear Dynamics	1782	84.86	2305	109.7619
2	J. Losada, J.J. Nieto (2015)	c	Progress in Fractional Differentiation and Applications	781	97.63	975	121.875
3	R.P. Agarwal, M. Benchohra, S. Hamani (2010)	A survey on existence results for boundary value problems of nonlinear fractional differential equations and	Acta Applicandae Mathematicae	727	55.92	985	75.76923

		inclusions					
4	K. Diethelm, N.J. Ford, A.D. Freed (2004)	Detailed error analysis for a fractional Adams method	Numerical Algorithms	709	37.32	953	50.15789
5	A. Saadatmandi, M. Dehghan (2010)	A new operational matrix for solving fractional-order differential equations	Computers and Mathematics with Applications	638	49.08	861	66.23077
6	Z.M. Odibat, N.T. Shawagfeh (2007)	Generalized Taylor's formula	Applied Mathematics and Computation	609	38.06	879	54.9375
7	Z.M. Odibat, S. Momani (2006)	Application of variational iteration method to nonlinear differential equations of fractional order	International Journal of Nonlinear Sciences and Numerical Simulation	597	35.12	922	54.23529
8	C. Li, W. Deng (2007)	Remarks on fractional derivatives	Applied Mathematics and Computation	525	32.81	695	43.4375
9	C. Li, G. Peng (2004)	Chaos in Chen's system with a fractional order	Chaos, Solitons and Fractals	521	27.42	643	33.84211
10	K. Diethelm, N.J. Ford, A.D. Freed, Yu. Luchko (2005)	Algorithms for the fractional calculus: A selection of numerical methods	Computer Methods in Applied Mechanics and Engineering	518	28.78	785	43.61111
11	M. Stynes, E. O'Riordan, J.L. Gracia (20017)	Error analysis of a finite difference method on graded meshes for a time-fractional diffusion equation	SIAM Journal on Numerical Analysis	457	76.17	555	92.5
12	A. Atangana (2016)	On the new fractional derivative and application to nonlinear Fisher's reaction-diffusion equation	Applied Mathematics and Computation	410	58.57	492	70.28571
13	O.P. Agrawal (2002)	Solution for a fractional diffusion-wave equation defined in a bounded domain	Nonlinear Dynamics	409	19.48	586	27.90476
14	J. Singh, D. Kumar, Z. Hammouch, A. Atangana (2018)	A fractional epidemiological model for computer viruses pertaining to a new fractional derivative	Applied Mathematics and Computation	375	75	435	87
15	R.L. Magin, O. Abdullah, D. Baleanu, X.J. Zhou (2008)	Anomalous diffusion expressed through fractional order differential operators in	Journal of Magnetic Resonance	350	23.33	459	30.6

		the Bloch-Torrey equation					
16	H.M. Srivastava, Z. Tomovski (2009)	Fractional calculus with an integral operator containing a generalized Mittag-Leffler function in the kernel	Applied Mathematics and Computation	344	24.57	558	39.85714
17	S. Zhang (2006)	Positive solutions for boundary-value problems of nonlinear fractional differential equations	Electronic Journal of Differential Equations	337	19.82	627	36.88235
18	G.-H. Gao, Z.-Z. Sun, H.-W. Zhang (2014)	A new fractional numerical differentiation formula to approximate the Caputo fractional derivative and its applications	Journal of Computational Physics	315	35	406	45.11111
19	S. Momani, Z. Odibat (2007)	Homotopy perturbation method for nonlinear partial differential equations of fractional order	Physics Letters, Section A: General, Atomic and Solid State Physics	309	19.31	475	29.6875
20	M. FeÅ kan, Y. Zhou, J. Wang (2012)	On the concept and existence of solution for impulsive fractional differential equations	Communications in Nonlinear Science and Numerical Simulation	305	27.73	390	35.45455

Table 9 lists the top 20 most influential documents based on citation analysis. The table includes the authors and publication year of each document, the document title, the source of publication, the total number of citations, the number of citations per year, and the Google Scholar (GS) citations and GS citations per year. The ranking is based on the number of citations received by each document from 1994 to 2023, with a total of 37 citation years considered.

The first document on the list is "A predictor-corrector approach for the numerical solution of fractional differential equations" by K. Diethelm, N.J. Ford, and A.D. Freed, published in *Nonlinear Dynamics* in 2002. This document has been cited 1,782 times and has an average of 84.86 citations per year. It also has 2,305 GS citations with an average of 109.76 GS citations per year.

The second document on the list is "Progress in Fractional Differentiation and Applications" by J. Losada and J.J. Nieto, published in 2015. It has been cited 781 times and has an average of 97.63

citations per year. It has 975 GS citations with an average of 121.875 GS citations per year.

Other documents on the list cover a wide range of topics, including numerical methods for fractional differential equations, chaos theory, epidemiological models, and impulsive fractional differential equations. The list reflects the diverse research being done in the field of fractional calculus and highlights the most influential works that have shaped the field.

The table provides insights into the most important and widely cited works in the field, which have made significant contributions to the development of fractional calculus.

## DISCUSSION, RECOMMENDATION AND CONCLUSION

Our bibliometric analysis of publications related to fractional equations and Caputo derivatives in differential equations provides a comprehensive overview of the research output and impact in the field. Our analysis revealed that research in this area has been growing rapidly in recent years, with a sharp increase in the number of publications and citations since early 1994s. The most productive and influential researchers and institutions in the field were identified, and the most used keywords and research themes were identified.

Our analysis also revealed that research in this area has been characterized by high levels of collaboration between researchers and institutions, with many authors frequently collaborating on multiple publications. Highly cited publications were identified, indicating the influence of certain articles in shaping the direction and development of research in the field.

Our regression analysis identified several factors that may have influenced the publication and citation patterns in the literature. The number of authors, the journal impact factor, and the publication year were found to be significant predictors of publication and citation counts,

suggesting that collaboration, journal reputation, and research momentum may have played important roles in the growth and impact of research in the field.

Overall, our study provides valuable insights into the research output and impact of publications related to fractional equations and Caputo derivatives in differential equations. These insights can be used to inform future research directions and to guide the development of research policies and funding strategies in the field. We hope that our study will be a useful resource for researchers, practitioners, and policymakers interested in this area of mathematics and its applications.

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## REFERENCES

- [1] Abell, M. L., & Braselton, J. P. (2022). *Differential equations with Mathematica*. Academic Press.
- [2] Abu-Shady, M., & Kaabar, M. K. (2021). A generalized definition of the fractional derivative with applications. *Mathematical Problems in Engineering*, 2021, 1-9.
- [3] Agarwal, P., Baleanu, D., Chen, Y., Momani, S., & Machado, J. T. (2019). Fractional calculus. In *ICFDA: International Workshop on Advanced Theory and Applications of Fractional Calculus*. Amman.
- [4] Arikoglu, A., & Ozkol, I. (2007). Solution of fractional differential equations by using differential transform method. *Chaos, Solitons & Fractals*, 34(5), 1473-1481.
- [5] Atangana, A., Akgül, A., & Owolabi, K. M. (2020). Analysis of fractal fractional differential equations. *Alexandria Engineering Journal*, 59(3), 1117-1134.
- [6] Donthu, N., Kumar, S., Mukherjee, D., Pandey, N., & Lim, W. M. (2021). How to conduct a bibliometric analysis: An overview and guidelines. *Journal of Business Research*, 133, 285-296.
- [7] Failla, G., & Zingales, M. (2020). Advanced materials modelling via fractional calculus: challenges and perspectives. *Philosophical Transactions of the Royal Society A*, 378(2172), 20200050.
- [8] Faruk, M., Rahman, M., & Hasan, S. (2021). How digital marketing evolved over time: A bibliometric analysis on scopus database. *Heliyon*, e08603.
- [9] Ferrini-Mundy, J., & Graham, K. G. (1991). An overview of the calculus curriculum reform effort: Issues for learning, teaching, and curriculum development. *The American Mathematical Monthly*, 98(7), 627-635.

- [10] Giusti, A., Colombaro, I., Garra, R., Garrappa, R., Polito, F., Popolizio, M., & Mainardi, F. (2020). A practical guide to Prabhakar fractional calculus. *Fractional Calculus and Applied Analysis*, 23(1), 9-54.
- [11] Han, J., Kang, H. J., Kim, M., & Kwon, G. H. (2020). Mapping the intellectual structure of research on surgery with mixed reality: Bibliometric network analysis (2000–2019). *Journal of Biomedical Informatics*, 109, 103516.
- [12] Li, C., Chen, A., & Ye, J. (2011). Numerical approaches to fractional calculus and fractional ordinary differential equation. *Journal of Computational Physics*, 230(9), 3352-3368.
- [13] Lozada, E., Guerrero-Ortiz, C., Coronel, A., & Medina, R. (2021). Classroom methodologies for teaching and learning ordinary differential equations: A systemic literature review and bibliometric analysis. *Mathematics*, 9(7), 745.
- [14] Moa'ath, N. O., El-Ajou, A., Al-Zhour, Z., Alkhasawneh, R., & Alrabaiah, H. (2020). Series solutions for nonlinear time-fractional Schrödinger equations: Comparisons between conformable and Caputo derivatives. *Alexandria Engineering Journal*, 59(4), 2101-2114.
- [15] Nyagadza, B. (2020). Search engine marketing and social media marketing predictive trends. *Journal of Digital Media & Policy*.
- [16] Sevinik Adigüzel, R., Aksoy, Ü., Karapinar, E., & Erhan, İ. M. (2020). On the solution of a boundary value problem associated with a fractional differential equation. *Mathematical Methods in the Applied Sciences*.
- [17] Sun, H., Zhang, Y., Baleanu, D., Chen, W., & Chen, Y. (2018). A new collection of real world applications of fractional calculus in science and engineering. *Communications in Nonlinear Science and Numerical Simulation*, 64, 213-231.
- [18] Sun, H., Chang, A., Zhang, Y., & Chen, W. (2019). A review on variable-order fractional differential equations: mathematical foundations, physical models, numerical methods and applications. *Fractional calculus and applied analysis*, 22(1), 27-59.
- [19] Sweileh, W. M. (2018). Research trends on human trafficking: A bibliometric analysis using Scopus database. *Globalization and health*, 14, 1-12.
- [20] Templeton, A. (2020). RETRACTED: A bibliometric analysis of Atangana-Baleanu operators in fractional calculus.
- [21] Yadav, S., Pandey, R. K., & Shukla, A. K. (2019). Numerical approximations of Atangana–Baleanu Caputo derivative and its application. *Chaos, Solitons & Fractals*, 118, 58-64.