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Analyzing Power-Saving Techniques with Two Types of Vacations in Queueing Models: Impact on Server Performance and Power Consumption

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Abstract
In today's rapidly evolving world of technology, power conservation has become crucial. When a server is in idle mode, it still consumes a certain amount of power, making it necessary to analyze queueing models with power-saving techniques. This study focuses on a queueing model with two types of vacations, aiming to reduce power consumption when the server is idle. Additionally, the model considers server failure and immediate repair scenarios.
To assess the performance of the system, essential performance measures are derived using probability generating techniques. These measures help evaluate the efficiency of the power-saving approach and quantify the impact of server failure and repair on the overall system performance. Numerical analysis is conducted to validate the system performance metrics. The analysis provides numerical results that demonstrate the effectiveness of the power-saving strategies and their impact on power consumption, queueing behavior, and overall system performance. This study contributes to the understanding and optimization of power consumption in idle servers, offering insights into the benefits of incorporating power-saving techniques such as different types of vacations and immediate repairs. The findings can guide the design and management of server systems, ensuring energy efficiency and improved overall performance.

Keywords: Energy Saving, M/M/1 Model, Vacation, Breakdown, Repair

1. Introduction And Related Work

A queueing model is a mathematical framework used to analyze the behavior of queues or waiting lines in various systems, including Wireless Sensor Networks (WSNs). In the context of WSNs, a queueing model allows us to study the performance and efficiency of data transmission and processing within the network. It considers the arrival and departure of data packets, the processing capabilities of the sensor nodes, and potential congestion or delays in the network. By utilizing queueing models in WSNs, researchers and engineers can gain insights into the network's capacity, response time, and resource utilization, enabling them to optimize system design, improve data delivery, and enhance overall network performance.

Vacation models are used to address various real-life congestion scenarios that occur in our daily lives, including situations found in manufacturing, production, computer systems, and communication systems. In a vacation queueing system, the server may not be available for a brief period of time due to reasons such as undergoing maintenance, attending to other queues, or searching for new tasks, which is a common feature in many systems. During this period of unavailability, known as a vacation, the server is unable to serve its primary users. In most queuing models, the server begins serving customers as soon as they arrive. The server's vacation state is indicated by the absence of a customer, meaning that the server is not attending to its main queue. Instead, it may be engaged in other tasks, thus making efficient use of energy consumption.

In recent times, there has been significant research conducted by various authors on queueing models that incorporate server vacation. Vijayashree and Ambika [7] focused on non-persistent clients who experienced distinct disruptions and interruptions during their vacations. Yue et al. [8] investigated the impact of customer impatience on different vacation queueing strategies. Ibe and Isijola [1] classified vacations into two types: Type 1 vacations, which occur after a busy period of nonzero length, and Type 2 vacations, which occur when there are no clients waiting for the server to return from vacation. Additionally, Kalite et al. [2] modified the vacation policy of the M/M/1 queuing model to accommodate two different types of vacations. Madheswari and Suganthi [3, 4] studied a retrial queue with optional second service under Bernoulli vacation. Majid et al. [5] examined both multiple vacations and single vacation queueing models. Tuen [6] conducted an in-depth analysis of a power-saving queueing approach with a single server and a working vacation, where vacations were associated with reduced power consumption when the server was not in use. Gupta and Kumar [9] analyzed a waiting server in an M/M/1 retrial queueing model that accounted for server malfunction and repair during vacations, and developed probabilities for different server states and other performance measures of the system. GnanaSekar and Kandaiyan [10] investigated a single server retrial queueing system with delayed repair, feedback, and a working vacation policy, including an asymmetric transition representation.

This paper examines two categories of vacations: vacation type I and vacation type II. When there are no customers in the queue, the server takes vacation type I. Once vacation I is finished, the server proceeds to vacation type II. Throughout the vacation duration, the server engages in additional tasks, ensuring that its idle time is effectively utilized.

2. Model Description

We consider two types of vacation queueing model, where the customer arrives according to Poisson distribution with rates $\lambda^b, \lambda_1^v, \lambda_2^v$ andλ^d during busy period, type I vacation, type II vacation and repair respectively. The time to serve a customer is assumed to be exponentially distributed with mean $1/\mu$.

We examine two variations of a vacation queueing model, where customers arrive based on a Poisson distribution with arrival rates $\lambda^b, \lambda_1^v, \lambda_2^v$ andλ^d . In the busy period, there are three types of activities: type I vacation, type II vacation, and repair. The service time for each customer is assumed to follow an exponential distribution with a mean of 1/μ.

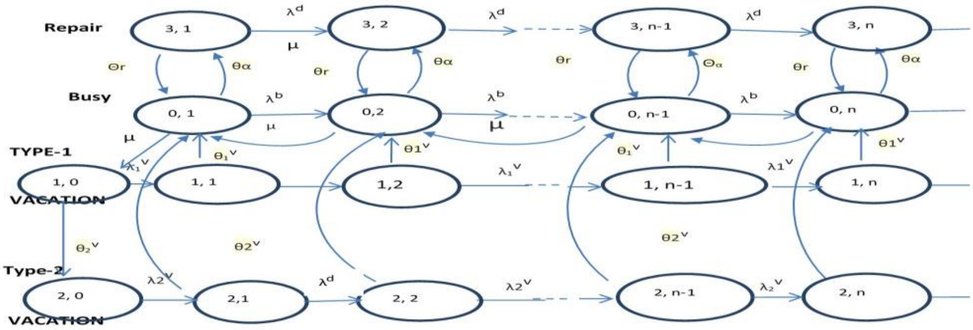

Figure 1 State Transition Diagram

In the proposed model two types of vacation is assumed: (i) Type I vacation begins after completion of exhaustive service and there are no customer in the system, (ii) Type II vacation starts, at end of the type I vacation there are no customers in the system. Both the vacations are exponentially distributed with rates θ_1^v and θ_2^v. During the busy time the server may breakdown with rateθ_α. Immediate server repair is considered with rateθ_r. Server breakdown and repair follow the exponential distribution. Arrival rates, service rate, repair and breakdown rates are independent to each other.

Let the state of the system is denoted by (n, i), where n is the number of customers in the system and i is the server state. Letp_n^i be the probabilities of the different server states (n, i), where

$$i = \begin{cases} b, & server\ is\ busy \\ v_1, & server\ is\ on\ type\ I\ vacation \\ v_2, & server\ is\ on\ type\ II\ vacation \\ d, & server\ is\ on\ down\ state \end{cases}$$

This system can be modelled by a continuous time Markov chain whose steady state transition diagram is shown in the Figure 1.

3. Steady state analysis

Let $p_n^i(t)$ denoted the probability that sever state (n, i) at time t and let$p_n^i = \lim_{t\to\infty} p_n^i(t)$. The main results of the model are stated in the following theorem:

Theorem: The steady state PGF of the different server state respectively busy, type I vacation, type II vacation and downstate is given by

$$P_b(z) = \frac{z\left[\lambda \quad (1-z)+\theta_r\right]\begin{bmatrix}\lambda_1^v\{\lambda_2^v(1-z)[\lambda_1^v+2\theta_1^v]+\lambda_1^v\theta_2^v+\theta_1^v\theta_2^v[2-z]\}\\ +\theta_1^{v^2}[\lambda_2^v+\theta_2^v]\end{bmatrix}}{\{(\mu-\lambda^b z)[\lambda^d(1-z)+\theta_r]-z\lambda^d\theta_\alpha\}[\lambda_1^v(1-z)+\theta_1^v][\lambda_2^v(1-z)+\theta_2^v]} p_0^{v_1} \quad (1)$$

$$P_{v_1}(z) = \frac{\lambda_1^v+\theta_1^v}{\lambda_1^v(1-z)+\theta_1^v} p_0^{v_1} \quad (2)$$

$$P_{v_2}(z) = \frac{\theta_1^v(\lambda_2^v+\theta_2^v)}{\lambda_2^v[\lambda_2^v(1-z)+\theta_2^v]} p_0^{v_1} \quad (3)$$

$$P_r(z) = \frac{\theta_\alpha P_b(z)}{\lambda^d(1-z)+\theta_r} \quad (4)$$

Where $p_0^{v_1}$is probability that the server is in type I vacation when the system is empty.

$$p_0^{v_1} = \frac{\lambda_2^v\theta_1^v\theta_2^v[(\mu-\lambda)\theta_r-\lambda^d\theta_\alpha]}{(\theta_\alpha+\theta_r)(\lambda_2^v)\left\{\lambda_1^v[\lambda_1^v\theta_1^v+\theta_1^v\theta_2^v]+\theta_1^{v^2}(\lambda_2^v+\theta_2^v)\right\} + [(\mu-\lambda^b)\theta_r-\theta_\alpha\lambda^d]\left\{\lambda_2^v\theta_2^v(\lambda_1^v+\theta_1^v)+\theta_1^{v^2}(\lambda_2^v+\theta_2^v)\right\}}. \quad (5)$$

Proof*:* The steady - state equations for $p_n^b, p_n^{v_1}, p_n^{v_2} and p_n^d$are as follows:

$$(\lambda^b + \mu + \theta_d)p_1^b = \theta_1^v p_1^{v_1} + \theta_2^v p_1^{v_2} + \theta_r^b p_1^{v_2} \quad (6)$$

$$(\lambda^b + \mu + \theta_d)p_n^b = \lambda^b\, p_{n-1}^b + \theta_1^v p_n^{v_1} + \theta_2^v p_n^{v_2} + \mu P_{n+1}^b + \theta_r p_n^d \quad (7)$$

$$(\lambda_1^v + \theta_1^v)p_0^{v_1} = \mu p_1^b \quad (8)$$

$$\left(l_1^v + q_1^v\right)p_n^{v_1} = \lambda_1^v p_{n-1}^{v_1} \quad (9)$$

$$\left(l_2^v p_0^{v_2}\right) = \theta_1^v p_0^{v_1} \quad (10)$$

$$\left(l_2^v + q_2^v\right)p_n^{v_2} = \lambda_2^v p_{n-1}^{v_2} \quad (11)$$

$$\left(l^d + q_r\right)p_1^d = \theta_\alpha p_1^b \quad (12)$$

$$\left(l^d + q_r\right)p_n^d = \theta_\alpha p_n^b + \lambda^d p_{n-1}^d \quad (13)$$

The following PGF's are defined for serve is busy, type I vacation, type II vacation and repair as

$P_b(z) = \sum_{n=1}^{\infty} z^n p_n^b$ $\quad P_{v_1}(z) = \sum_{n=0}^{\infty} z^n p_n^{v_1}$

$P_{v_2}(z) = \sum_{n=0}^{\infty} z^n p_n^{v_2}$ $\quad P_r(z) = \sum_{n=0}^{\infty} z^n p_n^d$.

The results (1) - (4) were derived by solving equations (6) - (13) and simplifying them algebraically using the probability generating function (PGF). This enabled obtaining an analytical solution expressed in closed-form expressions.

Let z = 1 in (1) – (4) yields

$$P_b(1) = \frac{\theta_r\{\lambda_1^v[\lambda_1^v\theta_2^v+\theta_1^v\theta_2^v]+\theta_1^{v^2}(\lambda_2^v+\theta_2^v)\}}{[(\mu-\lambda^b)\theta_r-\lambda^d]\theta_1^v\theta_2^v} p_0^{v_1} \quad (15)$$

$$P_{v_1}(1) = \frac{(\lambda_1^v+\theta_2^v)}{\theta_1^v} p_0^{v_1} \quad (16)$$

$$P_{v_2}(1) = \frac{\theta_1^v(\lambda_2^v+\theta_2^v)}{\lambda_2^v\theta_2^v} p_0^{v_1} \quad (17)$$

$$P_r(1) = \frac{\theta_\alpha P_b(1)}{\theta_r} \quad (18)$$

Applying the normalizing condition $P_b(1) + P_{v_1}(1) + P_{v_2}(1) + P_r(z) = 1$, we obtain $P_0^{v_1}$ represented in equation (5).

3. Performance Measures

For the proposed model performance measures are obtained as follows:

1. The mean number of customers in the queue:

$$L = P_b^{'}(1) + P_{v_1}^{'}(1) + P_{v_2}^{'}(1) + P_r^{'}(1)$$

2. The number of customers in the busy system:

$$P_b^{'}(1) = \left[\frac{u^{'}}{v} - \frac{uv^{'}}{v^2}\right] p_0^{v_1}$$

where

$$u = \theta_r\{\theta_2^v\lambda^b + \theta_1^v\theta_2^v[\lambda_1^v + \theta_1^v] + (\lambda_1^v)^2\theta_2^v\}$$

$$v = \{\theta_r(\mu - \lambda^b) - \theta_\alpha\lambda^d\}\theta_1^v\theta_2^v$$

$$u' = \{\lambda_2^v(\theta_1^v)^2 + (\theta_1^v\theta_2^v)[\lambda_1^v + \theta_1^v] + (\lambda_1^v)^2\theta_2^v\}\{\theta_r - \lambda^d\} - \theta_r\lambda_1^v\{\theta_1^v[2\lambda_2^v + \theta_2^v] + \lambda_1^v\lambda_2^v\}$$

$$v^{'} = -(\mu - \lambda^b)\{\theta_r\lambda_1^v\theta_2^v + \theta_r\lambda_2^v\theta_1^v + \lambda_3^v\theta_1^v\theta_2^v\} + \theta_\alpha\lambda_3^v[\lambda_1^v\theta_2^v + \lambda_2^v\theta_1^v - \theta_1^v\theta_2^v] - \theta_r\lambda^b\theta_1^v\theta_2^v$$

3. The number of customers in the system when the server is in type I vacation:

$$P_{v_1}^{'}(1) = \frac{\lambda_1^v(\lambda_1^v + \theta_2^v)}{(\theta_1^v)^2} p_0^{v_1}$$

4. The number of customers in the system when the server is in type II vacation:

$$P_{v_2}^{'}(1) = \frac{\theta_1^v(\lambda_1^v + \theta_2^v)}{(\theta_2^v)^2} p_0^{v_1}$$

5. The number of customers in the system when the server is in downstate:

$$P_r^{'}(1) = \left\{\frac{\theta_\alpha\lambda_3^v uv + \alpha u^{'}v - \alpha uv^{'}}{\theta_r v^2}\right\} p_0^{v_1}$$

5. Numerical illustration

In this section, the numerical analysis is carried out for the suitable values of the system parameters under the stability condition $p_0^{v_1} < 1$.

Table 1: Impact of λ^b and μ system performance measures

μ =10					
λ^b	$P_b'(1)$	$P_{v1}'(1)$	$P_{v2}'(1)$	$P_r'(1)$	L
0.1	0.017663	0.003708	0.018765	0.088416	0.128551
0.15	0.018592	0.008272	0.027232	0.093194	0.14729
0.2	0.019421	0.014557	0.035105	0.097525	0.166608
0.25	0.020148	0.02248	0.042404	0.101407	0.186438
0.3	0.020774	0.031948	0.049146	0.104841	0.206709
0.35	0.021297	0.042863	0.055356	0.107826	0.227343
0.4	0.021719	0.055121	0.061057	0.110362	0.24826
0.45	0.022038	0.068617	0.066275	0.112449	0.269378
0.5	0.022253	0.083243	0.071034	0.114086	0.290617
μ=15					
0.1	0.012182	0.003724	0.018843	0.060979	0.095727
0.15	0.012812	0.008325	0.027406	0.064214	0.112757
0.2	0.013375	0.014683	0.03541	0.067155	0.130623
0.25	0.013871	0.022728	0.042873	0.0698	0.149273
0.3	0.014301	0.032381	0.049812	0.07215	0.168643
0.35	0.014663	0.043554	0.056248	0.074204	0.188669
0.4	0.014958	0.056156	0.062203	0.075964	0.209281
0.45	0.015185	0.070092	0.0677	0.077429	0.230406
0.5	0.015345	0.085268	0.072762	0.0786	0.251975
μ=30					
0.1	0.006309	0.003739	0.018921	0.031579	0.060548
0.15	0.006628	0.008378	0.027579	0.03322	0.075805
0.2	0.006915	0.014809	0.035714	0.034715	0.092153
0.25	0.007168	0.022977	0.043341	0.036063	0.109549
0.3	0.007389	0.032812	0.050475	0.037266	0.127942
0.35	0.007576	0.044241	0.057136	0.038323	0.147276
0.4	0.00773	0.057185	0.063343	0.039235	0.167492
0.45	0.00785	0.07156	0.069118	0.040001	0.188529
0.5	0.007937	0.087282	0.074481	0.040622	0.210323

The chosen parameters are $l^b = l$, $l_1^v = a_1 l$, $l_2^v = a_2 l$, $\lambda^d = a_3\lambda$, where $a_1 = 0.3$, $a_2 = 0.2$, $a_3 = 0.1$ $\theta_1^v = .4$ $\theta_2^v = 1$ $q_r = .2$ $\theta_\alpha = 1$. Table 1 show for the increasing arrival rate from 0.1 to 0.5 for various service rates (10, 15, 30). This table confirms that, the numbers of arrivals in the system raises, the number of customers in the different server state and queue size are increased.

For increasing service rates queue size is decreased when the server is on busy and repair, but in the vacation state queue size is increased. In the suggested model, Figure 2 verifies the table values, indicating that as the service rate increases, the queue size decreases as expected.

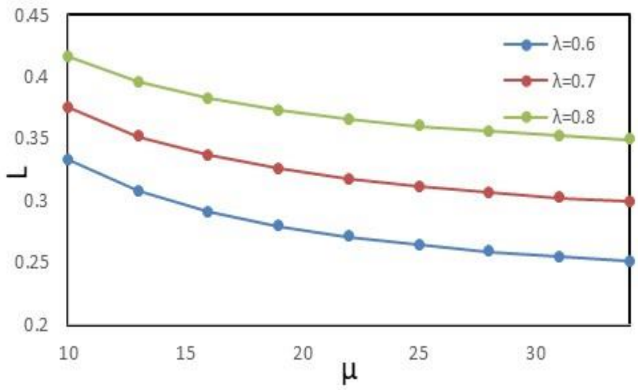

Figure 2: μ versus L

Figure 3 shows that increase in repair rate, the queue size lowered for different service rates. Figure 4 depicts, how the system queue size grows monotonically for escalation of breakdown rates.

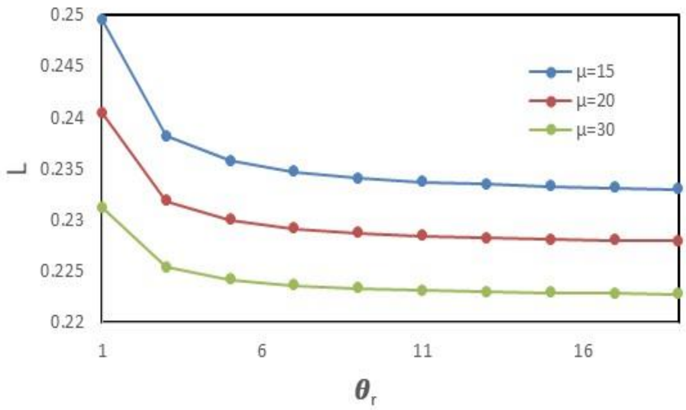

Figure 3: θ versus L

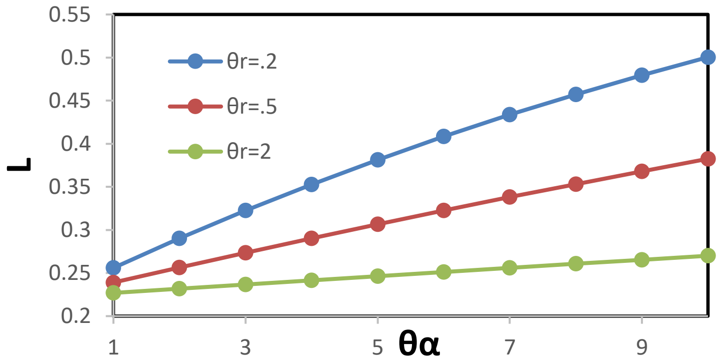

Figure 4: θ_α versus L

6. Conclusion

This proposed model examines two types of vacation: server breakdown and immediate repair. The model calculates performance measures for the system and analyzes its behaviour through numerical analysis. When in vacation modes, the server is turned off, effectively utilizing power consumption in two ways: type I and type II vacations. The study concludes that instead of investing time in the repair process, it is more beneficial to focus on improving the quality of service (QoS) to enhance overall system performance.

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