

# Influence of the Relative Weight of the Cost Variability on the Performance and Robustness of Condition-Based Maintenance Strategies and Time-Based Maintenance Strategies

Khamiss Cheikh<sup>1</sup>, EL Mostapha Boudi<sup>2</sup>

<sup>1,2</sup> Department of Mechanical Engineering, Energetic Team, Mechanical and Industrial, Systems (EMISys),  
Mohammadia School of Engineers, Mohammed V University, Rabat, Morocco

## Abstract

The continuous economic growth and resulting infrastructural and technological development have drawn attention to asset management. The tied up high capital in equipment and resources has driven organizations to seek for more effective maintenance strategies to asset management problems. The maintenance process involves a combination of preventive and corrective actions that aim to retain or restore a system to its operating condition. It is a complex and critical process, particularly for manufacturing firms. The cost of maintenance can account for a significant portion, ranging from 15 to 70% of total production costs [1]. The objective of optimal maintenance strategies is to ensure optimal system reliability, availability, and safety performance while minimizing maintenance costs [2]. In the literature, two main types of maintenance techniques are discussed: time-based maintenance (TBM) and condition-based maintenance (CBM) [3,4]. TBM relies on the age of the system and statistical information about its lifetime to make preventive maintenance decisions [5,6]. However, this approach does not take into account the realistic operating conditions of the system over time. On the other hand, CBM is an advanced maintenance technique that considers diagnostic and prognostic information about the system's condition over time. It has gained popularity in the literature and is now recognized as an interesting approach for maintenance optimization [7,8,9,10]. Extensive research has been conducted on both TBM and CBM strategies, resulting in a large number of strategies that have been investigated, developed, and successfully applied to monocomponent systems. For example, some strategies utilize the current equipment condition, such as the deterioration level [11, 12, 13], while others consider the future equipment health state for making maintenance decisions [14,15,16]. These approaches have proven effective in optimizing maintenance processes, so in this article we will evaluate the influence of the relative weight of the cost variability on the Performance and Robustness of Condition-Based Maintenance Strategies CBM and Time-Based Maintenance Strategies TBM.

**Keywords:** Maintenance, Optimization, diagnostic, prognostic, time-based maintenance, condition-based maintenance.

## 1. Introduction

The escalating significance of asset management is underscored by the sustained economic expansion and technological advancements. In response to capital being tied up in equipment, organizations are actively pursuing more efficient maintenance strategies to tackle the challenges associated with asset management. The efficacy of these strategies is frequently gauged through key objectives, including but not limited to Performance and Robustness. Once the criterion is chosen, the maintenance strategy must be evaluated. There are many evaluation methods in the literature. This research endeavors to introduce the most commonly employed stochastic assessment techniques, with a detailed overview available in [17]. All methods are presented considering the asymptotic average cost criterion, although they are applicable to other criteria. The relative weight of the cost variability

can be varied among these evaluation methods, which plays a very interesting role in increasing the performance and robustness of the Condition-Based Maintenance Strategies and Time-Based Maintenance Strategies that will be evaluated.

Analytical evaluation of the performance criterion of a maintenance strategy is essentially based on the regeneration and semi-regeneration properties of the maintained system evolution process [18]. When maintenance models are sufficiently simple (monotone degradation, static decision rules, periodic inspection, perfect replacement, minimal repair, etc.), it is possible to identify renewal instants (or regeneration instants), that is, instants at which the system is exactly in the same state (and with the same laws governing its evolution) with a probability of 1 after a finite time. Renewal instants generally correspond to the dates of equipment renewal (preventively or correctly). If the

duration of the intervention for the renewal is negligible, and if the system is actually new at the initial instant, we will speak of a simple renewal process for the renewal dates [19]. In this case, the regeneration property of the renewal process can be used to calculate the asymptotic average cost  $C_\infty$  [20,21]. In particular,  $C_\infty$  is equal to the ratio of the expected cost per renewal cycle to the average cycle length. Its formula is widely used in the literature to optimize maintenance costs [22,23]. One can also refer to the articles [24,25] for the application of the formula of the asymptotic average cost  $C_\infty$  in maintenance optimization. Some maintenance models evaluated by Monte Carlo simulation are found in [26,27].

In this paper, we focus on the construction of models to evaluate the performance of maintenance strategies. This requires determining performance criteria and their evaluation methods. The economic criterion is the most widely used to optimize the performance of maintenance strategies [28,29]. We first present the different cost criteria available in the literature, and then the methods for evaluating them. Finally, we show the chosen criterion and the evaluation methods applied in this paper, and more specifically, we will evaluate the influence of the relative weight of the cost variability on the Performance and Robustness of Condition-Based Maintenance Strategies and Time-Based Maintenance Strategies. Finally, in Section VI, we conclude our findings, emphasizing the value of our research in providing insights into the Performance and Robustness of TBM and CBM strategies.

## 2. Degradation and Failure Model

This paper introduces the scalar random variable  $X_t$  to represent the system's degradation level at any given time  $t \geq 0$ . In the absence of maintenance interventions,  $X_t$  exhibits an increasing trend, starting from  $X_0 = 0$ . The degradation increment between two time points  $t$  and  $s$  ( $t \leq s$ ), denoted as  $X_s - X_t$ , is independent of degradation levels before  $t$ . Any monotonic stochastic process from the Lévy family [30] can be employed to model the system's degradation evolution.

This paper adopts the well-established homogeneous Gamma process to model system degradation [31]. The Gamma process is characterized by shape parameter  $\alpha$  and scale parameter  $\beta$  [32]. This choice is supported by extensive practical applications and its mathematical tractability [33]. Therefore, for  $t \leq s$ , the degradation

increment  $X_s - X_t$  follows a Gamma distribution with a probability density function.

$$f_{\alpha,(s-t),\beta}(x) = \frac{\beta^{\alpha(s-t)} x^{\alpha(s-t)-1} e^{-\beta x}}{\Gamma(\alpha(s-t))} \cdot 1_{\{x \geq 0\}}, \quad (1)$$

And survival function:

$$\bar{F}_{\alpha,(s-t),\beta}(x) = \frac{\Gamma(\alpha(s-t), \beta x)}{\Gamma(\alpha(s-t))}, \quad (2)$$

$\Gamma(\alpha) = \int_0^\infty z^{\alpha-1} e^{-z} dz$   
Where  $\Gamma(\alpha)$  represents the complete Gamma function.

$$\Gamma(\alpha, x) = \int_x^\infty z^{\alpha-1} e^{-z} dz$$

And  $\Gamma(\alpha, x)$  represents the upper incomplete Gamma function.

And " $1\{\cdot\}$ " denotes the indicator function, which evaluates to 1 if the argument is true and 0 otherwise.

To define system failure, we consider the random failure time of the system  $\tau_L$ , which can be expressed as:

$$\tau_L = \inf\{t \in \mathbb{R}^+ | X_t \geq L\} \quad (3)$$

Where  $L$  represents the critical threshold.

The density function of  $\tau_L$  at time  $t \geq 0$  is given by:

$$f_{\tau_L}(t) = \frac{\alpha}{\Gamma(\alpha t)} \int_{t\beta}^\infty (\ln(z) - \psi(\alpha t)) z^{\alpha t-1} e^{-z} dz, \quad (4)$$

Where  $\psi(v)$  is known as digamma function and can be expressed as:

$$\psi(v) = \frac{\partial}{\partial v} \ln(\Gamma(v)) \quad (5)$$

## 3. Maintenance Strategies and Cost Models

This Section introduces the two main maintenance strategies, Block Replacement (BR) and Periodic Inspection and Replacement (PIR), and outlines the assumptions associated with the maintained system.

### A. Maintenance Assumptions

The system in focus employs two maintenance alternatives: Preventive Replacement (PR) and Corrective Replacement (CR). Replacement involves either physical replacement or a comprehensive repair,

restoring the system to a condition equivalent to being brand new. However, in practice, PR and CR activities may have unequal costs. CR, often unexpected and potentially causing environmental harm, generally incurs higher costs than PR [34]. Even when employing the same maintenance activities, the system may accumulate different costs due to the intricacy of maintenance on a more deteriorated system.

Let  $C(X_t)$  and  $C_c(X_t)$  denote PR and CR costs at time  $t$ , respectively. These costs increase with the degradation level  $X_t$  and adhere to the relationship  $0 < C_i < C(X_t) < C_c(X_t)$ , where  $C_i$  represents the cost of inspection. Furthermore, since replacement occurs at discrete times (inspection time in the PIR strategy and calendar time bloc  $T$  for BR strategy), downtime occurs after a failure. An additional cost is incurred from the moment of failure until the next replacement time at a constant cost rate  $C_d > 0$ .

In our scenario, we treat  $C(X_t)$  and  $C_c(X_t)$  as fixed parameters. This approach allows us to delve into the impact of the relative weighting of cost variability on both the Performance and Robustness of Condition-Based Maintenance Strategies and Time-Based Maintenance Strategies.

## B. Maintenance Strategies

1) *Block replacement strategy (BR):* In This strategy, The decision-making process is simple, based on a time block  $T$ . The system is replaced at regular intervals of  $kT$ , where  $k$  is any positive integer. The replacement occurs proactively if the system is still operational at that time ( $XkT < L$ ), or reactively if it malfunctions ( $XkT \geq L$ ).

2) *Periodic Inspection and Replacement strategy (PIR):*

The PIR (Periodic Inspection Replacement) strategy involves regularly inspecting a system at fixed intervals, regardless of its condition or age. Inspection times are denoted as  $T_k = k\delta$ , where  $k$  is a positive integer and  $\delta$  is the inter-inspection time interval.

During inspections, the system's degradation level,  $XkT$ , is assessed. Based on this observed degradation level, a decision is made:

- If  $XkT \geq L$ , the system is considered to have failed and is replaced correctively (CR) with a new one at time  $T_k$ .

- If  $M \leq XkT < L$ , the system is still functioning but is deemed too degraded and should be preventively replaced (PR) with a new one at time  $T_k$ .
- If  $XkT < M$ , the system is considered healthy, and no action is taken at  $T_k$ .

## 4. Maintenance Cost Model

The long-run expected maintenance cost rate criterion is a widely used method for assessing the effectiveness of maintenance strategies [35]. It considers the average maintenance cost per unit of time over an extended period. This criterion is defined as follows [36]:

$$C_{\infty} = \lim_{t \rightarrow \infty} \frac{E[C(t)]}{t} = \frac{E[C(S)]}{E[S]} \quad (6)$$

Where  $S$  is the length of a renewal cycle,  $C(S)$  is the total maintenance cost incurred over the cycle  $S$ .

To evaluate how robust maintenance strategies are, we suggest using a criterion called the standard deviation of the MCPRC and that is defined as follows:

$$K = \frac{C(S)}{S}. \quad (7)$$

Where  $K$  is a random variable, that is evaluated by the mean value  $\mu = E(K)$  and the standard deviation.

$$\sigma = \sqrt{E(K^2) - E^2(K)} = \sqrt{E(K^2) - \mu^2}. \quad (8)$$

To measure both the performance and robustness of maintenance strategies, we use a combination of two metrics, the formula for combining these metrics might look like this:

$$\varphi = C_{\infty} + \lambda \cdot \sigma; \quad \lambda \geq 0. \quad (9)$$

The coefficient  $\lambda$  in equation (9) functions as a tool for balancing the emphasis on cost variability (robustness) in comparison to mean cost (performance) during maintenance strategy decisions. In interpretation:

- If  $\lambda$  is 1 or less ( $\lambda \leq 1$ ), decision makers give greater priority to the performance of maintenance strategies, emphasizing the minimization of expected cost.
- If  $\lambda$  is greater than 1 ( $\lambda > 1$ ), decision makers prioritize the robustness of maintenance strategies. This indicates a willingness to tolerate a slightly higher expected cost in exchange for reducing the variability or uncertainty in the costs.

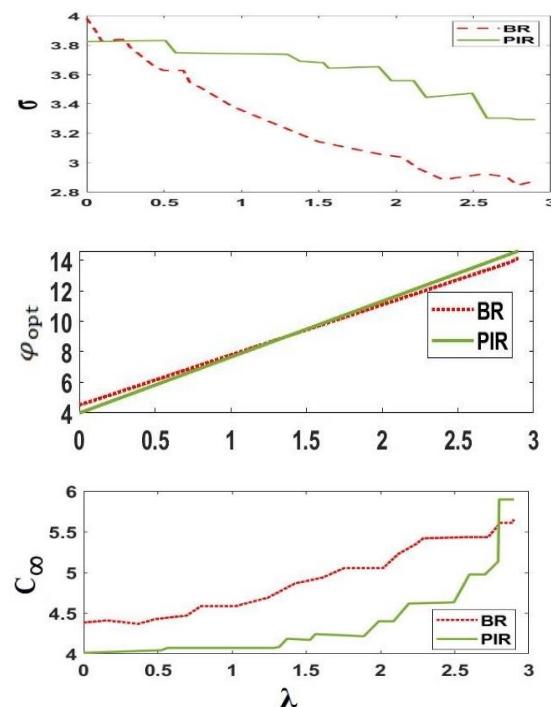
In our study, we delve the impact of the relative weighting of cost variability  $\lambda$  on both the Performance and Robustness of Condition-Based Maintenance Strategies and Time-Based Maintenance Strategies.

## 5. Maintenance Strategies Comparison

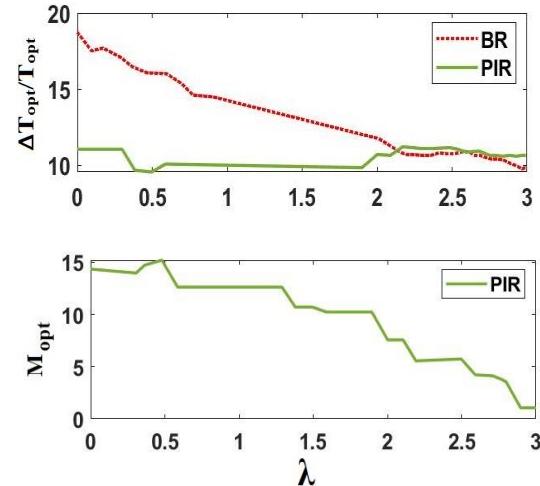
This section focuses on a comparative analysis of the performance and robustness of the two examined strategies, namely the BR and PIR strategies, across different setups of the relative weight parameter  $\lambda$ . This examination allows us to delve the impact of the relative weighting of cost variability on both the Performance and Robustness of this two strategies BR and PIR.

### A. Sensitivity to the relative weight of the cost variability

In evaluating maintenance programs, the relative weight  $\lambda$  reflects decision-makers' financial considerations and risk tolerance. A quantitative assessment of how  $\lambda$  influences the performance and robustness of maintenance strategies is crucial. System characteristics are held constant, and maintenance costs are fixed, while varying  $\lambda$  from 0 to 3.



(a)  $\sigma$ ,  $\varphi_{opt}$  and  $C_\infty$



(b)  $T_{opt}$ ,  $\Delta T_{opt}$  and  $M_{opt}$

Fig. 1. Varied relative weight of the cost variability

We hold the system characteristics at  $\alpha = \beta = 0.1$ ,  $L = 29$ , and  $M_s = 14$ , and the maintenance costs at  $C_i = 7$ ,  $C_d = 19$ ,  $C_c = 98$ , and  $C_0 = 48$ . We then vary  $\lambda$  from 0 to 3 in increments of 0.1, the impact on cost functions  $\varphi_{opt}$ ,  $C_\infty$ ,  $\sigma$ , and decision variables  $T_{opt}$ ,  $\Delta T_{opt}$ ,  $M_{opt}$  for PIR and BR strategies is observed and presented in Fig. 1. At  $\lambda = 1.4$ , optimal cost criteria  $\varphi_{opt}$  for both PIR and BR strategies align, indicating equivalence. Long-term expected cost rates  $C_\infty$  and standard deviations of MCPRC  $\sigma$  show trends of increase and decrease, respectively, concerning  $\lambda$ . PIR excels in performance but lags in robustness, emphasizing the inherent trade-off. Overall objective functions of both strategies are nearly equivalent (Fig. 1a), highlighting the challenge of achieving high performance and robustness simultaneously.

Fig. 1b reveals that  $M_{opt}$  and  $T_{opt}$  decrease with increasing  $\lambda$ , while  $\Delta T$  remains relatively constant. This underscores the crucial roles of condition-based ( $M_{opt}$ ) and time-based ( $T_{opt}$ ) aspects in balancing performance and robustness.

Amplification of  $\lambda$  signals a deliberate focus on prioritizing robustness over performance. Applying the BR strategy results in decreased  $T_{opt}$  as  $\lambda$  increases, emphasizing the importance of minimizing downtimes. The PIR strategy adapts to resemble BR ( $\Delta T_{opt} \approx T_{opt}$ ) as  $\lambda$  increases, showcasing comparable robustness. Figure 1b depicts opposing trends in  $C_\infty$  and  $\sigma$ , confirming the inherent trade-off. Despite challenges, the PIR strategy consistently outperforms in  $\varphi_{opt}$ , showcasing its ability to balance performance and robustness.

## 6. Conclusion

Through this study, we have evaluated the influence of the relative weight of the cost variability on the Performance and Robustness of Condition-Based Maintenance Strategies and Time-Based Maintenance Strategies using a new criterion, which combines the long-term expected maintenance cost rate  $C^\infty$ , the standard deviation of the MCPRC  $\sigma$ , and the relative weight of the cost variability  $\lambda$ .

In this regard, the PIR strategy explored in this study presents itself as a promising option for our system in terms of performance and robustness, and it is better than the BR strategy.

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