

Fermatean Fuzzy Generalized π Closed Sets in Fermatean Fuzzy Topological Spaces

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Abstract

In this paper, we introduce the notion of Fermatean fuzzy π closed sets and open sets, Fermatean fuzzy generalized π closed sets and open sets in Fermatean fuzzy topological space. Further, we established its properties.

Keywords: Fermatean fuzzy set (FFS), closed Fermatean fuzzy set (CFFS), open Fermatean fuzzy set (OFFS), Fermatean fuzzy generalized closed set (FFGCS), Fermatean fuzzy generalized open set (FFGOS), Fermatean fuzzy regular closed set (FFRCS), Fermatean fuzzy regular open set (FFROS), Fermatean fuzzy generalized regular closed set (FFGRCS) and Fermatean fuzzy generalized regular open set (FFGROS), Fermatean fuzzy π closed set (FF π CS), Fermatean fuzzy π open set (FF π OS), Fermatean fuzzy generalized π closed set (FFG π CS) and Fermatean fuzzy generalized π open set (FFG π OS).

1. Introduction

Fuzzy set was introduced by L.A.Zadeh [16] in 1965. A lot of research work has been done on this subject since 1965. In 1968, C. L. Chang [8] defined the concept of fuzzy topological space and generalized some basic notions of topology such as open set, closed set, continuity and compactness in fuzzy topological spaces. The idea of intuitionistic fuzzy set was first introduced by K. Atanassov [3] and many works by the same author and his colleagues appeared in the literature [4, 5]. B. Boomathi, M. Palanisamy [7] defined generalized regular closed set in Intuitionistic fuzzy topological spaces in 2017. D. Coker [9] subsequently initiated a study of intuitionistic fuzzy topological spaces. S.S. Thakur and Rekha Chaturvedi [14, 15] defined regular generalized closed sets in Intuitionistic fuzzy topological spaces and generalized closed sets regular generalized closed sets in Intuitionistic fuzzy topological spaces in 2006 and in 2008. Fermatean fuzzy set was proposed by T. Senapati

and R. R. Yager [13] in 2020, and handles uncertain information more easily in the process of decision making. Hariwan Z. Ibrahim [11] introduced the concept of Fermatean fuzzy topological space and defined open sets, closed sets and continuity to Fermatean fuzzy topological spaces. M.Andal, T. Thiripurasundari [1, 2] introduced the concept of Fermatean fuzzy generalized closed set and Fermatean fuzzy generalized regular closed set in 2022. K. Kadambavanam and K. Vaithiyalingam [12] introduced the concept of weakly π generalized closed sets in Intuitionistic fuzzy topological spaces. The aim of this paper is to introduce the concept of Fermatean π closed set and Fermatean fuzzy generalized π closed set in a Fermatean fuzzy topological spaces.

2. Preliminaries

Definition 2.1. The intuitionistic fuzzy sets [3] are defined on a non - empty set X as objects having the form $I = \{\langle x, \alpha_1(x), \beta_1(x) \rangle : x \in X\}$, where

$\alpha_1(x): X \rightarrow [0, 1]$ and $\beta_1(x): X \rightarrow [0, 1]$ denote the degree of membership and the degree of non-membership of each element $x \in X$ to the set I , respectively, and $0 \leq \alpha_1(x) + \beta_1(x) \leq 1$, for all $x \in X$.

Definition 2.2. Let X be a universe of discourse. A Fermatean fuzzy set (FFS) [13] F in X is an object having the form $F = \{(x, \alpha_F(x), \beta_F(x)) : x \in X\}$, where $\alpha_F(x): X \rightarrow [0, 1]$ and $\beta_F(x): X \rightarrow [0, 1]$ including the condition $0 \leq (\alpha_F(x))^3 + (\beta_F(x))^3 \leq 1$, for all $x \in X$. The numbers $\alpha_F(x)$ and $\beta_F(x)$ denote, respectively, the degree of membership and the degree of non-membership of the element x in the set F .

For any FFS F and $x \in X$, $\pi_F(x) = \sqrt[3]{1 - (\alpha_F(x))^3 - (\beta_F(x))^3}$ is identified as the degree of indeterminacy of x to F . In the interest of simplicity, we shall mention the symbol $F = (\alpha_F, \beta_F)$ for the FFS $F = \{(x, \alpha_F(x), \beta_F(x)) : x \in X\}$.

Definition 2.3. Let $F = (\alpha_F, \beta_F)$, $F_1 = (\alpha_{F_1}, \beta_{F_1})$ and $F_2 = (\alpha_{F_2}, \beta_{F_2})$ be three Fermatean fuzzy sets [13] (FFSs), then their operations are defined as follows:

- (1) $F_1 \cap F_2 = (\min\{\alpha_{F_1}, \alpha_{F_2}\}, \max\{\beta_{F_1}, \beta_{F_2}\})$.
- (2) $F_1 \cup F_2 = (\max\{\alpha_{F_1}, \alpha_{F_2}\}, \min\{\beta_{F_1}, \beta_{F_2}\})$.
- (3) $F^c = (\beta_F, \alpha_F)$.

We say F_1 is a subset of F_2 or F_2 contains F_1 and we write $F_1 \subset F_2$ or $F_2 \supset F_1$ if $\alpha_{F_1} \leq \alpha_{F_2}$ and $\beta_{F_1} \geq \beta_{F_2}$.

Remark 2.4. If $\alpha_{F_1} = \alpha_{F_2}$ and $\beta_{F_1} = \beta_{F_2}$, then $F_1 = F_2$.

Note 2.5. Here, if the union and the intersection are infinite, then we use supremum "sup" and infimum "inf" instead of maximum "max" and minimum "min", respectively.

Notations 2.6. For the sake of simplicity, we use the notion 1_x for the Fermatean fuzzy subset $(1, 0)$ and we use the notion 0_x for the Fermatean

fuzzy subset $(0, 1)$, that is, $\alpha_{1_x} = 1$, $\beta_{1_x} = 0$, $\alpha_{0_x} = 0$ and $\beta_{0_x} = 1$.

A Fermatean fuzzy subset F of a non-empty set X is a pair (α_F, β_F) of a membership function $\alpha_F(x): X \rightarrow [0, 1]$ and a non-membership function $\beta_F(x): X \rightarrow [0, 1]$ with $(\alpha_F(x))^3 + (\beta_F(x))^3 = (\gamma_F(x))^3$ for any $x \in X$ where $\gamma_F(x): X \rightarrow [0, 1]$ is a function which is called the strength of commitment at point x .

Definition 2.7. Let X be a non-empty set and τ be a family of Fermatean fuzzy subsets of X . If

- (1) 1_x and $0_x \in \tau$,
- (2) for any $F_1, F_2 \in \tau$, we have $F_1 \cap F_2 \in \tau$,
- (3) for any $\{F_i\}_{i \in I} \subset \tau$, we have $\bigcup_{i \in I} F_i \in \tau$ where I is an arbitrary index set then τ is called a Fermatean fuzzy topology [11] on X .

The pair (X, τ) is said to be Fermatean fuzzy topological space. Every member of τ is called an open Fermatean fuzzy subset. The complement of an open Fermatean fuzzy subset is called a closed Fermatean fuzzy subset.

Definition 2.8. Let (X, τ) be a Fermatean fuzzy topological space and $F = \{(x, \alpha_F(x), \beta_F(x)) : x \in X\}$ be a Fermatean fuzzy set in X . Then, the Fermatean fuzzy interior and Fermatean fuzzy closure [11] of F are defined by

- (1) $cl(F) = \bigcap \{H : F \subset H \text{ and } H \text{ is a closed Fermatean fuzzy set in } X\}$.
- (2) $int(F) = \bigcup \{G : G \subset F \text{ and } G \text{ is an open Fermatean fuzzy set in } X\}$.

Definition 2.9. An intuitionistic fuzzy topology (shortly IFT) [14] on a non empty set X is a family of an intuitionistic fuzzy sets in X satisfying

- (1) $0_{\sim}, 1_{\sim} \in \tau$,
- (2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,
- (3) $\bigcup G_j \in \tau$ for any $\{G_j | j \in J\} \subset \tau$.

In this case, the pair (X, τ) is called an intuitionistic fuzzy topological space and any

intuitionistic fuzzy set in τ is called an intuitionistic fuzzy open set (shortly, IFOS) in X .

Definition 2.10. An intuitionistic fuzzy set $A = \langle x, \alpha_A, \beta_A \rangle$ in an intuitionistic fuzzy topological space (X, τ) is said to be an

- (1) Intuitionistic fuzzy regular closed set [7] (shortly, IFRCS) if $A = \text{cl}(\text{int}(A))$.
- (2) Intuitionistic fuzzy regular open set [7] (shortly, IFROS) if $A = \text{int}(\text{cl}(A))$.
- (3) Intuitionistic fuzzy π closed set [12] (shortly IF π CS) if A is the finite intersection of Intuitionistic fuzzy regular closed sets.
- (4) Intuitionistic fuzzy π open set [12] (shortly IF π OS) if A is the finite union of intuitionistic fuzzy regular open sets.

Definition 2.11. Let (X, τ) be an intuitionistic fuzzy topological space. An intuitionistic fuzzy set $A = \langle x, \alpha_A, \beta_A \rangle$ in (X, τ) is said to be

- (1) a generalized intuitionistic fuzzy closed set [14] (in shortly GIF - closed set) if $\text{IFcl}(A) \subseteq G$ whenever $A \subseteq G$ and G is intuitionistic fuzzy open set in X .

The complement of a GIF - closed set is GIF - open set.

- (2) a Intuitionistic fuzzy generalized regular closed set (in shortly IFGR - closed set) [7] if $\text{rcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is intuitionistic fuzzy open set in X .
- (3) a regular generalized intuitionistic fuzzy closed set (in shortly IFRG - closed set) [15] if $\text{IFcl}(A) \subseteq G$ whenever $A \subseteq G$ and G is intuitionistic fuzzy open set in X .

Definition 2.12. Let (X, τ) be a Fermatean fuzzy topological space. A Fermatean fuzzy set $A = (\alpha_A, \beta_A)$ in (X, τ) is said to be

- (1) a Fermatean fuzzy regular open set [2] if $A = \text{int}(\text{cl}(A))$.
- (2) a Fermatean fuzzy regular closed set [2] if $A = \text{cl}(\text{int}(A))$.

Definition 2.13. Let (X, τ) be a Fermatean fuzzy topological space. A Fermatean fuzzy set $A = (\alpha_A, \beta_A)$ in (X, τ) is said to be

- (1) a Fermatean fuzzy generalized closed set [1] (in shortly FFG - closed) if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open Fermatean fuzzy set. The complement of a FFG - closed set is FFG - open set.
- (2) a Fermatean fuzzy generalized regular closed set [2] (in shortly FFGR - closed) if $\text{rcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open Fermatean fuzzy set. The complement of a FFGR - closed set is FFGR - open set.

Theorem 2.14. Let (X, τ) be a Fermatean fuzzy topological space [12] and F_1 and F_2 be Fermatean fuzzy sets in X . Then the following are hold:

- (1) $F_1 \subseteq \text{cl}(F_1)$,
- (2) $\text{int}(F_1) \subseteq F_1$,
- (3) If $F_1 \subseteq F_2$, then $\text{cl}(F_1) \subseteq \text{cl}(F_2)$,
- (4) If $F_1 \subseteq F_2$, then $\text{int}(F_1) \subseteq \text{int}(F_2)$,
- (5) $\text{int}(F_1 \cap F_2) = \text{int}(F_1) \cap \text{int}(F_2)$,
- (6) $\text{cl}(F_1 \cup F_2) = \text{cl}(F_1) \cup \text{cl}(F_2)$,
- (7) F_1 is open Fermatean fuzzy set if and only if $F_1 = \text{int}(F_1)$,
- (8) F_1 is closed Fermatean fuzzy set if and only if $F_1 = \text{cl}(F_1)$,
- (9) $\text{Int}(F_1) \cup \text{int}(F_2) \subseteq \text{int}(F_1 \cup F_2)$,
- (10) $\text{cl}(F_1 \cap F_2) \subseteq \text{cl}(F_1) \cap \text{cl}(F_2)$.

3. Fermatean Fuzzy π Closed Sets

In this section, we introduce the concept of Fermatean fuzzy regular closed sets in Fermatean fuzzy topological space.

Definition 3.1. A Fermatean fuzzy set $A = (\alpha_F, \beta_F)$ of a Fermatean fuzzy topological space (X, τ) is said to be Fermatean fuzzy π closed if A is the finite intersection of Fermatean fuzzy regular closed sets, symbolically A is a FF π CS in X .

Example 3.2. Let $X = \{e_1, e_2\}$. Consider the following family of Fermatean fuzzy subsets $\tau = \{1_X, 0_X, A_1, A_2, A_3, A_1 \cap A_3, A_1 \cup A_3\}$ where

$$1_X = \{ \langle e_1, \alpha_{1_X}(e_1) = 1, \beta_{1_X}(e_1) = 0 \rangle, \langle e_2, \alpha_{1_X}(e_2) = 1, \beta_{1_X}(e_2) = 0 \rangle \},$$

$$0_X = \{ \langle e_1, \alpha_{0_X}(e_1) = 0, \beta_{0_X}(e_1) = 1 \rangle, \langle e_2, \alpha_{0_X}(e_2) = 0, \beta_{0_X}(e_2) = 1 \rangle \},$$

$$A_1 = \{ \langle e_1, \alpha_{A_1}(e_1) = 0.3, \beta_{A_1}(e_1) = 0.7 \rangle, \langle e_2, \alpha_{A_1}(e_2) = 0.6, \beta_{A_1}(e_2) = 0.6 \rangle \},$$

$$A_2 = \{ \langle e_1, \alpha_{A_2}(e_1) = 0.7, \beta_{A_2}(e_1) = 0.5 \rangle, \langle e_2, \alpha_{A_2}(e_2) = 0.7, \beta_{A_2}(e_2) = 0.6 \rangle \},$$

and

$$A_3 = \{ \langle e_1, \alpha_{A_3}(e_1) = 0.7, \beta_{A_3}(e_1) = 0.8 \rangle, \langle e_2, \alpha_{A_3}(e_2) = 0.7, \beta_{A_3}(e_2) = 0.7 \rangle \}.$$

Then (X, τ) is a Fermatean fuzzy topological space. Here $FF\pi CS = \{1_X, 0_X, A_1^c, A_2^c, A_3^c, A_1^c \cap A_3^c, A_1^c \cup A_3^c\}$.

Theorem 3.3. Every Fermatean fuzzy regular closed set is a Fermatean fuzzy π closed set.

Proof. Let A be a Fermatean fuzzy regular closed set in (X, τ) . Since, A is a finite intersection of Fermatean fuzzy regular closed set is a Fermatean fuzzy π closed set. Therefore, A is a Fermatean fuzzy π closed set in (X, τ) .

Remark 3.4. The converse part of the above theorem 3.3 need not be true as seen from the following example.

Example 3.5. From example 3.2, $A_1^c \cap A_3^c$ is a Fermatean fuzzy π closed set. But it is not a Fermatean fuzzy regular closed set in (X, τ) .

Theorem 3.6. Every Fermatean fuzzy π closed set is a closed Fermatean fuzzy set.

Proof. Let A be a Fermatean fuzzy π closed set. Since every Fermatean fuzzy regular closed set is a Fermatean fuzzy π closed set and every fuzzy regular closed set is a closed Fermatean fuzzy set, it follows that A is a Fermatean fuzzy π closed set in (X, τ) .

Remark 3.7. The converse part of above theorem 3.6 need not be true as seen from the following example.

Example 3.8. Let $X = \{e_1, e_2\}$. Consider the following family of Fermatean fuzzy subsets $\tau = \{1_X, 0_X, A_1, A_2, A_3, A_1 \cap A_2, A_1 \cup A_2, A_1 \cap A_3\}$ where

$$1_X = \{ \langle e_1, \alpha_{1_X}(e_1) = 1, \beta_{1_X}(e_1) = 0 \rangle, \langle e_2, \alpha_{1_X}(e_2) = 1, \beta_{1_X}(e_2) = 0 \rangle \},$$

$$0_X = \{ \langle e_1, \alpha_{0_X}(e_1) = 0, \beta_{0_X}(e_1) = 1 \rangle, \langle e_2, \alpha_{0_X}(e_2) = 0, \beta_{0_X}(e_2) = 1 \rangle \},$$

$$A_1 = \{ \langle e_1, \alpha_{A_1}(e_1) = 0.3, \beta_{A_1}(e_1) = 0.7 \rangle, \langle e_2, \alpha_{A_1}(e_2) = 0.7, \beta_{A_1}(e_2) = 0.6 \rangle \},$$

$$A_2 = \{ \langle e_1, \alpha_{A_2}(e_1) = 0.7, \beta_{A_2}(e_1) = 0.7 \rangle, \langle e_2, \alpha_{A_2}(e_2) = 0.6, \beta_{A_2}(e_2) = 0.6 \rangle \},$$

and

$$A_3 = \{ \langle e_1, \alpha_{A_3}(e_1) = 0.7, \beta_{A_3}(e_1) = 0.8 \rangle, \langle e_2, \alpha_{A_3}(e_2) = 0.6, \beta_{A_3}(e_2) = 0.7 \rangle \}.$$

Then (X, τ) is a Fermatean fuzzy topological space. Here, A_1^c is a closed Fermatean fuzzy set, but it is not a Fermatean fuzzy π closed set in (X, τ) .

Theorem 3.9. If A and B are two Fermatean fuzzy π closed sets in (X, τ) , then $A \cup B$ is a Fermatean fuzzy π closed set in (X, τ) .

Proof. w.k.t, finite union of Fermatean fuzzy regular closed sets is a Fermatean fuzzy regular closed set and every Fermatean fuzzy regular closed set is a Fermatean fuzzy π closed set. It follows that $A \cup B$ is a Fermatean fuzzy π closed set in (X, τ) .

Remark 3.10. If A and B are two Fermatean fuzzy π closed sets in (X, τ) , then $A \cap B$ is also a Fermatean fuzzy π closed set in (X, τ) . Since Fermatean fuzzy π closed is a finite intersection of Fermatean fuzzy regular closed sets.

4. Fermatean Fuzzy Generalized π Closed Sets

In this section, we introduce the concept of Fermatean fuzzy generalized π

closed sets in Fermatean fuzzy topological space.

Definition 4.1. Let (X, τ) be a Fermatean fuzzy topological space. A Fermatean fuzzy set A in (X, τ) is said to be a Fermatean fuzzy generalized π closed set (in shortly FFG π - closed set) if $\pi\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open Fermatean fuzzy set.

Example 4.2. Consider the Fermatean fuzzy topological space in example 3.2. Take, $A = \{(e_1, \alpha_A(e_1) = 0.4, \beta_A(e_1) = 0.7), (e_2, \alpha_A(e_2) = 0.5, \beta_A(e_2) = 0.7)\}$ is a Fermatean fuzzy set in (X, τ) . Clearly, A is a Fermatean fuzzy generalized π closed set in (X, τ) .

Definition 4.3. Let A be a Fermatean fuzzy set in a Fermatean fuzzy topological space (X, τ) . Then Fermatean fuzzy generalized π closure and Fermatean fuzzy generalized π interior of A are defined by

- (1) $\text{FFG}\pi\text{cl}(A) = \bigcap \{H : A \subseteq H \text{ and } H \text{ is Fermatean fuzzy generalized } \pi \text{ closed set in } X\}$.
- (2) $\text{FFG}\pi\text{int}(A) = \bigcup \{G : G \subseteq A \text{ and } G \text{ is Fermatean fuzzy generalized } \pi \text{ open set in } X\}$.

Theorem 4.4. Every Fermatean fuzzy π closed set is a Fermatean fuzzy generalized π closed set.

Proof. Let A be a Fermatean fuzzy π closed set in a Fermatean fuzzy topological space (X, τ) . Let U be a open Fermatean fuzzy set in (X, τ) such that $A \subseteq U$. Since A is Fermatean fuzzy π closed set, we have $\pi\text{cl}(A) \subseteq U$. Therefore $\pi\text{cl}(A) \subseteq U$. Hence A is a Fermatean fuzzy generalized π closed set.

Remark 4.5. The converse part of the above theorem 4.4 need not be true as seen from the following example.

Example 4.6. From example 4.2, A is a Fermatean fuzzy generalized π closed set. But it is not a Fermatean fuzzy π closed set in (X, τ) .

Theorem 4.7. Every Fermatean fuzzy regular closed set is a Fermatean fuzzy generalized π closed set.

Proof. Let A be a Fermatean fuzzy regular closed set in a Fermatean fuzzy topological space (X, τ) . Let U be a open Fermatean fuzzy set in (X, τ) such that $A \subseteq U$. Since every Fermatean fuzzy regular closed set is a Fermatean fuzzy π closed set, we have $\pi\text{cl}(A) \subseteq \text{rcl}(A) = A \subseteq U$. Therefore $\pi\text{cl}(A) \subseteq U$. Hence A is a Fermatean fuzzy generalized π closed set.

Remark 4.8. The converse part of the above theorem 4.7 need not be true as seen from the following example.

Example 4.9. From example 4.2, A is a Fermatean fuzzy generalized π closed set. But it is not a Fermatean fuzzy regular closed set in (X, τ) .

Theorem 4.10. Every Fermatean fuzzy generalized regular closed set is a Fermatean fuzzy generalized π closed set.

Proof. Let A be a Fermatean fuzzy generalized regular closed set in a Fermatean fuzzy topological space (X, τ) . Let U be a open Fermatean fuzzy set in (X, τ) such that $A \subseteq U$. Since every Fermatean fuzzy generalized regular closed set is a Fermatean fuzzy π closed set, we have $\pi\text{cl}(A) \subseteq \text{rcl}(A) = A \subseteq U$. Therefore $\pi\text{cl}(A) \subseteq U$. Hence A is a Fermatean fuzzy generalized π closed set. □

Remark 4.11. The following shows that the converse part of the above theorem 4.10 need not be true as seen from the following example.

Example 4.12. From example 3.2, $A^c \cap A^c$ is a Fermatean fuzzy generalized π closed set, but it is not a Fermatean fuzzy generalized regular closed set in (X, τ) .

Theorem 4.13. Every Fermatean fuzzy generalized π closed set is a Fermatean fuzzy generalized closed set.

Proof. Let A be a Fermatean fuzzy generalized π closed set in (X, τ) and U be an open Fermatean fuzzy set such that $A \subseteq U$. By our assumption, $\pi\text{cl}(A) \subseteq U$. Also, $\text{cl}(A) \subseteq \pi\text{cl}(A) \subseteq U$. Hence A is a Fermatean fuzzy generalized closed set in (X, τ) .

Remark 4.14. The following example shows that the converse of the above theorem 4.7 is not true as seen from the following example.

Example 4.15. From example 3.8, A_1^c is a Fermatean fuzzy generalized closed set, but it is not a Fermatean fuzzy generalized π closed set in (X, τ) .

Theorem 4.16. If A and B are Fermatean fuzzy generalized π closed sets in (X, τ) , then $A \cup B$ is also a Fermatean fuzzy generalized π closed set in (X, τ) .

Proof. Let U be an open Fermatean fuzzy set in (X, τ) such that $A \cup B \subseteq U$. Now $A \subseteq U$ and $B \subseteq U$. Since A and B are Fermatean fuzzy generalized π closed sets in (X, τ) , $\pi\text{cl}(A) \subseteq U$ and $\pi\text{cl}(B) \subseteq U$. Therefore $\pi\text{cl}(A) \cup \pi\text{cl}(B) \subseteq U$. Since $\pi\text{cl}(A) \cup \pi\text{cl}(B) = \pi\text{cl}(A \cup B)$, $\pi\text{cl}(A \cup B) \subseteq U$. Therefore $A \cup B$ is a Fermatean fuzzy generalized π closed set in (X, τ) .

Remark 4.17. If A and B are Fermatean fuzzy generalized π closed sets in (X, τ) , then $A \cap B$ is not a Fermatean fuzzy generalized π closed set in (X, τ) as seen from the following example.

Example 4.18. Consider the Fermatean fuzzy topological space in example 3.8. Take any two Fermatean fuzzy generalized π closed sets $A = \{\langle 0.4, 0.7 \rangle, \langle 0.5, 0.7 \rangle\}$ and $B = \{\langle 0.6, 0.8 \rangle, \langle 0.5, 0.6 \rangle\}$. Then $A \cap B =$

$\{\langle 0.4, 0.8 \rangle, \langle 0.5, 0.7 \rangle\}$ is not a Fermatean fuzzy generalized π closed set in (X, τ) .

Theorem 4.19. Let (X, τ) be a Fermatean fuzzy topological space. If A is a Fermatean fuzzy generalized π closed set and $A \subseteq B \subseteq \pi\text{cl}(A)$, then B is a Fermatean fuzzy generalized π closed set in (X, τ) .

Proof. Let (X, τ) be a Fermatean fuzzy topological space. Let U be an open Fermatean fuzzy set such that $B \subseteq U$. Since $A \subseteq B$, $A \subseteq U$ and A is a Fermatean fuzzy generalized π closed set, $\pi\text{cl}(A) \subseteq U$. But $\pi\text{cl}(B) \subseteq \pi\text{cl}(A)$. Since $B \subseteq \pi\text{cl}(A)$ and also $\pi\text{cl}(B) \subseteq U$. Therefore B is a Fermatean fuzzy generalized π closed set in (X, τ) .

Theorem 4.20. If a Fermatean fuzzy set A is both a Fermatean fuzzy π open and a Fermatean fuzzy generalized π closed set in (X, τ) then A is a Fermatean fuzzy generalized π closed set in (X, τ) .

Proof. Let A be both a Fermatean fuzzy π open and a Fermatean fuzzy generalized π closed set in (X, τ) . Since every Fermatean fuzzy π open set is an open Fermatean fuzzy set and $A \subseteq A$, $\pi\text{cl}(A) \subseteq A$. Also $A \subseteq \pi\text{cl}(A)$. Therefore, $\pi\text{cl}(A) = A$. i.e., A is a Fermatean fuzzy generalized π closed set in (X, τ) .

Theorem 4.21. If a Fermatean fuzzy set A is both an open Fermatean fuzzy set and a Fermatean fuzzy generalized π closed set in (X, τ) then A is a closed Fermatean fuzzy set in (X, τ) .

Proof. Suppose that A is both an open Fermatean fuzzy set and a Fermatean fuzzy generalized π closed set in (X, τ) . Now, $A \subseteq A$, we have $A \subseteq \pi\text{cl}(A)$. Therefore, $\pi\text{cl}(A) = A$ and hence A is a closed Fermatean fuzzy set in (X, τ) .

Theorem 4.22. In a Fermatean fuzzy topological space (X, τ) if $\text{FFO}(X) = \{1_x, 0_x\}$, where $\text{FFO}(X)$ is the family of all open Fermatean fuzzy sets then every Fermatean fuzzy subset of (X, τ) is a Fermatean fuzzy generalized π closed set.

Proof. Let (X, τ) be a Fermatean fuzzy topological space and $\text{FFO}(X) = \{1_x, 0_x\}$. Let A be any Fermatean fuzzy subset of (X, τ) . Suppose that $A = 0_x$. Then 0_x is a Fermatean fuzzy generalized π closed set in (X, τ) . Suppose that $A \neq 0_x$. Then 1_x is the only open Fermatean fuzzy set containing A and so $\pi\text{cl}(A) \subseteq 1_x$. Hence A is a Fermatean fuzzy generalized π closed set in (X, τ) .

Theorem 4.23. Let A be a Fermatean fuzzy generalized π closed set in (X, τ) and $\pi\text{cl}(A) \cap (1_x - \pi\text{cl}(A)) = 0_x$. Then $\pi\text{cl}(A) - A$ does not contain any non-zero closed Fermatean fuzzy set in (X, τ) .

Proof: Let A be a Fermatean fuzzy generalized π closed set in (X, τ) and $\pi\text{cl}(A) \cap (1_x - \pi\text{cl}(A)) = 0_x$. Now to prove the result by contradiction method. Let B be a closed Fermatean fuzzy set such that $\pi\text{cl}(A) - A \supseteq B$ and $B \neq 0_x$.

Now $B \subseteq \pi\text{cl}(A) - A$. ie., $B \subseteq 1_x - A$ which implies $A \subseteq 1_x - B$. Since B is closed Fermatean fuzzy set in (X, τ) , $1_x - B$ is open Fermatean fuzzy set in (X, τ) . Since A is Fermatean fuzzy generalized π closed set in (X, τ) , by the definition $\pi\text{cl}(A) \subseteq 1_x - B$. Therefore $B \subseteq 1_x - \pi\text{cl}(A)$. Since $B \subseteq \pi\text{cl}(A)$ and $B \subseteq \pi\text{cl}(A) \cap (1_x - \pi\text{cl}(A)) = 0_x$, by hypothesis. This shows that $B = 0_x$. Which is contradiction to our assumption. Hence $\pi\text{cl}(A) - A$ does not contain any non-zero closed Fermatean fuzzy set in (X, τ) .

Theorem 4.24. Let A be a Fermatean fuzzy generalized π closed set in (X, τ) and $\pi\text{cl}(A) \cap (1_x - \pi\text{cl}(A)) = 0_x$. Then $\pi\text{cl}(A) - A$ does not contain any non-zero Fermatean fuzzy π closed set in (X, τ) .

Proof. Follows from theorem 4.23 and the fact that every Fermatean fuzzy π closed set is a closed Fermatean fuzzy set in (X, τ) . Theorem 4.25. If $A \subseteq B \subseteq \text{cl}(A)$ and A is a Fermatean fuzzy generalized π closed set in (X, τ) , then B

is also a Fermatean fuzzy generalized π closed set in (X, τ) .

Proof. Let A be a Fermatean fuzzy generalized π closed set in (X, τ) . Therefore, $\pi\text{cl}(A) \subseteq U$, whenever $A \subseteq U$ and U is open Fermatean fuzzy set in (X, τ) . Since every Fermatean fuzzy generalized π closed set is a Fermatean fuzzy generalized closed set, $\text{cl}(A) \subseteq \pi\text{cl}(A) \subseteq U$. Clearly $\text{cl}(A) \subseteq U$. Since $B \subseteq \text{cl}(A)$, $B \subseteq U$. Therefore, $\pi\text{cl}(B) \subseteq U$. Hence B is a Fermatean fuzzy generalized π closed set in (X, τ) .

Theorem 4.26. If $A \subseteq B \subseteq \text{cl}(A)$ and A is a Fermatean fuzzy generalized π closed set in (X, τ) , then B is also a Fermatean fuzzy generalized closed set in (X, τ) .

Proof. Follows from the theorem 4.25, as the fact that every Fermatean fuzzy generalized π closed set is a Fermatean fuzzy generalized closed set in (X, τ) .

Remark 4.27. The intersection of Fermatean fuzzy generalized π closed and Fermatean fuzzy π closed set need not be a Fermatean fuzzy generalized π closed set in (X, τ) .

Example 4.28. Consider the Fermatean fuzzy topological space in example 3.8. Take Fermatean fuzzy π closed set and Fermatean fuzzy generalized π closed set $A = \{ \langle 0.7, 0.7 \rangle, \langle 0.6, 0.7 \rangle \}$ and $B = \{ \langle 0.6, 0.8 \rangle, \langle 0.5, 0.6 \rangle \}$. Then $A \cap B = \{ \langle 0.6, 0.8 \rangle, \langle 0.5, 0.7 \rangle \}$. Which is not a Fermatean fuzzy generalized π closed set in (X, τ) .

Theorem 4.29. The union of Fermatean fuzzy generalized π closed set and Fermatean fuzzy π closed set is a Fermatean fuzzy generalized π closed set in (X, τ) .

Proof. Let A be a Fermatean fuzzy generalized π closed set, B be a Fermatean fuzzy π closed set and U be a open Fermatean fuzzy set in π closed set. Since A is a Fermatean fuzzy generalized π closed set, $\pi\text{cl}(A) \subseteq U$ whenever $A \subseteq U$. Since B is a

Fermatean fuzzy π closed set in (X, τ) , $B \subseteq U$. Clearly $\pi cl(B) \subseteq U$. Since every Fermatean π closed set is a Fermatean fuzzy generalized π closed set, B is a Fermatean fuzzy π closed set. Also w.k.t, the union of two Fermatean fuzzy generalized π closed set is a Fermatean fuzzy generalized π closed set. Therefore, $A \cup B$ is a Fermatean fuzzy generalized π closed set.

Corollary 4.30. The union of Fermatean fuzzy generalized π closed set and closed Fermatean fuzzy set is a Fermatean fuzzy generalized closed set in (X, τ) .

Proof. Let A be a Fermatean fuzzy generalized π closed set, B be a closed Fermatean fuzzy set and U be an open Fermatean fuzzy set in (X, τ) . Since A is a Fermatean fuzzy generalized π closed set, $\pi cl(A) \subseteq U$ whenever $A \subseteq U$. Since B is a closed Fermatean fuzzy set in (X, τ) , $B \subseteq U$. Clearly $cl(B) \subseteq U$. Since every closed Fermatean fuzzy set is a Fermatean fuzzy generalized closed set, B is a Fermatean fuzzy generalized closed set. Also w.k.t, every Fermatean fuzzy generalized π closed set is a Fermatean fuzzy generalized closed and the union of two Fermatean fuzzy generalized sets is a Fermatean fuzzy generalized closed set. Therefore $A \cup B$ is a Fermatean fuzzy generalized closed set.

Theorem 4.31. Let A and B be two Fermatean fuzzy sets in (X, τ) . If B is a Fermatean fuzzy generalized π closed set and $A \subseteq \pi cl(B)$, then $A \cup B$ is a Fermatean fuzzy generalized π closed set in (X, τ) .

Proof. Let B be a Fermatean fuzzy generalized π closed set and $A \subseteq \pi cl(B)$. Since B is a Fermatean fuzzy generalized π closed set and, $\pi cl(B) \subseteq U$ whenever $B \subseteq U$ and U is an open Fermatean fuzzy set in (X, τ) . Clearly, $B \subseteq \pi cl(B)$ and $A \subseteq \pi cl(B)$, it follows that $A \cup B \subseteq \pi cl(B)$. Therefore $B \subseteq A \cup B \subseteq \pi cl(B)$. By theorem 4.19, $A \cup B$ is a Fermatean fuzzy generalized π closed set in (X, τ) .

5. Fermatean Fuzzy π Open Sets

Definition 5.1. A Fermatean fuzzy set $A = (\alpha_F, \beta_F)$ of a Fermatean fuzzy topological space (X, τ) is said to be Fermatean fuzzy π open if A is a finite union of Fermatean fuzzy regular open sets.

Example 5.2. Consider a Fermatean fuzzy topological space (X, τ) in example 3.2. Then $FF\pi OS = \{1_X, 0_X, A_1, A_2, A_3, A_1 \cap A_3, A_1 \cup A_3\}$ is a Fermatean fuzzy π open sets in (X, τ) .

Theorem 5.3. Every Fermatean fuzzy regular open set is a Fermatean fuzzy π open set.

Proof. Let A be a Fermatean fuzzy regular open set in (X, τ) . Since a finite union of Fermatean fuzzy regular open sets is a Fermatean fuzzy π open set. It follows that A is a Fermatean fuzzy π open set in (X, τ) .

Remark 5.4. The converse part of above theorem 5.3 need not be true as seen from the following example.

Example 5.5. From example 5.2, $A_1 \cup A_3$ is a Fermatean fuzzy π open set, but it is not a Fermatean fuzzy regular open set in (X, τ) .

Theorem 5.6. Every Fermatean fuzzy π open set is an open Fermatean fuzzy set.

Proof. Let A be a Fermatean fuzzy π open set. Since every Fermatean fuzzy regular open set is a Fermatean fuzzy π open set and every Fermatean fuzzy regular open set is an open Fermatean fuzzy set, it follows that A is a Fermatean fuzzy π open set in (X, τ) .

Remark 5.7. The converse part of above theorem 5.6 need not be true as seen from the following example.

Example 5.8. From example 3.8, A_1 is an open Fermatean fuzzy set. But it is not a Fermatean fuzzy π open set in (X, τ) .

Theorem 5.9. If A and B are two Fermatean fuzzy π open sets in (X, τ) , then $A \cap B$ is a Fermatean fuzzy π open set in (X, τ) .

Proof. w.k.t, intersection of two Fermatean fuzzy regular open sets is a Fermatean fuzzy

regular open set and every Fermatean fuzzy regular open set is a Fermatean fuzzy π open set. It follows that, $A \cap B$ is a Fermatean fuzzy π closed set in (X, τ) .

Remark 5.10. If A and B are two Fermatean fuzzy π open sets in (X, τ) , then $A \cup B$ is also a Fermatean fuzzy π open set in (X, τ) .

Example 5.11. From example 5.2, $A_1 \cup A_3$ is a Fermatean fuzzy π open set in (X, τ) .

6. Fermatean Fuzzy Generalized π Open Sets

Definition 6.1. A Fermatean fuzzy set A in a Fermatean fuzzy topological space (X, τ) is said to be Fermatean fuzzy generalized π open set if A^c is Fermatean fuzzy generalized π closed set in (X, τ) .

Example 6.2. From example 4.2, A^c is a Fermatean fuzzy generalized π open set in (X, τ) .

Theorem 6.3. Every Fermatean fuzzy π open set is a Fermatean fuzzy generalized π open set.

Proof. Let A^c be a Fermatean fuzzy π open set in a Fermatean fuzzy topological space (X, τ) . Therefore A is a Fermatean fuzzy π closed set. From theorem 4.4, A is a Fermatean fuzzy generalized π closed set in (X, τ) . Therefore, A^c is a Fermatean fuzzy generalized π open set in (X, τ) .

Remark 6.4. The converse part of the above theorem 6.3 need not be true as seen from the following example.

Example 6.5. From example, 6.2, A^c is a Fermatean fuzzy generalized π open set in (X, τ) . But it is not a Fermatean fuzzy π open set in (X, τ) .

Theorem 6.6. Every Fermatean fuzzy regular open set is a Fermatean fuzzy generalized π open set.

Proof. Let A^c be a Fermatean fuzzy regular open set in (X, τ) . Therefore, A be a Fermatean fuzzy regular closed set. By theorem 4.7, A is a Fermatean fuzzy generalized π closed set in (X, τ) . Therefore, A^c is a Fermatean fuzzy generalized π open set in (X, τ) .

Remark 6.7. The converse part of the above theorem 6.6 need not be true as seen from the following example.

Example 6.8. From example, 6.2, A^c is a Fermatean fuzzy generalized π open set in (X, τ) . But it is not a Fermatean fuzzy regular open set in (X, τ) .

Theorem 6.9. Every Fermatean fuzzy generalized regular open set is a Fermatean fuzzy generalized π open set.

Proof. Let A^c be a Fermatean fuzzy generalized regular open set in (X, τ) . Therefore, A is a Fermatean fuzzy generalized regular closed set. By theorem 4.10, A is a Fermatean fuzzy generalized π closed set in (X, τ) . Therefore, A^c is a Fermatean fuzzy generalized π open set in (X, τ) .

Remark 6.10. The converse part of the above theorem 6.9 need not be true as seen from the following example.

Example 6.11. From example 6.2, A^c is a Fermatean fuzzy generalized π open set, but it is not a Fermatean fuzzy generalized regular closed set in (X, τ) .

Theorem 6.12. Every Fermatean fuzzy generalized π open set is a Fermatean fuzzy generalized open set.

Proof. Let A^c be a Fermatean fuzzy generalized π open set in (X, τ) . Therefore, A is a Fermatean fuzzy generalized π closed set in (X, τ) . w.k.t., Every Fermatean fuzzy generalized π closed set is a Fermatean fuzzy generalized closed set. It follows that, A^c is a Fermatean fuzzy generalized open set in (X, τ) .

Remark 6.13. The converse part of the above theorem 6.12 need not be true as seen from the following example.

Example 6.14. From example 4.15, A_1 is a Fermatean fuzzy generalized open set. But it is not a Fermatean fuzzy generalized π open set.

Theorem 6.15. Let A be a Fermatean fuzzy generalized π open set in (X, τ) . If B is a Fermatean fuzzy set in (X, τ) such that $\pi \text{int}(A) \subseteq B \subseteq A$, then B is also a Fermatean fuzzy generalized π open set in (X, τ) .

Proof. Now A^c is a Fermatean fuzzy generalized π closed set and B is a Fermatean fuzzy set in (X, τ) such that $A^c \subseteq B^c \subseteq \pi \text{int}(A)^c$. By theorem 4.19, $A^c \subseteq B^c \subseteq \text{int}(A)^c = \pi \text{cl}(A^c)$. Hence by theorem 4.19, B^c is Fermatean fuzzy generalized π closed set in (X, τ) . Hence by definition 6.1, B is a Fermatean fuzzy generalized π open set in (X, τ) .

Theorem 6.16. A Fermatean fuzzy set A of a Fermatean fuzzy topological space (X, τ) is Fermatean fuzzy generalized π open iff $B \subseteq \pi \text{int}(A)$ whenever $B \subseteq A$ and B is closed Fermatean fuzzy set in (X, τ) .

Proof. Suppose that A is a Fermatean fuzzy set such that $B \subseteq \pi \text{int}(A)$ whenever $B \subseteq A$ and B is closed Fermatean fuzzy set in (X, τ) . Let $A^c \subseteq U$ and U is an open Fermatean fuzzy set in (X, τ) . Since $A^c \subseteq U$, $U^c \subseteq A$. Hence by assumption we have $U^c \subseteq \pi \text{int}(A)$. i.e, $\pi \text{int}(A)^c \subseteq U$. Also, $\pi \text{int}(A)^c = \pi \text{cl}(A^c)$. Therefore, $\pi \text{cl}(A^c) \subseteq U$, which implies that A^c is a Fermatean fuzzy generalized π closed set in (X, τ) . By definition 6.1, A is Fermatean fuzzy generalized π open set in (X, τ) .

Conversely, assume that A is Fermatean fuzzy generalized π open set in (X, τ) and B be a closed Fermatean fuzzy set in (X, τ) . Then B^c is open Fermatean fuzzy set in (X, τ) and $A^c \subseteq B^c$. Since A^c is Fermatean fuzzy generalized π closed set in (X, τ) , $\pi \text{cl}(A^c) = \pi \text{int}(A)^c \subseteq B^c$ which implies that $B \subseteq \pi \text{int}(A)$.

Theorem 6.17. If A and B are Fermatean fuzzy generalized π open sets in (X, τ) , then $A \cap B$ is a Fermatean fuzzy generalized π open set in (X, τ) .

Proof. Let A and B be two Fermatean fuzzy generalized π open sets in (X, τ) . Then by definition 6.1, A^c and B^c are Fermatean fuzzy generalized π closed sets in (X, τ) . By theorem 4.16, $A^c \cup B^c$ is also a Fermatean fuzzy generalized π closed set in (X, τ) . That is, $A^c \cup B^c = (A \cap B)^c$. Therefore $(A \cap B)^c$ is a Fermatean fuzzy generalized π closed set in (X, τ) . By definition 6.1, $A \cap B$ is a Fermatean fuzzy generalized π open set in (X, τ) .

Remark 6.18. If A and B are Fermatean fuzzy generalized regular π sets in (X, τ) , then $A \cup B$ need not be a Fermatean fuzzy generalized π open set in (X, τ) as seen from the following example.

Example 6.19. Consider the Fermatean fuzzy topological space (X, τ) in example 3.2. From example 4.18, $(A \cap B)^c$ is not a Fermatean fuzzy generalized π open set in (X, τ) .

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