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Multi-Objective Optimization of a Two-stage Helical Gearbox with Second Stage Double Gear Sets to Decrease Gearbox Bottom Area and Improve Gearbox Efficiency

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Abstract
Introduction: The purpose of this study is to investigate multi-target optimization of a two-stage helical gearbox with second stage double gear sets (SSDGS) in order to identify the optimum essential design parameters for reducing gearbox bottom area and increasing gearbox efficiency. The Taguchi approach and grey relation analysis (GRA) were employed to solve the problem in two steps. The single-objective optimization problem was handled first in order to decrease the gap among variable levels, and then the multi-objective optimization problem was tackled in order to identify the best primary design variables. The first and second stage coefficients of wheel face width (CWFW), permissible contact stresses (ACS), and first stage gear ratio were also computed. The outcomes of the study were used to determine the optimal values for five essential design parameters of a two-stage helical gearbox with SSDGS.

Keywords: Helical gearbox, Double gear sets, Optimization, Multi-objective, Gear ratio, Bottom area, Gearbox efficiency.

1. Introduction

The gearbox is a critical component of a mechanical drive system. It is used to slow the speed of the motor shaft while increasing the torque from the motor shaft to the working shaft. As a result, the optimal design of the gearbox has become a very significant problem that has drawn the attention of many scientists.

There has been several research on the best design of helical gearboxes up to that date. [1] investigates the effects of eleven input factors on second and third stage ratios in order to reduce the cost of a three-stage helical gearbox. [2] describes a research that used a chain and a two-stage helical reducer to optimize partial transmission ratios in mechanical drive systems. [3] performs a simulation experiment to reduce the volume of a two-stage helical gearbox. Also, [4] concentrated on developing hybrid composite gears for a two-stage constant mesh helical gearbox. It was discovered that by substituting EN36 steer gears with EN36/Carbon Fibre Composite material, the helical gear's weight and inertia were lowered, increasing the safety of gearbox use. Recognizing the significance of gearbox cost minimization in both design and manufacture, [5] calculated cost using component mass for two-stage helical gearboxes with second-stage double gear-sets. [6] suggested two approaches for identifying ideal gear ratios to decrease helical reducer cross-sectional area in the same region of concern. Furthermore, [7] carried out a simulation experiment to investigate the relationship between partial gear ratios and input parameters, from which models for splitting the total gear ratio of a two-step helical reducer were developed. [8] also looked at the use of airborne sound for condition monitoring of a multi-stage helical gearbox. [9] reported on a unique study that used optimization and regression techniques to determine optimal partial ratios of three-step helical gearboxes with second-step double gear-sets. Additionally, to obtain the smallest system length, [10] suggested suitable gear ratios for mechanically driven systems employing a two-stage helical gearbox with double gear sets in the first stage and a chain drive. [11] examines appropriate gear ratios for a system that employs a two-stage helical gearbox with double gear sets in the first stage.

The goal of this study is to investigate multi-target optimization learning for a two-stage helical gearbox using SSDGS. In this work, two single objectives were pursued: lowering gearbox bottom area and enhancing gearbox efficiency. In addition, the CWFW for both stages, the ACS for both stages, and the gear ratio for the first stage were also analyzed. Furthermore, the multi-objective optimization issue in gearbox design was solved in two phases by integrating the Taguchi approach with the GRA. The optimal values for five essential design factors were also offered for creating a two-stage helical gearbox with SSDGS.

2. Optimization problem

2.1 Gearbox bottom area calculation

The bottom area A_b of the gearbox can be calculated by (Fig. 1):

$$A_b = L \cdot B \quad (1)$$

In which, L, and B are determined by (Fig. 1):

$$L = \frac{d_{w11}}{2} + a_{w1} + a_{w2} + \frac{d_{w22}}{2} + 2 \cdot S_G + 2 \cdot k \quad (2)$$

$$B = b_{w1} + b_{w2} + 7 \cdot S_G \quad (3)$$

In (1), $k = 8 \div 12$ [12]; d_{w11}, d_{w22} are gear pitch diameters of the first and second stages which are found by [12]:

$$d_{w11} = 2\, a_{w1}/(u_1 + 1) \quad (4)$$

$$d_{w22} = 2\, a_{w2} u_2\, /(u_2 + 1) \quad (5)$$

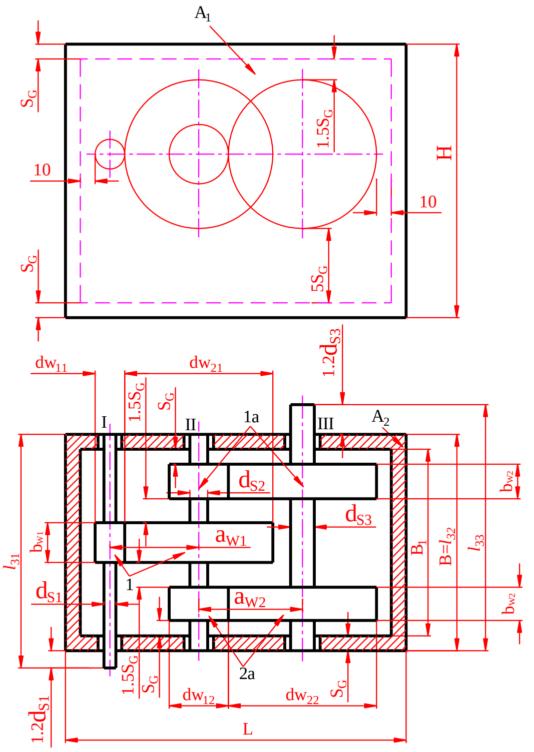

Fig. 1. Calculated schema

In the above equations, a_{w1} and a_{w2} are the center distances of the first and the second stages; b_{w1} and b_{w2} are the gear width of stage 1 and 2. These parts can be calculated by [12]:

$$a_{w1} = k_a(u_1 + 1)\sqrt[3]{T_{11}k_{H\beta1}/([\sigma_{H1}]^2 u_1 X_{ba1})}$$

$$a_{w2} = k_a(u_2 + 1)\sqrt[3]{T_{12}k_{H\beta2}/([\sigma_{H2}]^2 u_2 X_{ba2})}$$

$$b_{w1} = X_{ba1} \cdot a_{w1}$$

$$b_{w2} = X_{ba2} \cdot a_{w1}$$

In (4) and (5), $k_a = 43$ is the material coefficient [12]; $k_{H\beta1}$ and $k_{H\beta2}$ are the contacting load ratio for pitting resistance of the first and second stages; $k_{H\beta1} = 1.0 \div 1.06$ and $k_{H\beta2} = 1.02 \div 1.28$ [12]. $[\sigma_{H1}]$ and $[\sigma_{H2}]$ are ACS of stages 1 and 2 (MPa); u_1 and u_2 are the gear ratios of stages 1 and 2. X_{ba1} and X_{ba2} are CWFW and T_{11} and T_{12} are the torque on the pinion (Nmm) of stages 1 and 2:

$$T_{11} = T_{out}/\left(u_g \cdot \eta_{hg}^2 \cdot \eta_b^3\right)$$

$$T_{12} = T_{out}/\left(2 \cdot u_2 \cdot \eta_{hg} \cdot \eta_{be}^2\right)$$

In which, T_{out} is the output torque (N.mm); η_{hg} is the helical gear efficiency ($\eta_{hg} = 0.96 \div 0.98$); η_b is the rolling bearing efficiency (η_h=0.99÷0.995) [12].

2.2 Gearbox efficiency calculation

The gearbox's efficiency can be determined by:

$$\eta_{gb} = \frac{100 \cdot P_l}{P_{in}}$$

Where, P_l is the overall gearbox power loss[13]:

$$P_l = P_{lg} + P_{lb} + P_{ls}$$

Where, P_{lg} is the total gear power loss; P_{lb} is the bearing power loss; P_{ls} is the seal power loss. These elements can be found by:

+) The power loss in gears:

$$P_{lg} = \sum_{i=1}^{2} P_{lgi}$$

With P_{lgi} is the power losses in gears of i stage:

$$P_{lgi} = P_{gi} \cdot \left(1 - \eta_{gi}\right)$$

In which, η_{gi} is the anticipated gear efficiency of the i stage [14]:

$$\eta_{gi} = 1 - \left(\frac{1 + 1/u_i}{\beta_{ai} + \beta_{ri}}\right) \cdot \frac{f_i}{2} \cdot (\beta_{ai}^2 + \beta_{ri}^2)$$

In which, u_i is the gear ratio of i stage; f is the friction coefficient; β_{ai} and β_{ri} are the arcs of approach and retreat on the i stage which can be determined by [14]:

$$\beta_{ai} = \frac{\left(R_{e2i}^2 - R_{02i}^2\right)^{1/2} - R_{2i} \cdot \sin\alpha}{R_{01i}}$$

$$\beta_{ri} = \frac{\left(R_{e1i}^2 - R_{01i}^2\right)^{1/2} - R_{1i} \cdot \sin\alpha}{R_{01i}}$$

In the above equations, R_{e1i} and R_{e2i} are the outer radiuses of the pinion and gear; R_{1i} and R_{2i} are the pitch radiuses of the pinion and gear; R_{01i} and R_{01i} are the base-circle radiuses of the pinion and gear; α is the pressure angle.
In (12), f is friction coefficient which can be found by [15]:

- When the sliding velocity v ≤ 0.424 (m/s):

$$f = -0.0877 \cdot v + 0.0525 \quad (19)$$

- When the sliding velocity v > 0.424 (m/s) :

$$f = 0.0028 \cdot v + 0.0104 \quad (20)$$

+) The power loss in bearings can be calculated by [13]:

$$P_{lb} = \sum_{i=1}^{6} f_b \cdot F_i \cdot v_i \quad (21)$$

In which, $f_b = 0.0011$ is the bearing friction coefficient (for radical ball bearings with angular contact) [13]; F is the bearing load (N); v is the peripheral speed; i is the bearing ordinal number (i = 1÷6).
+) The overall power loss in seals can be determined by [13]:

$$P_s = \sum_{i=1}^{2} \mathrm{P_{si}} \quad (22)$$

Where P_{si} is the power loss in a single seal (w):

$$P_{si} = [145 - 1.6 \cdot t_{oil} + 350 \cdot log\, log(VG_{40} + 0.8)] \cdot d_s^2 \cdot n \cdot 10^{-7} \quad (23)$$

With VG_{40} is the ISO Viscosity Grade number.

2.3 Objective function and constrains

2.3.1. Objectives Functions

In this work, the multi-objective optimization problem consists of two distinct targets:
Minimizing the gearbox bottom area:

$$\min f_2(X) = \mathrm{A}b \quad (24)$$

Maximizing the gearbox efficiency:

$$\min f_1(X) = \eta_{gb} \quad (25)$$

In which X is design variable vector. In this study, five main design factors including u_1, Xba_1, Xba_2, AS_1, and AS_2 have been chosen as variables. Therefore, gets:

$$X = \{u_1, Xba_1, Xba_2, AS_1, AS_2\}$$

2.3.2. Constrains

The multi-objective function has the following constraints:

$$1 \le u_1 \le 9 \text{ and } 1 \le u_2 \le 9$$
$$0.25 \le X_{ba1} \le 0.3 \text{ and } 0.25 \le X_{ba2} \le 0.4$$
$$350 \le AS_1 \le 420 \text{ and } 350 \le AS_2 \le 420 \quad (29)$$

3. Methods

In this study, five main design factors were chosen for analysis. The lowest and maximum values for various variables are shown in Table 1. The Taguchi approach and grey relation analysis were used to tackle the optimization problem. To optimize the number of levels for each variable, the L25 (5^5) design was employed. However, among the variables evaluated, u1 has a rather broad range (in this case u_1=1÷9 - Table 1). Even with five levels, the difference in the values of these qualities remained significant (in this example, the difference is ((9-1)/4 = 2).
The 2-stage multi-objective optimization problem solution approach [16] was used to help close the gap between variable values scattered throughout a large range. The first step of this approach deals with a single-objective optimization issue, while the second stage deals with a multi-objective optimization problem to identify the best key design elements.

Table 1. Main design parameters and their maximum and minimum limits

Factor	Notation	Minimum limit	Maximum limit
Gear ratio of stage 1	u_1	1	9
CWFW of stage 1	X_{ba1}	0.25	0.3
CWFW of stage 2	X_{ba2}	0.25	0.4
ACS of stage 1 (MPa)	AS_1	350	420
ACS of stage 2 (MPa)	AS_2	350	420

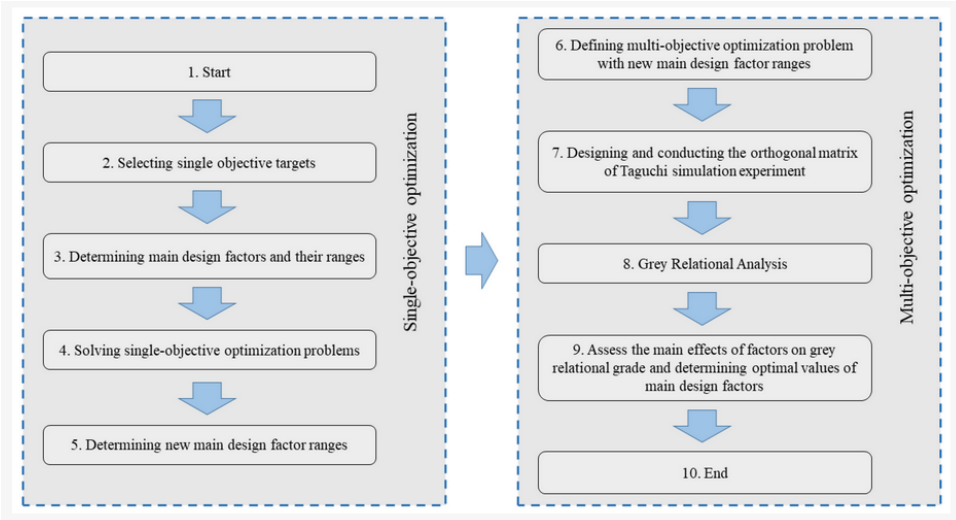

Fig. 2. Method for solving multi-objective problem [15]

4. Single-objective Optimization

In this study, the direct search approach is utilized to solve the single-objective optimization problem. A computer program based on Matlab was also developed to tackle two single-objective problems: lowering gearbox bottom area and optimizing gearbox efficiency. Based on the program's findings, Figure 3 demonstrates a relationship between the ideal gear ratio of the first stage u_1 and the total gearbox ratio u_t. In addition, additional constraints for the variable u_1 have been devised, as indicated in Table 2.

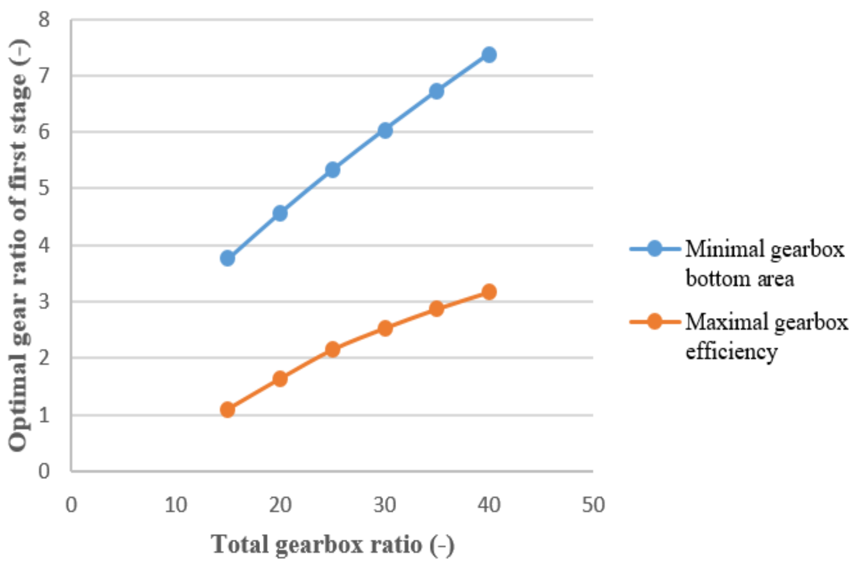

Fig. 3. Optimal gear ratio of stage u_1 versus total gearbox ratio

Table 2. New constraints of u_1

u_t	u_1	
	Minimum limit	Maximum limit
15	1.09	3.77
20	1.63	4.58
25	2.14	5.34
30	2.52	6.05
35	2.86	6.73
40	3.16	7.38

5. Multi-objective Optimization

The goal of this work's multi-objective optimization problem for a two-stage helical gearbox with SSDGS is to find the best primary design variables with a given total gearbox ratio that meet two single-objective functions: minimizing gearbox bottom area and maximizing gearbox efficiency. To accomplish this, a computer experiment was carried out. Table 3 shows the main design elements and their values for u_t = 20. The experimental design was built with the Taguchi approach and the L25 (5^5) design, and the data was analyzed with Minitab R18 software. Table 4 shows the experimental plan and outcomes for u_t = 20.

Table 3. Main design factors and their levels for u_t = 20

Factor	Notation	Level				
		1	2	3	4	5
Gear ratio of stage 1	u_1	1.63	2.3675	3.105	3.8425	5.34
CWFW of stage 1	X_{ba1}	0.25	0.2675	0.275	0.2875	0.3
CWFW of stage 2	X_{ba2}	0.25	0.2875	0.325	0.3625	0.4
ACS of stage 1 (MPa)	AS_1	350	368	386	404	420
ACS of stage 2 (MPa)	AS_2	350	368	386	404	420

Table 4. Experimental plan and output results for u_t = 20

Exp. No.	Input Factors					Ab	η_{gb}
	u_1	X_{ba1}	X_{ba2}	AS_1	AS_2	(dm²)	(%)
1	1.6300	0.2500	0.2500	350	350	6.408	95.572
2	1.6300	0.2625	0.2875	368	368	6.155	95.540
3	1.6300	0.2750	0.3250	386	386	5.924	95.534
4	1.6300	0.2875	0.3625	404	404	5.711	95.500
5	1.6300	0.3000	0.4000	420	420	5.545	95.498
6	2.3675	0.2500	0.2875	386	404	5.168	95.286
7	2.3675	0.2625	0.3250	404	420	5.010	95.306
8	2.3675	0.2750	0.3625	420	350	5.832	95.339
9	2.3675	0.2875	0.4000	350	368	6.035	95.281
10	2.3675	0.3000	0.2500	368	386	5.475	95.359
11	3.1050	0.2500	0.3250	420	368	5.282	95.130
12	3.1050	0.2625	0.3625	350	386	5.532	95.127
13	3.1050	0.2750	0.4000	368	404	5.317	95.109
14	3.1050	0.2875	0.2500	386	420	4.887	95.129
15	3.1050	0.3000	0.2875	404	350	5.591	95.153
16	3.8425	0.2500	0.3625	368	420	5.047	94.951
17	3.8425	0.2625	0.4000	386	350	5.753	94.989
18	3.8425	0.2750	0.2500	404	368	5.232	94.998
19	3.8425	0.2875	0.2875	420	386	5.040	94.998
20	3.8425	0.3000	0.3250	350	404	5.335	94.979
21	5.3400	0.2500	0.4000	404	386	5.315	94.640
22	5.3400	0.2625	0.2500	420	404	4.889	94.669
23	5.3400	0.2750	0.2875	350	420	5.269	94.648
24	5.3400	0.2875	0.3250	368	350	5.890	94.683
25	5.3400	0.3000	0.3625	386	368	5.642	94.664

The Taguchi and GRA approaches are used for dealing with multi-objective optimization problems. The following are the major steps in this approach:

+) Using the following equations, calculate the signal-to-noise ratio (S/N):

The greater the S/N, the lower the gearbox bottom area:

$$SN = -10log_{10}(\frac{1}{n}\sum_{i=1}^{m} y_i^2) \quad (30)$$

The larger the S/N, the more efficient the gearbox:

$$SN = -10log_{10}(\frac{1}{n}\sum_{i=1}^{m} \frac{1}{y_i^2}) \quad (31)$$

In which, y_i is the output result and m í the experimental repetition number. As this is a simulation, m = 1; That means no repetitions are needed. Table 5 presents the calculated S/N values for the objectives.

Table 5. S/N values for each experiment when u_t=20

No.	Input Factors					Ab		η_{gb}	
	u_1	X_{ba1}	X_{ba2}	AS_1	AS_2	(dm²)	S/N	(%)	S/N
1	1.6300	0.2500	0.2500	350	350	6.408	-16.1345	95.572	39.6066
2	1.6300	0.2625	0.2875	368	368	6.155	-15.7846	95.540	39.6037
3	1.6300	0.2750	0.3250	386	386	5.924	-15.4523	95.534	39.6032
4	1.6300	0.2875	0.3625	404	404	5.711	-15.1342	95.500	39.6001
5	1.6300	0.3000	0.4000	420	420	5.545	-14.8780	95.498	39.5999
6	2.3675	0.2500	0.2875	386	404	5.168	-14.2665	95.286	39.5806
7	2.3675	0.2625	0.3250	404	420	5.010	-13.9968	95.306	39.5824
8	2.3675	0.2750	0.3625	420	350	5.832	-15.3164	95.339	39.5854
9	2.3675	0.2875	0.4000	350	368	6.035	-15.6135	95.281	39.5801
10	2.3675	0.3000	0.2500	368	386	5.475	-14.7677	95.359	39.5872
11	3.1050	0.2500	0.3250	420	368	5.282	-14.4560	95.130	39.5663
12	3.1050	0.2625	0.3625	350	386	5.532	-14.8576	95.127	39.5661
13	3.1050	0.2750	0.4000	368	404	5.317	-14.5133	95.109	39.5644
14	3.1050	0.2875	0.2500	386	420	4.887	-13.7808	95.129	39.5663
15	3.1050	0.3000	0.2875	404	350	5.591	-14.9498	95.153	39.5684
16	3.8425	0.2500	0.3625	368	420	5.047	-14.0607	94.951	39.5500
17	3.8425	0.2625	0.4000	386	350	5.753	-15.1979	94.989	39.5535
18	3.8425	0.2750	0.2500	404	368	5.232	-14.3734	94.998	39.5543
19	3.8425	0.2875	0.2875	420	386	5.040	-14.0486	94.998	39.5543
20	3.8425	0.3000	0.3250	350	404	5.335	-14.5427	94.979	39.5526
21	5.3400	0.2500	0.4000	404	386	5.315	-14.5101	94.640	39.5215
22	5.3400	0.2625	0.2500	420	404	4.889	-13.7844	94.669	39.5242
23	5.3400	0.2750	0.2875	350	420	5.269	-14.4346	94.648	39.5222
24	5.3400	0.2875	0.3250	368	350	5.890	-15.4023	94.683	39.5254
25	5.3400	0.3000	0.3625	386	368	5.642	-15.0287	94.664	39.5237

The data quantities for the two single-objective functions are different. To ensure comparability, the data must be normalized, or brought to a standard scale. The normalization value Z_{ij}, which ranges from 0 to 1, is used to normalize the data. This value is obtained using the following formula:

$$Z_i = \frac{SN_i - min\ (SN_i, = 1,2,..n)}{max(SN_i, j = 1,2,..n) - min\ (SN_i, = 1,2,..n)} \quad (32)$$

With n=25 is the experimental number.

+) The grey relational factor can be found by:

$$y_i(k) = \frac{\Delta_{min} + \xi.\Delta_{max}(k)}{\Delta_i(k) + \xi.\Delta_{max}(k)} \quad (33)$$

Where, i=1,2,...,n; k=2 is the objective number; $\Delta_j(k) = \|Z_0(k) - Z_j(k)\|$ with Z0(k) and Zj(k) are the reference and particular comparison sequence; Δ_{min} and Δ_{max} are the minimum and maximum values of i(k); ζ=0.5 is the characteristic coefficient.

+) Calculating the coefficient of grey relations by:

$$\overline{y_i} = \frac{1}{k}\sum_{j=0}^{k} y_{ij}\ (k)$$

In which y_{ij} is the grey relation value of the j^th^ output aim of the i^th^ experiment. For every test, the predicted grey relation number y_i as well as the average grey relation value $\overline{y_i}$ was presented in table 6.

A greater average grey relation value is suggested to promote harmony among the output elements. As a consequence, a multi-objective problem's objective function may be reduced to a single-objective optimization problem, generating the mean grey relation value.

Table 6. Values of $\Delta_i(k)$ and $\overline{y_i}$

No.	S/N		Zi		Δ_i (k)		Grey relation value y_i		$\overline{y_i}$
			Ab	ngb					
	Ab	η_{gb}	Reference values		Ab	η_{gb}	Ab	η_{gb}	
			1.000	1.000					
1	-16.1345	39.6066	0.0000	1.0000	1.000	0.000	0.333	1.000	0.667
2	-15.7846	39.6037	0.1487	0.9658	0.851	0.034	0.370	0.936	0.653
3	-15.4523	39.6032	0.2898	0.9594	0.710	0.041	0.413	0.925	0.669
4	-15.1342	39.6001	0.4250	0.9231	0.575	0.077	0.465	0.867	0.666
5	-14.8780	39.5999	0.5338	0.9210	0.466	0.079	0.518	0.863	0.691
6	-14.2665	39.5806	0.7937	0.6942	0.206	0.306	0.708	0.620	0.664
7	-13.9968	39.5824	0.9083	0.7156	0.092	0.284	0.845	0.637	0.741
8	-15.3164	39.5854	0.3476	0.7509	0.652	0.249	0.434	0.667	0.551
9	-15.6135	39.5801	0.2213	0.6888	0.779	0.311	0.391	0.616	0.504
10	-14.7677	39.5872	0.5807	0.7723	0.419	0.228	0.544	0.687	0.616
11	-14.4560	39.5663	0.7132	0.5270	0.287	0.473	0.635	0.514	0.575
12	-14.8576	39.5661	0.5425	0.5238	0.458	0.476	0.522	0.512	0.517
13	-14.5133	39.5644	0.6888	0.5044	0.311	0.496	0.616	0.502	0.559
14	-13.7808	39.5663	1.0000	0.5259	0.000	0.474	1.000	0.513	0.757
15	-14.9498	39.5684	0.5033	0.5516	0.497	0.448	0.502	0.527	0.514
16	-14.0607	39.5500	0.8811	0.3348	0.119	0.665	0.808	0.429	0.619
17	-15.1979	39.5535	0.3979	0.3756	0.602	0.624	0.454	0.445	0.449
18	-14.3734	39.5543	0.7483	0.3853	0.252	0.615	0.665	0.449	0.557
19	-14.0486	39.5543	0.8862	0.3853	0.114	0.615	0.815	0.449	0.632
20	-14.5427	39.5526	0.6763	0.3649	0.324	0.635	0.607	0.440	0.524
21	-14.5101	39.5215	0.6902	0.0000	0.310	1.000	0.617	0.333	0.475
22	-13.7844	39.5242	0.9985	0.0313	0.002	0.969	0.997	0.340	0.669
23	-14.4346	39.5222	0.7222	0.0086	0.278	0.991	0.643	0.335	0.489
24	-15.4023	39.5254	0.3111	0.0464	0.689	0.954	0.421	0.344	0.382
25	-15.0287	39.5237	0.4698	0.0259	0.530	0.974	0.485	0.339	0.412

Table 7 displays the results of an ANOVA test run to assess the effect of the key design factors on the average grey relation value $\overline{y_i}$. According to Table 7, u_1 has the greatest influence on $\overline{y_i}$ (38.79%), followed by AS_2 (28.5%), X_{ba2} (16.86%), AS_1 (8.04%), and X_{ba1} (4.49%). Table 8 demonstrates the order of the effect of the main design variables on $\overline{y_i}$ using ANOVA analysis.

Table 7. Analysis of variance for means

Analysis of Variance for Means

Source	DF	Seq SS	Adj SS	Adj MS	F	P	C (%)
u1	4	0.093275	0.093275	0.023319	11.66	0.018	38.79
Xba1	4	0.010786	0.010786	0.002697	1.35	0.390	4.49
Xba2	4	0.040537	0.040537	0.010134	5.07	0.073	16.86
AS1	4	0.019338	0.019338	0.004834	2.42	0.207	8.04
AS2	4	0.068521	0.068521	0.017130	8.57	0.030	28.50
Residual Error	4	0.007999	0.007999	0.002000			3.33
Total	24	0.240455					

Model Summary

S	R-Sq	R-Sq(adj)
0.0447	96.67%	80.04%

+) *Calculating optimum main design factors:* In theory, the best factor set would include core design elements with the greatest S/N values. As a result, the influence of the main design aspects on the S/N ratio (Fig. 4) was calculated. Furthermore, the optimal set of multi-objective parameters (corresponding to the red points) may be simply deduced from the Figure 4 chart. Table 9 displays the proper levels and values for the key design variables of the multi-objective function.

+) Evaluating proposal modeling: Figure 5 displays the results of the Anderson-Darling approach, which is used to evaluate the adequacy of the proposed model. The data points corresponding to the experimental observations (shown in the graph as blue dots) are within the 95% standard deviation zone specified by the top and bottom limits. Furthermore, the p-value of 0.498 is much higher than the significance level of α = 0.05. These findings show that the empirical model used in this work is suitable for evaluation.

Table 8. Response table for means

Response Table for Means

Level	u1	Xba1	Xba2	AS1	AS2
1	0.669	0.5999	0.6529	0.5401	0.5126
2	0.6151	0.6059	0.5905	0.5657	0.5401
3	0.5844	0.565	0.5782	0.5903	0.5817
4	0.556	0.588	0.5529	0.5908	0.6164
5	0.4855	0.5513	0.5356	0.6232	0.6592
Delta	0.1835	0.0546	0.1173	0.0832	0.1465
Rank	1	5	3	4	2

Average of grey analysis value: 0.582

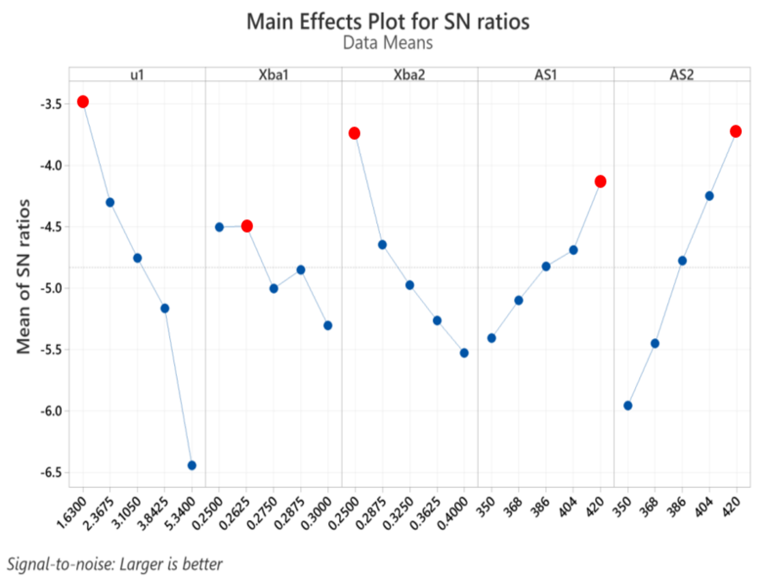

Fig. 4. Main effects plot for S/N ratios

Table 9. Optimum values of main design parameters

No.	Input Parameters	Code	Optimum Level	Optimum Value
1	Gear ratio of first stage	u_1	1	1.63
2	CWFW of stage 1	Xba1	2	0.2625
3	CWFW of stage 2	Xba2	1	0.25
4	ACS of stage 1 (MPa)	AS1	5	420
5	ACS of stage 2 (MPa)	AS2	5	420

Continue in the same manner as with u_t=20, but with the u_t values 15, 25, 30, 35, and 40. Table 10 shows the ideal values of the five main design parameters for each of the five main design parameters at different ut. Figure 6 depicts the relationship between the ideal first-stage gear ratio and the overall gearbox ratio. In addition, to calculate the ideal values of u_1, the following regression formula (with R^2=0.999) was proposed:

$u_1 = 2.1241 \cdot \ln(u_t) - 4.694$

After having u_1, the optimal gear ratio of second stage u_2 is determined by $u_2=u_t/u_1$.

Table 10. Optimal values of main design factors

No.	u_t					
	15	20	25	30	35	40
u_1	1.09	1.63	2.14	2.52	2.86	3.16
X_{ba1}	0.25	0.2625	0.2265	0.2265	0.2625	0.2625
X_{ba2}	0.25	0.25	0.25	0.25	0.25	0.25
AS_1	420	420	420	420	420	420
AS_2	420	420	420	420	420	420

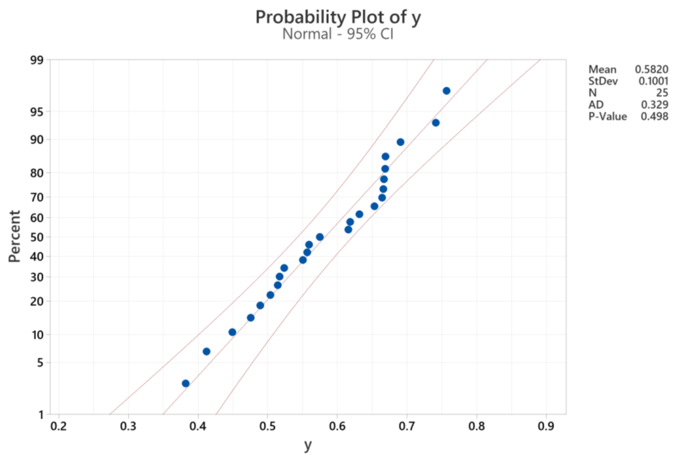

Fig. 5. Probability plot of $\bar{y}$

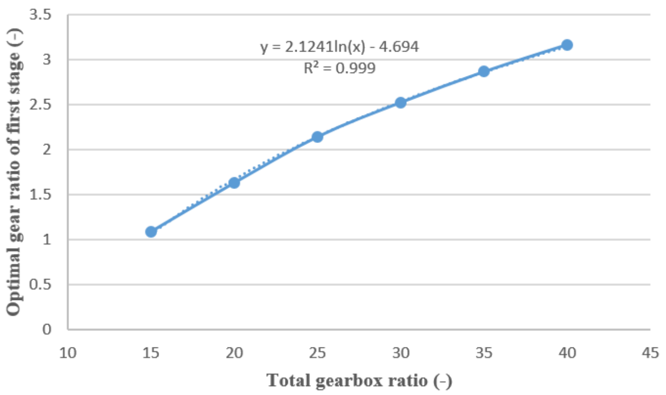

Fig. 6. Optimal gear ratio of first stage versus total gearbox ratio

6. Conclusion

This article discusses the outcomes of a multi-objective optimization study on optimizing a two-stage helical gearbox with SSDGS to decrease gearbox bottom area and enhance gear-box efficiency. This research improved the gear ratio in the first stage, the efficiency of wheel face width in stages 1 and 2, and the allowable contact stress in stages 1 and 2. To address this issue, a simulation experiment based on the Taguchi L25 type was developed and carried out. The impact of significant design elements on the multi-objective aim was also studied. It was found that u_1 has the greatest influence on $\bar{y}_i$ (38.79%), followed by AS_2 (28.5%), X_{ba2} (16.86%), AS_1 (8.04%), and X_{ba1} (4.49%). Additionally, the ideal settings for the essential gearbox parameters have been recommended. For identifying the appropriate first stage u_1 gear ratio, a regression technique (Equation (35)) was also presented.

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