Rectangular Cone S –Metric space and Fixed Point Theorems of Contractive Mappings

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Abstract: In this paper, we introduced the rectangular cone S - metric space which is a generalization of rectangular S –metric space and prove some fixed point theorems of different type of contractive conditions furnished with examples.
Keywords: Rectangular cone S- metric space, contraction mapping, fixed point.
Subject Classification: 47H10; 54H25.

1. Introduction and Preliminaries

Life is full of changes is a constant, But in some certain situations the amount of distance between two point must be exists. For this consideration, in 1906 Frecht [7] introduced a metric space. After that Banach introduced the Banach contraction principle in metric space, which is the most important key tool in fixed point theory. Many researchers extended and generalized the Banach contraction principle in different ways and different fields [17, 18, 19, 21, 23]. In 2000, Branciari [3] initiated the rectangular metric space. Many originators generalized the concept of rectangular metric space and discussed the in fixed point results in different type of contractive conditions [1-14]. Sedghi et al. [20] initiated the concept of S- metric space in 2012. Recently O.K. Adewale and C. Iluno [1] introduced rectangular S - metric space and proved some fixed point results.

In this paper we introduce a concept of Rectangular cone S - metric space and proved some fixed point results in various contractive conditions.

Definition 1.1. [3] Let X be a nonempty set and a function $\delta : X^2 \rightarrow [0,\infty)$ satisfying the following conditions.

(1) $\delta(\alpha_1,\alpha_2) = 0$ if and only if $\alpha_1 = \alpha_2$ for all $\alpha_1,\alpha_2 \in X$.

(2) $\delta(\alpha_1,\alpha_2) = \delta(\alpha_2,\alpha_1)$ for all $\alpha_1,\alpha_2 \in X$.

(3) $\delta(\alpha_1,\alpha_2) \leq \delta(\alpha_1,\alpha_3) + \delta(\alpha_3,\alpha_4) + \delta(\alpha_4,\alpha_2)$ for all $\alpha_1,\alpha_2 \in X$ and all distinct points $\alpha_3,\alpha_4 \in X - \{\alpha_1,\alpha_2\}$ δ is called a rectangular metric in X and (X, δ) is called rectangular metric space.

Definition 1.2. [20] Let X be a nonempty set and a function $\delta: X^3 \rightarrow [0, \infty)$ satisfies the following properties.

1. $\delta(\alpha_1,\alpha_2,\alpha_3) \geq 0$.
2. $\delta(\alpha_1,\alpha_2,\alpha_3) = 0$ if and only if $\alpha_1 = \alpha_2 = \alpha_3$.
3. $\delta(\alpha_1,\alpha_2,\alpha_3) \leq \delta(\alpha_1,\alpha_1,\beta) + \delta(\alpha_2,\alpha_2,\beta) + \delta(\alpha_3,\alpha_3,\beta)$ for all $\alpha_1,\alpha_2,\alpha_3,\beta \in X$. Then δ is called S-metric on X and the pair (X, δ) is called an S-metric space.

Definition 1.3. [1] Let X be a nonempty set and define a function $\bar{\delta} : X^3 \rightarrow [0, \infty)$ satisfies the following properties.

(i) $\bar{\delta}(\alpha_1, \alpha_2, \alpha_3) = 0$ iff $\alpha_1 = \alpha_2 = \alpha_3$.

(ii) $\bar{\delta}(\alpha_1,\alpha_2,\alpha_3) \leq \bar{\delta}(\alpha_1,\alpha_1,\beta)+ \bar{\delta}(\alpha_2,\alpha_2,\beta)+ \bar{\delta}(\alpha_3,\alpha_3,\beta)$ for all $\alpha_1,\alpha_2,\alpha_3 \in X$ and distinct points $\beta \in X - \{\alpha_1,\alpha_2,\alpha_3\}$

The pair $(X, \bar{\delta})$ is called the rectangular S - metric space.

Definition 1.4. [9] hung and zung Let E be the Banach space and a subset M of E is called cone if and only if

(i) M is closed and non-empty

(ii) $p\alpha_1 + q\alpha_2 \in M$ for all $\alpha_1, \alpha_2 \in M$

(iii) $M \cap (-M) = 0$

A cone $M \subset E$ define a partial ordering << with respect to M by $\alpha << \beta$ if and only if $\beta - \alpha \in M$. The cone M called normal if there is a number $K > 0$ such that $0 \leq \alpha \leq \beta$ implies that $|\alpha| \leq K |\beta|$ for all α, β ∈ E. The least positive number K is called a normal constant.

Definition1.5. [11] Suppose E be the Banach space then M be a cone in E. Let X be a non-empty set and define a function Γ: $X^3 \rightarrow E$ satisfying the following properties

(i) Γ($\alpha_1,\alpha_2,\alpha_3$) ≥ 0

(ii) Γ($\alpha_1,\alpha_2,\alpha_3$) = 0 if and only if $\alpha_1 = \alpha_2 = \alpha_3$

(iii) Γ($\alpha_1,\alpha_2,\alpha_3$) ≤ Γ($\alpha_1,\alpha_1,\beta$)+Γ($\alpha_2,\alpha_2,\beta$)+Γ($\alpha_3,\alpha_3,\beta$)

Then Γ is called the cone - S-metric and the pair (X, Γ) is called the cone S- metric space.

2. Main Result

In this section we introduce a rectangular cone S-metric space and prove some fixed point theorems in rectangular cone S – metric space.

Definition 2.1. Let X be a non-empty set, $\mathcal{E}$ be a real Banach space and $\mathcal{M}$ be the cone of $\mathcal{E}$.

Define $\bar{\Gamma}$: $X^3 \rightarrow \mathcal{E}$ is satisfying the following properties,

(i) $\bar{\Gamma}$ ($\alpha_1, \alpha_2, \alpha_3$) ≥ 0.

(ii) $\bar{\Gamma}$ ($\alpha_1, \alpha_2, \alpha_3$) = 0 iff $\alpha_1 = \alpha_2 = \alpha_3$.

(iii) $\bar{\Gamma}$ ($\alpha_1,\alpha_2,\alpha_3$) ≤ $\bar{\Gamma}$ (α_1,α_1,β)+ $\bar{\Gamma}$ (α_2,α_2,β)+ $\bar{\Gamma}$ (α_3,α_3,β) for all $\alpha_1,\alpha_2,\alpha_3$ ∈X and distinct points β ∈ X −{$\alpha_1,\alpha_2,\alpha_3$}

Then (X, $\bar{\Gamma}$) is called the rectangular cone S - metric space.

Example 2.2. Let $\mathcal{E} = R^2$, $\mathcal{M}$ = {(α_1, α_2): $\alpha_1, \alpha_2 \geq 0$}, for X = N ∪{0}, $\bar{\Gamma}$: $X^3 \rightarrow \mathcal{E}$ such that

$$\bar{\Gamma}(\alpha_1, \alpha_2, \alpha_3) = \begin{cases} 0 & If\ \alpha_1 = \alpha_2 = \alpha_3 \\ d(\alpha_1,\alpha_3) + d(\alpha_2,\alpha_3), cd(\alpha_1,\alpha_3) + d(\alpha_2,\alpha_3) & for\ all\ c > 0\ and\ \alpha_1, \alpha_2\alpha_3 \in X \end{cases}$$

is a rectangular cone S- metric space.

Definition 2.3. Let (X , $\bar{\Gamma}$) ne a rectangular cone S-metric space and a sequence $\{\alpha_n\}$ in X converges to α if and only if $\bar{\Gamma}(\alpha_n, \alpha_n, \alpha) \rightarrow 0$ as n → ∞ then there exists n_0 ∈ N such that n ≥ n_0 and $\bar{\Gamma}(\alpha_n, \alpha_n, \alpha) \ll c\ for\ all\ c \in \mathcal{E}$, 0 << c.

Definition 2.4. Let (X , $\bar{\Gamma}$) be a rectangular cone S-metric space and a sequence $\{\alpha_n\}$ is called a Cauchy sequence , if $\bar{\Gamma}(\alpha_n, \alpha_n, \alpha_m) \rightarrow 0$ as n, m→∞, (i.e) there exists n_0 ∈ N such that $\bar{\Gamma}(\alpha_n, \alpha_n, \alpha_m)$<< c for each c ∈ $\mathcal{E}$ and 0 << ∈.

Definition 2.5. The rectangular cone S-metric space is complete, if every Cauchy sequence is convergent.

Theorem 2.6. Let (X, $\bar{\Gamma}$) be a complete rectangular cone S-metric space and $\mathcal{M}$ be the normal cone with normal constant k. The mapping T: X → X satisfies the following condition

$$\bar{\Gamma}(T\alpha, T\alpha, T\beta) \leq c\bar{\Gamma}(\alpha, \alpha, \beta) \quad (1)$$

For all α, β ∈ X and $c \in [0,1)$, then T has a unique fixed point γ ∈ X, and we have $\lim_{n\to\infty} T^n\alpha = \gamma$ for each α ∈ X.

Proof: Let α_0 ∈ X and the sequence $\{\alpha_n\}$ be defined as $T^n\alpha_0 = \alpha_n$.

Then
$$\begin{aligned} \bar{\Gamma}(\alpha_n, \alpha_n, \alpha_{n+1}) &= \bar{\Gamma}(T\alpha_{n-1}, T\alpha_{n-1}, T\alpha_n) \\ &\leq c\,\bar{\Gamma}(\alpha_{n-1}, \alpha_{n-1}, \alpha_n) \\ &\ldots\ldots\ldots\ldots \\ &\leq c^n\bar{\Gamma}(\alpha_0, \alpha_0, \alpha_1) \quad (2) \end{aligned}$$

Taking limit n → ∞, we get $\lim_{n\to\infty} \bar{\Gamma}(\alpha_n, \alpha_n, \alpha_{n+1}) = 0$, since c ∈ [0,1). The ϵ >0 implies that $\lim_{n\to\infty} \bar{\Gamma}(\alpha_n, \alpha_n, \alpha_{n+1}) = 0$.

Next to show that $\{\alpha_n\}$ is Cauchy. Assume that $\{\alpha_n\}$ is not Cauchy, then there exists ϵ >0 and subsequence $\{\alpha_{m_k}\}$ and $\{\alpha_{n_k}\}$ such that $m_k < n_k < m_{k+1}$,

$$\bar{\Gamma}(\alpha_{m_k}, \alpha_{m_k}, \alpha_{n_k}) \gg \epsilon \quad (3)$$

And
$$\bar{\Gamma}(\alpha_{m_k}, \alpha_{m_k}, \alpha_{n_{k-1}}) < \epsilon \quad (4)$$

Then
$$\begin{aligned} &\bar{\Gamma}(\alpha_{m_{k-1}}, \alpha_{m_{k-1}}, \alpha_{n_{k-1}}) \\ &\leq \bar{\Gamma}(\alpha_{m_{k-1}}, \alpha_{m_{k-1}}, \alpha_{m_k}) + \bar{\Gamma}(\alpha_{m_{k-1}}, \alpha_{m_{k-1}}, \alpha_{m_k}) + \bar{\Gamma}(\alpha_{n_{k-1}}, \alpha_{n_{k-1}}, \alpha_{m_k}) \\ &\leq 2\bar{\Gamma}(\alpha_{m_{k-1}}, \alpha_{m_{k-1}}, \alpha_{m_k}) + \bar{\Gamma}(\alpha_{n_{k-1}}, \alpha_{n_{k-1}}, \alpha_{m_k}) \\ &\leq 2\bar{\Gamma}(\alpha_{m_{k-1}}, \alpha_{m_{k-1}}, \alpha_{m_k}) + \epsilon \end{aligned}$$

Taking limit k →∞,

$$\lim_{k\to\infty} \bar{\Gamma}(\alpha_{m_{k-1}}, \alpha_{m_{k-1}}, \alpha_{n_{k-1}}) \leq \epsilon$$

From (1), (2), (3)

$\epsilon \leq \bar{\Gamma}\left(\alpha_{m_k}, \alpha_{m_k}, \alpha_{n_k}\right) \leq \bar{\Gamma}\left(\alpha_{m_{k-1}}, \alpha_{m_{k-1}}, \alpha_{n_{k-1}}\right) \leq c\epsilon$

Which is a contradiction, since c ∈ $(0,1)$. Hence $\{\alpha_n\}$ is a Cauchy since X is complete. There exists $\gamma \in X$ such that $\lim_{n\to\infty} T^n\alpha = \gamma$

From (1)

$\bar{\Gamma}(\mathrm{T}\gamma, \mathrm{T}\gamma, \alpha_{n+1}) = \bar{\Gamma}(\mathrm{T}\gamma, \mathrm{T}\gamma, \alpha_n)$

$\leq c\,\bar{\Gamma}(\gamma, \gamma, \alpha_n)$

$\lim_{n\to\infty} \|\bar{\Gamma}(\mathrm{T}\gamma, \mathrm{T}\gamma, \alpha_{n+1})\| \leq c\,K\|\bar{\Gamma}(\mathrm{T}\gamma, \mathrm{T}\gamma, \gamma)\|$

Since c ∈ $(0,1)$, then $\bar{\Gamma}(\mathrm{T}\gamma, \mathrm{T}\gamma, \gamma) = 0$ which implies that $\bar{\Gamma}(\mathrm{T}\gamma, \mathrm{T}\gamma, \gamma) \ll 0$ thus Tγ = γ.

Next to prove that T has a unique fixed point. Let γ, γ_1 be two fixed points of T such that $\gamma \neq \gamma_1$

From (1) $\bar{\Gamma}(\gamma, \gamma, \gamma_1) = \bar{\Gamma}(\mathrm{T}\gamma, \mathrm{T}\gamma, \gamma_1) \leq c\bar{\Gamma}(\gamma, \gamma, \gamma_1)$

Where c ∈ $(0,1)$, which implies that $\bar{\Gamma}(\gamma, \gamma, \gamma_1) = 0$

Thus $\gamma = \gamma_1$.

Corollary 2.7. Let (X , $\bar{\Gamma}$) be a complete rectangular cone S-metric space and $\mathcal{M}$ be the normal cone with normal constant k. The mapping T: X → X satisfies the following condition

$$\bar{\Gamma}(T^n\alpha, T^n\alpha, T^n\beta) \leq c\,\bar{\Gamma}(\alpha, \alpha, \beta)$$

Where c∈ $(0,1)$ is a constant then T has a unique fixed point.

Proof: Form theorem 2.6, T^n has a unique fixed point γ, but $T^n(T\gamma) = T(T^n\gamma)$ = T(γ).

Therefore Tγ is also a fixed point of T^n. Hence Tγ = γ , γ is a fixed point of T and also a fixed point of T^n.

Theorem 2.8. Let (X , $\bar{\Gamma}$) be a complete rectangular cone S-metric space and T: X → X be a mapping there exists a real number b satisfying 0 ≤ b ≤ ½ such that α, β, γ ∈ X.

$$\bar{\Gamma}(\mathrm{T}\alpha, \mathrm{T}\beta, \mathrm{T}\gamma,) \leq b\,[\,\bar{\Gamma}(\alpha, \mathrm{T}\alpha,, \mathrm{T}\alpha,) + \bar{\Gamma}(\beta, \mathrm{T}\beta, \mathrm{T}\beta,) + \bar{\Gamma}(\gamma, \mathrm{T}\gamma, \mathrm{T}\gamma,)] \qquad (5)$$

Then T has a unique fixed point.

Proof: Let $\alpha_0 \in X$, define a sequence $\{\alpha_n\}$ by $\alpha_n = T^n\alpha_0$.

Then $\bar{\Gamma}(\alpha_n, \alpha_n, \alpha_{n+1}) = \bar{\Gamma}(T\alpha_{n-1}, \mathrm{T}\alpha_{n-1}, \mathrm{T}\alpha_n)$

$\leq b[\bar{\Gamma}(\alpha_{n-1}, \mathrm{T}\alpha_{n-1}, \mathrm{T}\alpha_{n-1}) + \bar{\Gamma}(\alpha_{n-1}, \mathrm{T}\alpha_{n-1}, \mathrm{T}\alpha_{n-1}) + \bar{\Gamma}(\alpha_n, \mathrm{T}\alpha_n, \mathrm{T}\alpha_n)$

$\leq b[\bar{\Gamma}(\alpha_{n-1}, \alpha_n, \alpha_n) + \bar{\Gamma}(\alpha_{n-1}, \alpha_n, \alpha_n) + \bar{\Gamma}(\alpha_n, \alpha_{n+1}, \alpha_{n+1})$

$\leq 2b\bar{\Gamma}(\alpha_{n-1}, \alpha_n, \alpha_n) + b\bar{\Gamma}(\alpha_n, \alpha_{n+1}, \alpha_{n+1})$

$\leq 2b\bar{\Gamma}(\alpha_{n-1}, \alpha_n, \alpha_n) + b\bar{\Gamma}(\alpha_{n+1}, \alpha_n, \alpha_n)$

$= \frac{2b}{(1-b)}\,\bar{\Gamma}(\alpha_{n-1}, \alpha_{n-1}, \alpha_n)$

Since $\bar{\Gamma}$ is symmetric and $\rho = \frac{2b}{(1-b)} < \frac{1}{2}$

$\bar{\Gamma}(\alpha_n, \alpha_n, \alpha_{n+1}) \leq \rho\bar{\Gamma}(\alpha_{n-1}, \alpha_{n-1}, \alpha_n)$

Continuing the process, we get

$\bar{\Gamma}(\alpha_n, \alpha_n, \alpha_{n+1}) \leq \rho^n\,\bar{\Gamma}(\alpha_0, \alpha_0, \alpha_1)$

By (ii) of definition 2.1, we have

$\bar{\Gamma}(\alpha_n, \alpha_m, \alpha_m) \leq \bar{\Gamma}(\alpha_n, \alpha_n, \alpha_{n+1}) + \bar{\Gamma}(\alpha_m, \alpha_m, \alpha_{n+1}) + \bar{\Gamma}(\alpha_m, \alpha_m, \alpha_{n+1})$

$= \bar{\Gamma}(\alpha_n, \alpha_n, \alpha_{n+1}) + 2\bar{\Gamma}(\alpha_m, \alpha_m, \alpha_{n+1})$

$\leq \bar{\Gamma}(\alpha_n, \alpha_n, \alpha_{n+1}) + 2\,\bar{\Gamma}(\alpha_{m+1}, \alpha_{m+1}, \alpha_{n+2}) + 2^2\bar{\Gamma}(\alpha_m, \alpha_m, \alpha_{n+2})$

$\leq \bar{\Gamma}(\alpha_n, \alpha_n, \alpha_{n+1}) + 2\,\bar{\Gamma}(\alpha_{m+1}, \alpha_{m+1}, \alpha_{n+2}) + 2^2\bar{\Gamma}(\alpha_{n+2}, \alpha_{n+2}, \alpha_{n+3}) + \ldots\ \ldots\ldots\ldots. + 2^{m-1}\bar{\Gamma}(\alpha_m, \alpha_m, \alpha_{m+1})$

$\leq \rho^n\,\bar{\Gamma}(\alpha_0, \alpha_0, \alpha_1) + 2\rho^{n+1}\,\bar{\Gamma}(\alpha_0, \alpha_0, \alpha_1) + 2^2\rho^{n+2}\,\bar{\Gamma}(\alpha_0, \alpha_0, \alpha_1) + \ldots\ldots + 2^{m-1}\rho^{m-1}\,\bar{\Gamma}(\alpha_0, \alpha_0, \alpha_1)$

$\leq \rho^n(1 + 2\rho + (2\rho)^2 + \ldots\ldots\ldots + (2\rho)^{m-n-1})\bar{\Gamma}(\alpha_0, \alpha_0, \alpha_1)$

$\bar{\Gamma}(\alpha_n, \alpha_m, \alpha_m) \leq \rho^n(1-2\rho)^{-1}\bar{\Gamma}(\alpha_0, \alpha_0, \alpha_1)$

Taking the limit n, m → ∞ , we have

$$\lim_{n,m\to\infty} \bar{\Gamma}(\alpha_n, \alpha_m, \alpha_m) = \lim_{n,m\to\infty} \rho^n(1-2\rho)^{-1}\bar{\Gamma}(\alpha_0, \alpha_0, \alpha_1) = 0.$$

For n,m, l ∈ N with n > m > l

$\bar{\Gamma}(\alpha_n, \alpha_m, \alpha_l) \leq \bar{\Gamma}(\alpha_n, \alpha_n, \alpha_{n-1}) + \bar{\Gamma}(\alpha_m, \alpha_m, \alpha_{n-1}) + \bar{\Gamma}(\alpha_l, \alpha_l, \alpha_{n-1})$

Taking the of $\bar{\Gamma}(\alpha_n, \alpha_m, \alpha_l)$ $as\ n.m.l. \to \infty$

$$\lim_{n,m,l \to \infty} \bar{\Gamma}(\alpha_n, \alpha_m, \alpha_l) = 0.$$

Therefore$\{\alpha_n\}$ is Cauchy sequence and (X , $\bar{\Gamma}$) be a complete rectangular cone S- metric space such that $\{\alpha_n\}$ is $\bar{\Gamma}$ – convergent to α. Suppose $T\alpha \neq \alpha$

From (5)

$$\bar{\Gamma}(\alpha_n, T\alpha, T\alpha) \leq b[\bar{\Gamma}(\alpha_{n-1}, \alpha_n, \alpha_n) + \bar{\Gamma}(\alpha, T\alpha, T\alpha) + \bar{\Gamma}(\alpha, T\alpha, T\alpha)$$
$$\leq b[\bar{\Gamma}(\alpha_{n-1}, \alpha_n, \alpha_n) + 2\bar{\Gamma}(\alpha, T\alpha, T\alpha)$$

Taking the limit as n→ ∞,

$$\bar{\Gamma}(\alpha_n, T\alpha, T\alpha) \leq 2b\,\bar{\Gamma}(\alpha_n, T\alpha, T\alpha)$$

Hence $\bar{\Gamma}(\alpha_n, T\alpha, T\alpha) \leq 0$, which is a contradiction.

Therefore $T\alpha = \alpha$, to show that the uniqueness γ≠ α such that Tγ = γ, then

$$\bar{\Gamma}(T\alpha, T\gamma, T\gamma,) \leq b\,[\bar{\Gamma}(\alpha, T\gamma, T\gamma) + \bar{\Gamma}(\gamma, T\gamma, T\gamma,) + \bar{\Gamma}(\gamma, T\gamma, T\gamma,)]$$

Since $T\alpha = T\alpha$ and $T\gamma = \gamma$ implies that $\alpha = \gamma$.

Example 2.9: Let X be a non- empty set, $\mathcal{E}=R^2$ be the real Banach space, and $\mathcal{M}$ be the normal cone in $\mathcal{E}$. Then the function $\bar{\Gamma}: X^3 \to \mathcal{E}$ is defined by $\bar{\Gamma}(\alpha, \beta, \gamma,) = |\alpha - \gamma| + |\beta - \gamma|$ for all α, β, γ $\in X$ then (X, $\bar{\Gamma}$) be a complete rectangular cone S-metric space. Consider the mapping T : X→X is defined by $T\alpha = \frac{\alpha+1}{3}$ then

$$\begin{aligned}\bar{\Gamma}(T\alpha, T\alpha, T\beta) &= |T\alpha - T\beta| + |T\alpha - T\beta| \\ &= 2|T\alpha - T\beta| \\ &= \frac{2}{3}|\alpha - \beta| \\ &\leq \frac{1}{2}[2|\alpha - \beta|] \\ &= c\bar{\Gamma}(\alpha, \alpha, \beta)\end{aligned}$$

Since c=1/2 < 1. Hence T satisfies all the conditions of theorem 2.7 and $0 \in X$ is a unique fixed point.

3. Conclusion

In this paper, we introduced a rectangular cone e S- metric space and proved some fixed point results. Our results extends several fixed point results in existing literature.

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