Forcing *S*-Geodetic Number of A FuzzyGraph

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***Abstract*—** Chartrand and Zhang in 1999 introduced the idea of forcing geode-tic number of crisp graphs and determined it for several classes of graphs. In this paper, the concept is extended to fuzzy graphs using sum distance and is called the forcing s-geodetic number. A characterization of the forcing s-geodetic number depending on the s-geodetic bases present in the fuzzy graph is identified. The forcing s-geodetic number of fuzzy trees, complete fuzzy graphs and of fuzzy cycles subject to certain conditions are identified.

Index Terms— s-geodetic set, s-geodetic basis, s-geodetic number, forcing sub- set, forcing s-geodetic number

I. Introduction

The concept of fuzzy sets was brought into existence by Zadeh in 1965 [29] which gave a platform for describing the uncertainties existing in day-to-day life situations. The theory of fuzzy graphs was later on developed by Rosenfeld in the year 1975 [21] along with Yeh and Bang [28]. The fuzzy analogue of several graph theoretic concepts like paths, cycles, trees and connectedness was also obtained by Rosenfeld [21] and the concept of fuzzy trees [18], auto-morphism of fuzzy graphs [2], fuzzy interval graphs [15], cycles and cocycles of fuzzy graphs [16] etc has been established by several authors during the course of time.

Fuzzy groups and the notion of a metric in fuzzy graphs were introduced by Bhattacharya [1]. The concept of strong arcs [5] and geodesic distance infuzzy graphs [4] were introduced by Bhutani and Rosenfeld in the year 2003. The definition of fuzzy end nodes and some of their properties were established by the same authors in [3]. Several other important works on fuzzy graphs can be found in [19, 13, 26]. Studies in fuzzy graphs using μ-distance was carried out by Rosenfeld [22] in 1975 and was further studied by Sunitha and Vijayakumar in [26]. In crisp graph, the concept of geodetic iterationnumber was first introduced by

Harary and Nieminen in 1981 [11]. This con- cept along with that of geodetic numbers in graphs was .

again discussed by several authors in [6], [9] and [8]. Later on, these concepts were extended to fuzzy graphs using geodesic distance by Suvarna and Sunitha in [27] and the same based on μ-distance was introduced by Linda and Sunitha in [12]. The concept of sum distance and some of its metric aspects was introduced by Mini Tom and Sunitha in [14]. Based on sum distance, *s*-geodetic number ofa fuzzy graph was introduced in [20] and certain properties satisfied by them were identified by Sameeha and Sunitha in 2020. In this paper, the concept of forcing *s*-geodetic number of a fuzzy graph isintroduced using sum distance. A characterization of the forcing *s*-geodetic number of a fuzzy graph depending on the *s*-geodetic bases present is identified. The forcing *s*-geodetic number of fuzzy trees, of complete fuzzy graphsand of fuzzy cycles subject to certain conditions is obtained and a necessary condition for the forcing *s*-geodetic number of a fuzzy graph to be less than 2 is identified

II. Preliminaries

A **fuzzy graph** [17] is a triplet $G : (V, \sigma, \mu)$ where V is a vertex set, σ a fuzzy subset of V and μ a fuzzy relation on σ such that $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$, $\forall u, v \in V$. We assume that V is finite and non-empty, μ is reflexive (*i.e.,* $\mu(x, x) = \sigma(x), \forall x$) and symmetric (*i.e.,* $\mu(x, y) = \mu(y, x), \forall (x, y)$). Also we denote the underlying crisp graph [10] by $G^* : (\sigma^*, \mu^*)$ where $\sigma^* = \{ u \in V / \sigma(u) > 0\}$ and $\mu^* = \{(u, v) \in VXV / \mu(u, v) > 0\}$. Here we assume $\sigma^* = V$.

A fuzzy graph $H : (V, \tau, \nu)$ is called a **partial fuzzy sub graph** [17] of $G : (V, \sigma, \mu)$ if τ

$(u) \leq \sigma(u)$ for every $u \in \tau^*$ and $\nu(u, v) \leq \mu(u, v)$ $\forall (u, v) \in \nu^*$.

In particular, we call $H:(V, \tau, \nu)$ a **fuzzy sub graph** of $G : (V, \sigma, \mu)$ if $\tau(u) = \sigma(u)$, $\forall u \in \tau^*$ and $\nu(u, v) = \mu(u, v)$, $\forall (u, v) \in \nu^*$ and if in addition $\tau^* = \sigma^*$, then H is called a spanning fuzzy sub graph of G.

A fuzzy graph $H : (P, \tau, \nu)$ is called a fuzzy sub graph of $G : (V, \sigma, \mu)$ induced by P if $P \subseteq V$, $\tau(u) = \sigma(u)$ for all u in P and $\nu(u, v) = \mu(u, v)$ for all u, v in P. A fuzzy graph $G : (V, \sigma, \mu)$ is called **trivial** if $\sigma^* = 1$. Otherwise it is called **non-trivial**.

A fuzzy graph $G : (V, \sigma, \mu)$ is a **complete fuzzy graph** [17] if $\mu(u, v) = \sigma(u) \wedge \sigma(v)$ $\forall u, v \in \sigma^*$.

A **weakest arc** of $G:(V, \sigma, \mu)$ is an arc with least non zero membership value. A **path** P of length n is a sequence of distinct nodes $u_0, u_1, ..., u_n$ such that $\mu(u_{i-1}, u_i) > 0$, $i = 1, 2, 3, ..., n$ and the degree of membership of a weakest arc in the path is defined as its **strength**. The path becomes a **cycle** if $u_0 = u_n$, $n \geq 3$ and a cycle is called a **fuzzy cycle** [18] if it contains more than one weakest arc. The **strength of connectedness** between two nodes u and v is defined as the maximum of the strengths of all paths between u and v, and is denoted by $CONN_G(u, v)$. A fuzzy graph $G : (V, \sigma, \mu)$ is **connected** if for every u, v in σ^*, $CONN_G(u, v)>0$.

An arc (u, v) of a fuzzy graph is called **strong** if its weight is at least as great as the strength of connectedness of its end nodes u, v when the arc (u, v) is deleted and a u-v path P is called a **strong path** if P contains only strong arcs [5].

Two nodes u and v in a fuzzy graph $G : (V, \sigma, \mu)$ are **neighbors** if $\mu(u, v) > 0$ and v is called a **strong neighbor** of u if the arc (u, v) is strong. Also $N(u)$ denotes the set of neighbors of u other than u and degreeof u is $deg(u) = N(u)$. A node u with $deg(u) = 1$ is an **end node** and a node u with $deg(u) > 1$ is an **internal node**. A node v is called a **fuzzy end node** of G if it has exactly one strong neighbor in G [3].

A connected fuzzy graph $G : (V, \sigma, \mu)$ is called a **fuzzy tree** [21] if it hasa spanning fuzzy sub graph $F : (V, \sigma, \nu)$ which is a tree such that for all arcs(u, v) not in F, $CONN_F(u, v) > \mu(u, v)$.

For any path P: $u_0 - u_1 - u_2 - ... - u_n$, **length** of P, $L(P)$, is defined as the sum of the weights of the arcs in P. That is, $L(P) = \Sigma^n \mu(u_{i-1}, u_i)$.

If $n = 0$, define $L(P) = 0$ and for $n \geq 1$, $L(P) > 0$.

For any two nodes u, v in $G : (V, \sigma, \mu)$, if $P = \{P_i : P_i$ is a u-v path, i =1, 2, 3, ...}, then the **sum distance** between u and v is defined as $d_s(u, v) = Min\{L(P_i) : P_i \in P, i = 1, 2, 3, ...\}$[14].

The **eccentricity** $e_s(u)$ of a node u in the connected fuzzy graph $G : (V, \sigma, \mu)$ is the sum distance to a node farthest from u.

i.e, $e_s(u) = max\{d_s(u, v) : v \in V\}$. The **radius** $r_s(G)$ is the minimum eccentricity of the nodes, whereas the **diameter** $d_s(G)$ is the maximum eccentricity.

A node u is an s-**peripheral node** if $e_s(u) = d_s(G)$.

A **diametral path** of a fuzzy graph is a shortest path whose length is equal to the diameter of the fuzzy graph.

Throughout this paper we consider only connected fuzzy graphs.

Any path P from x to y whose length is $d_s(x,y)$ is called s-**geodesic** from x to y [14].

Let $S \subseteq V$ be a set of nodes of a connected fuzzy graph $G : (V, \sigma, \mu)$. Then the s-**geodetic closure** of S, with respect to sum distance, is the set of all nodes of S as well as all nodes that lie on s-geodesics between nodes of S and is denoted by (S) [14].

A set $S \subseteq V(G)$ such that every node of G is contained in an s-geodesic joining some pair of nodes in S is called an s-**geodetic cover (**s-**geodetic set)** of G. In other words if $(S) = V(G)$, then S is an s-geodetic cover of G [14].

The s-**geodetic number** of G, denoted by s-$gn(G)$, is the minimum order of its s-geodetic covers and any cover of order s-$gn(G)$ is an s-**geodetic basis** [14].

Example 2.1. Consider the fuzzy graph G given in Fig.1.

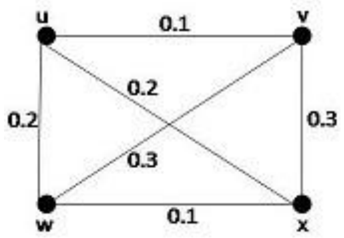
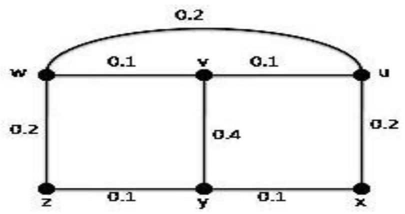


Fig.1

If $S = \{v, x, w\}$ then $(S) = \{u, v, x, w\} = V(G)$. Therefore S is an s-geodetic cover of G. The set $S = \{v, x, w\}$ is the unique s-geodetic basis of G and so $s\text{-}gn(G) = 3$.

The following results have been taken from [20].

Proposition 2.2. For any non-trivial connected fuzzy graph G on n nodes, $2 \leq s\text{-}gn(G) \leq n$.



Proposition 2.3. For a complete fuzzy graph G on 2 nodes, $s\text{-}gn(G) = 2$.

Proposition 2.4. Let $G : (V, \sigma, \mu)$ be a connected fuzzy graph on n nodes. Then the s-geodetic number, $s\text{-}gn(G) = n$ if and only if each pair of nodes in G is joined by an arc which is the unique s-geodesic between them.

Proposition 2.5. Let $G : (V, \sigma, \mu)$ be a fuzzy tree such that G^* is a tree. Then the set of all fuzzy end nodes of G form an s-geodetic basis for G and $s\text{-}gn(G)$ is the number of fuzzy end nodes of G.

Proposition 2.6. For a fuzzy cycle G on n nodes, each of whose arcs are having same strength, the s-geodetic number

$$s\text{-}gn(G) = \begin{cases} 2, & \text{when n is even} \\ 3, & \text{when n is odd} \end{cases}$$

Proposition 2.7. For any connected fuzzy graph G, $s\text{-}gn(G) = 2$ if and only if there exists s-peripheral nodes u and v such that every node of G lieson an s-geodesic joining u and v.

Also let P: $u = u_0, u_1, u_2, \ldots, u_n = v$ be an s-geodesic joining u and v. Then

$d_s(u, v) = d_s(u_0, u_1) + d_s(u_1, u_2) + \ldots + d_s(u_{n-1}, u_n)$.

III. FORCING S-GEODETIC NUMBER OF A FUZZY GRAPH

The concept of forcing geodetic number of crisp graphs was introduced by Gary Chartrand and Ping Zhang in 1999 [7]. In this section, we extend this concept to fuzzy graphs using sum distance.

Definition 3.1. For an s-geodetic basis S of a fuzzy graph $G : (V, \sigma, \mu)$, a subset T of S with the property that S is the unique s-geodetic basis containing T is called a forcing subset of S.

The minimum cardinality of a forcing subset for S is called the forcing s - geodetic number of S in G and is denoted by $s\text{-}gn_f(S)$. The forcing s-geodetic number of G, denoted by $s\text{-}gn_f(G)$, is defined as $s\text{-}gn_f(G) = min\{s\text{-}gn_f(S)\}$ where the minimum is taken over all s-geodetic bases S in G.

Example 3.2. Consider the fuzzy graph G given in Fig.2.

Fig.2

The s-geodetic bases of G are $S_1 = \{w, x\}$, $S_2 = \{u, z\}$ and $S_3 = \{v, y\}$.

Since S_1 is the only s-geodetic basis containing w, it follows that $s\text{-}gn_f(S_1) = 1$. By a similar argument,

$s\text{-}gn_f(S_2) = 1 = s\text{-}gn_f(S_3)$.

Hence $s\text{-}gn_f(G) = 1$.

Proposition 3.3. Let $G : (V, \sigma, \mu)$ be a connected fuzzy graph. Then,

1. $s\text{-}gn_f(G) = 0$ if and only if G has a unique s-geodetic basis.
2. $s\text{-}gn_f(G) = 1$ if and only if G has at least 2 s-geodetic bases, one of which is a unique s-geodetic basis containing one of the elements.
3. $s\text{-}gn_f(G) = s\text{-}gn(G)$ if and only if no s-geodetic basis of G is the unique s-geodetic basis containing any of its proper subsets.

Proof. 1. Let $s\text{-}gn_f(G) = 0$. Then, by definition 3.1, there exists some s-geodetic basis S of G such that $s\text{-}gn_f(S) = 0$ so that the empty set φ is the forcing subset for S of minimum cardinality.

Since the empty set φ is a subset of every set, it follows that S is the unique s-geodetic basis of G.
The converse is clearly true.

2. Let $s\text{-}gn_f(G) = 1$. Then by (1), G has at least two s-geodetic basis. Now since $s\text{-}gn_f(G) = 1$, there is a singleton subset T of an s-geodeticbasis S of G such that T is not a subset of any other s-geodetic basis of G. Thus S is the unique s-geodetic basis containing one of its elements.The converse is obvious.

3. Let $s\text{-}gn_f(G) = s\text{-}gn(G)$.
Then $s\text{-}gn_f(S) = s\text{-}gn(G)$ for every s-geodetic basis S in G. Also, by Proposition 2.2, since $s\text{-}gn(G) \geq 2$, we get $s\text{-}gn_f(G) \geq 2$. Then by (1), G has at least two s-geodetic bases and so the empty set φ is not a forcing subset for any s-geodetic basis of G.
Also since $s\text{-}gn_f(G) = s\text{-}gn(G)$, no proper subset of S is a forcing subset of S.
Thus no s-geodetic basis of G is the unique s-geodetic basis containing any of its proper subsets.
Conversely, the assumption implies that G contains more than one s- geodetic basis, and no subset of any s-geodetic basis S other than S is a forcing subset for S.
Therefore, $s\text{-}gn_f(G) = |S| = s\text{-}gn(G)$.

Corollary 3.4. *For a fuzzy graph G, the forcing s-geodetic number* $s\text{-}gn_f(G) \geq 2$ *if and only if every node of each s-geodetic basis belongs to at least two s-geodetic bases.*

Proposition 3.5. The forcing s-geodetic number of a fuzzy tree $G : (V, \sigma, \mu)$ such that G^* is a tree, is 0.

Proof. By Proposition 2.5, the fuzzy tree G has a unique s-geodetic basis consisting of its fuzzy end nodes. So by Proposition 3.3 (1), $s\text{-}gn_f(G) = 0$.

Proposition 3.6. The forcing s-geodetic number of a complete fuzzy graph $G : (V, \sigma, \mu)$ on 2 nodes is 0.

Proof. It follows from Proposition 2.3 that the s-geodetic number of a complete fuzzy graph G on 2 nodes is 2. Therefore $V(G)$ is the unique s-geodetic basis of G and so by Part(1) of Proposition 3.3, we get $s\text{-}gn_f(G) = 0$.

Proposition 3.7. The forcing s-geodetic number of a connected fuzzy graph $G : (V, \sigma, \mu)$ on n nodes in which each pair of nodes in G is joined by an arc which is the unique s-geodesic between them, is 0.

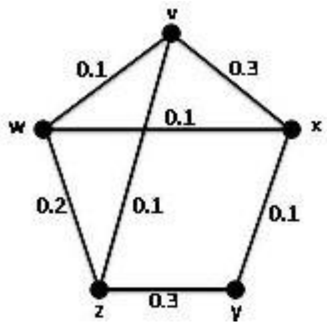
Proof. It follows from Proposition 2.4 that the s-geodetic number of a connected fuzzy graph G on n nodes in which each pair of nodes in G is joined byan arc which is the unique s-geodesic between them, is n. Therefore $V(G)$ is the unique s-geodetic basis of G and so by Proposition 3.3 (1), we get $s\text{-}gn_f(G) = 0$.

Proposition 3.8. For a fuzzy cycle $G : (V, \sigma, \mu)$ on n nodes, each of whose arcs are having same strength, the forcing s-geodetic number

$$s\text{-}gn_f(G) = \begin{cases} 1, \text{when n is even} \\ 2, \text{when n is odd} \end{cases}.$$

Proof. If n is even, then by Proposition 2.6, $s\text{-}gn(G) = 2$. Hence by Proposition 2.7, every s-geodetic basis of G consists of a pair of s-peripheral nodes. s-geodetic bases and it is clear that each singleton set is the minimum forcing subset for exactly one s-geodetic basis of G. Hence it follows from Proposition 3.3 (2) that the forcing s-geodetic number $s\text{-}gn_f(G) = 1$. If n is odd, then let $n = 2k + 1$, where k is an integer. Let the fuzzy cycle be $C : v_1, v_2, \ldots, v_k, v_{k+1}, v_{k+2}, \ldots, v_{2k+1}, v_1$.
If $S = \{u, v\}$ is any set of two nodes of $V(G)$, then no nodes of the u - v longest path other than u and v lies on the u - v s-geodesic in G and so no two element subset of G is an s-geodetic cover of G. Now, since n is odd, it is clear from Proposition 2.6 that $s\text{-}gn(G) = 3$ and so the sets $S_1 = \{v_1, v_{k+1}, v_{k+2}\}$, $S_2 = \{v_2, v_{k+2}, v_{k+3}\}, \ldots, S_{k+2} = \{v_{k+2}, v_1, v_2\}, \ldots, S_{2k+1} = \{v_{2k+1}, v_k, v_{k+1}\}$ are some of the s-geodetic bases of G. It is clear from these s-geodetic bases S_i $(1 \leq i \leq 2k + 1)$ that each set $\{v_i\}$ $(1 \leq i \leq 2k + 1)$ isa subset of more than one s-geodetic basis S_i. Hence by Corollary 3.4, we get $s - gn_f(G) \geq 2$. But since v_{k+1} and v_{k+2} are s-eccentric nodes of v_1, itis clear that S_1 is the unique s-geodetic basis containing $\{v_{k+1}, v_{k+2}\}$ and so $s\text{-}gn_f(G) = 2$.

Proposition 3.9. If $G : (V, \sigma, \mu)$ is a connected fuzzy graph with s-geodetic number $s\text{-}gn(G) = 2$, then its forcing s-geodetic number $s\text{-}gn_f$



(*G*) < 2.

Proof. Let s-$gn(G) = 2$. Then there exists a set $S = \{u, v\}$ which is an s-geodetic basis of G. Also then by Proposition 2.7, $d_s(u, v) = d_s(G)$, the diameter of G, and every node of G lies on some $u - v$ s-geodesic in G. Now we have to prove that s-$gn_f(G) < 2$. Suppose on the contrary that s-$gn_f(G) = 2$. Then it follows from Proposition 3.3(3) that S is not the unique s-geodetic basis containing the node u. Hence there exists some node $x \neq v$ such that $S = \{u, x\}$ is also an s-geodetic basis of G. Therefore we get $d_s(u, x) = d_s(G)$. Now since x lies on some $u - v$ s-geodesic in G, we get $d_s(u, x) < d_s(u, v) = d_s(G)$, which is a contradiction.

Definition 3.10. A node v of a connected fuzzy graph $G : (V, \sigma, \mu)$ is said to be an s-geodetic node of G if v belongs to every s-geodetic basis of G.

Example 3.11. Consider the fuzzy graph G given in Fig.3.

Fig.3 Here $S_1 = \{v, y, z\}$ and $S_2 = \{w, y, z\}$ are the only s-geodetic bases of G so that the nodes y and z are s-geodesic nodes of G.

Proposition 3.12. Let $G : (V, \sigma, \mu)$ be a connected fuzzy graph and W be the set of all s-geodetic nodes of G.
Then s-$gn_f(G) \leq s$-$gn(G) - |W|$.

Proof. Let S be any s-geodetic basis of G. Then s-$gn(G)=|S|$. Clearly $W \subseteq S$ and S is the unique s-geodetic basis containing $S - W$. Thus
s-$gn_f(G) \leq |S - W| \leq |S| - |W| = s$-$gn(G) - |W|$.
i.e, s-$gn_f(G) \leq s$-$gn(G) - |W|$.

IV. Conclusion

In this paper, the concept of forcing s-geodetic number of a fuzzy graph is introduced along with suitable examples. The forcing s-geodetic number of fuzzy graphs is characterized depending on the s-geodetic bases present in the fuzzy graph. The forcing s-geodetic number of fuzzy trees, of complete fuzzy graphs and of fuzzy cycles is identified. A necessary condition for the forcing s-geodetic number of a fuzzy graph to be less than 2 is established.

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