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Quantum Corrections Effect on Matter Wave Transport in Disordered Optical Potential

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Abstract

In this study, we present a numerical investigation into the average local current density, which describes the density propagation of a matter wave characterized by energy ε and wave vector k in a long-range disordered optical potential. Our approach begins with the application of the Bethe-Salpeter equation to calculate the static current density, denoted as $j(p)$, in the weak localization regime. This analysis helps us, to know how to pass from classical diffusion to quantum diffusion. The numerical results reveal a significant wave packet spectrum in the current density, demonstrating the impact of numerous counter-propagating amplitudes generated by multiple diffusion induced interferences. The solution of the Bethe-Salpeter equation, denoted as $F(p,\varepsilon)$, takes into account the quantum interferences that are responsible for the complete suppression of transport. At last, we examine the constant diffusion $D(\varepsilon)$ at finite disorder strength to find the Anderson localisation transition.

Keywords: Optical disorder, current density, diffusion coefficient, interferences. Anderson localisation transition

1. Introduction

The transport properties of a quantum particle in an optical disordered system are intrinsically determined by the interference of multiple scattering paths, which can lead to Anderson localization [1–4]. It was found that the quantum particle then remains localized around its initial position to leads to a total suppression of transport, consequently the diffusion is cancelling the transport and the conductivity strictly vanishes [5, 6]. Theoretical and numerical works have allowed the experimental observation of the Anderson transition with coherent matter waves in a disordered optical potential [7, 8]. In dimension three, the first experiment was carried out with a Bose-Einstein condensate [9], and it was shown in a fermionic gas [10]. The problem posed by the experimenters is that this localization is easily disturbed by the effects of decoherence or of the interaction between particles. Ultra-cold atom systems offer new services to investigate these problems. As a result, there are actually a large number of suggestions for knowing the position of a transition threshold. Various studies are still in progress on this theme which trigger several experiments and open up many perspectives on the effect between disorder and interactions [11–15].

The choice on the random optical potential for cold atoms was motivated in 2005 giving the first publication in the field of Bose-Einstein condensates [16]. The random potential to which atoms are exposed comes from the dipole force of the light-matter interaction [17]. This property gives it the particularity to controlling an experiment [18]. As well, the advantage to using the random optical potential is firstly the experimental flexibility to modify its characteristics parameters, on the other hand, to the exact knowledge of the statistical properties of these potentials [19–23]. The most interesting in the statistical properties of transport is the return probability to a given point at which two types of localization give rise[24]: a weak localization comes from the fact that the probability of the wave to return to its initial position is possible through loop paths, therefore interference effects come into play. Each loop can be traversed in one direction or in the other, which generates two multiple scattering paths along which exactly the same phase accumulates in successive scattering events. This coherent effect is valid for any specific realization of the disordered potential and thus survives the average of the disorders. Moreover, these two paths being in phase, this gives rise to constructive interference of the matter wave, which improves its probability of return. This effect leads a diffusive transport for which the diffusion coefficient is reduced. In the case where the disorder is strong, the propagation of a coherent wave is stopped after a certain time in any dimension d, this means that the diffusion coefficient equal to zero: $D(\varepsilon)$. The return probability decreases exponentially from a certain point in space with a characteristic length called the localization length.[25].

The remainder of the paper is organized as follows. In section 2 the fundamental concepts of quantum transport for matter waves in disordered media and present the Bethe-Salpeter equation as well as its associated eigenfunction. Furthermore, we present an analytical expression which characterizes the quantum corrections linked to the localization of the diffusion constant, in particular in the case of isotropic diffusion. Section 3 is devoted to the numerical simulation. We calculate of the current density of the cold atoms at long time $(\Omega \to 0)$ and large distance $(q \to \infty)$, the eigenfunction of the Bethe-Salpeter equation and diffusion constant. Our conclusions are drawn in section 4we note that the detailed calculations are already done in ref [26,27].

2. Model

Considering two particles of Green function $\varphi_{p,p\prime}(\varepsilon, \Omega, q)$ to describe the probability density of wave in a disorder potential. For a hydrodynamic expression, $\varphi_{p,p\prime}(\varepsilon, \Omega, q)$ is defined as[19,27]:

$$\varphi_{p,p\prime}(\varepsilon, \Omega, q) \equiv \overline{\langle p_+|G(\varepsilon_+)|p'_+\rangle\langle p_-|G^\dagger(\varepsilon_-)|p'_-\rangle} \quad (1)$$

where where G is the retarded Green operator, $\varepsilon_\pm = \varepsilon \pm \Omega/2$, $p_\pm = p \pm \Omega/2$, $p'_\pm = p' \pm \Omega/2$. and (q, Ω) are the Fourier conjugates of the space and time variables respectly.

The Bethe-Salpeter equation can be reformulated as a quantum kinetic equation for $\varphi_{p,p'}(\varepsilon, \Omega, q)$.

$$\varphi = \bar{G} \otimes \bar{G}^\dagger + \bar{G} \otimes \bar{G}^\dagger U \varphi \quad (2)$$

The first term in Eq. (2) represents the intensity propagation with uncorrelated average propagators. The second term involves the vertex function U, which includes all irreducible intensity and account for all correlations in the density propagation.

Conservation of quantum probability ensures the existence of a hydrodynamic pole for propagation. We expressed this as [26]

$$\varphi_{p,p\prime}(\varepsilon, \Omega, q) = \frac{2}{\sum_p -Im\, G_p(\varepsilon)} \frac{L_{p,\varepsilon}(q) L_{p',\varepsilon}(q)}{-i\Omega + Dq^2} \quad (3)$$

$\sum_p -Im\, G_p(\varepsilon)$ is the spectral function which encapsulates all the information concerning the diffusion of particles in a disordered medium. It is defined as follows [19]:

$$\sum_p -ImG_p(\varepsilon) = \frac{\Sigma_p(\varepsilon)}{|(\varepsilon - \varepsilon_p - \Sigma(\varepsilon)|^2} \quad (4)$$

The Green function is written in term of self-energy Σ: $G_p(\varepsilon) = 1/(\varepsilon - \varepsilon_p - \Sigma_p(\varepsilon))$.

Where $\varepsilon = {p^2}/{2m}$ is the kinetic energy of the atom and the self-energy Σ has the following form [28].

$$\Sigma_p(\varepsilon) = \sum_{p\prime} U_{p,p\prime}\, G_p(\varepsilon) \quad (5)$$

$L_{p,\varepsilon}(q)$ in equation 2 represents the eigenfunction of the Bethe-Salpeter equation related to hydrodynamic diffusion, while $D(\varepsilon)$ represents the diffusion constant. We expect that as time approaches infinity $(\Omega \to \infty)$ and distance becomes very large $q \to 0$, The function $L_{p,\varepsilon}(q)$ follows from [19,26]:

$$L_{p,\varepsilon}(q) = -ImG_p(\varepsilon) - i\,(p\,.\,q)\, F(p, \varepsilon) \quad (6)$$

Where

$$F(p, \varepsilon) = ImG(\varepsilon)\, 2\tau_p\, J\,(p, \varepsilon) - \frac{\partial\, Re\, G_p(\varepsilon)}{\partial p^2} + O(q^2) \quad (7)$$

With τ_p is the scattering mean-free time, It is already calculated in the reference.[28]:

$$\tau_p = \frac{\hbar}{2Im\Sigma_p(\varepsilon)}$$

In ref [27], we obtained a detailed formulation for the density current $J\,(p, \varepsilon)$when dealing with the potential of 3D laser spots generated through diffraction, a setup widely used in quantum gas experiments.

$$J\,(p, \varepsilon) = 1 + \sum_{p\prime\prime} \frac{p.p\prime\prime}{p^2} U_{p,p\prime\prime} |G(\varepsilon, p'')|^2 J\,(p'', \varepsilon) \quad (8)$$

We work with the same disordered correlation potential described in reference [28]. The correlation function is denoted as $\langle U(\mathrm{R})\rangle = V_R^2 \mathrm{sinc}^2(\mathrm{R}/\sigma)$, where $V_R$represents the amplitude disorder and σ is the correlation length. When $V_R \to \infty$ and $\sigma \to 0$, the speckle potentials are simplified to the random potentials of uncorrelated white noise. In Fourier space, this is written as follows:

$$U_{p,p\prime} = V_R^2 \int dR \sum_{p\prime} \frac{sin^2(\mathrm{R}/\sigma)}{(\mathrm{R}/\sigma)^2} e^{i(p,p\prime).R}$$

Knowing that $\sum_{p\prime} \equiv \int \frac{dp\prime}{(2\pi)^3}$

One found:

$$J\,(p,\varepsilon) = 1 + \frac{V_R^2}{(2\pi)^3}\, I(p,p')\, |G(\varepsilon,p')|^2\, J\,(p',\varepsilon) \qquad (9)$$

where

$$I(p,p') = \int dR \int dp' \frac{p.p'}{p^2} \frac{sin^2(\mathrm{R}/\sigma)}{(\mathrm{R}/\sigma)^2} e^{i(p,p').R} \qquad (10)$$

Equation (9) was solved by iteration, with spline interpolation between 100 points.

According to Fick's law, which establishes the relationship between the diffusive flux and the concentration gradient in the diffusive regime, the expression for the diffusion constant can be extracted as follows ref[27]:

$$D(\varepsilon) = \frac{1}{\pi\rho(\varepsilon)} \frac{2}{3} \sum_p p^2\, F(p,\varepsilon) \qquad (11)$$

The sum $\sum_p -ImG_p(\varepsilon)$ is connected to the density of states per unit volume $\rho(\varepsilon)$ as [27]:

$\sum_p -ImG_p(\varepsilon) = \pi\rho(\varepsilon)$. $\pi\rho(\varepsilon)$ measures the average number of states in the random medium per unit of volume.

when $p \to \infty$, we anticipate that $j(p) \longrightarrow 1$. We anticipate that $j(p)$ equals 1, indicating a transition to the classical regime.

In the next section, we will discuss the critical transition regime by numerically solving the previously mentioned Bethe-Salpeter equation.

3. Numerical results

In this section, we numerically examine the impact of quantum corrections on the transport of cold atoms. The numerical calculations were carried out at long range to investigate the position of mobility edge. We note that all energies are expressed in units of the quantum correlation energy$\varepsilon_\sigma = \frac{\hbar}{2m\sigma^2}$, momenta are expressed $as\, p = k$, where the de-Broglie wave number k is scaled in units of $1\,/\,\sigma$ (we recall that $\hbar = 1$).

We choose a constant value of the disorder amplitude (V_R=0.5ε_σ) for which the condition of a perturbative disorder to be valid $\varepsilon_p \gg V_R^2/\varepsilon_\sigma$[27].

We note that is the spectral function$\sum_p -Im\, G_p(\varepsilon)$ was already calculated in [26].

Figure 1 shows a numerical calculation of the current density $j(p)$ as a function of the momentum p. We observe that the current density reaches its maximum for small momenta at $p = 0.0\,1$, after which it gradually decreases and remains constant for higher momentum.

For $p = 0$, the calculation gives a current density $j(p)$ of approximately 1.7, which then rises to 1.8 before decreasing beyond $p = 0.01$. Examining the figure1, it becomes evident that the numerical simulation predicts a pronounced increase in current density around $p \sim 0.01$, indicating that more atoms can participate in multiple scattering. The calculations show that the current diffusion converges to 1 when the momentum becomes very large. In this regime, weak corrections disappear and classical diffusion occurs, as also indicated by equation (8).

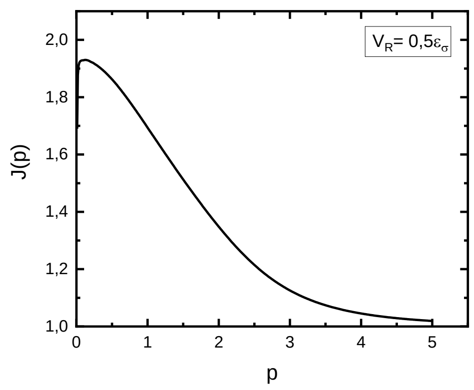

Figure 1. Current density as a function of momentum for $\varepsilon = -0.447$.

In this context, our primary focus is on the behavior of the diffusion coefficient $D(\varepsilon)$ in the presence of these quantum correction scattering effects, which play a crucial role in predicting the mobility edge. It is worth noting that $D(\varepsilon)$is a positive quantity; however, the second derivative of the Green function includes both negative and positive values (as seen in Figure 2). Here, typically, it has taken 100-200 iterations to achieve satisfactory convergence.

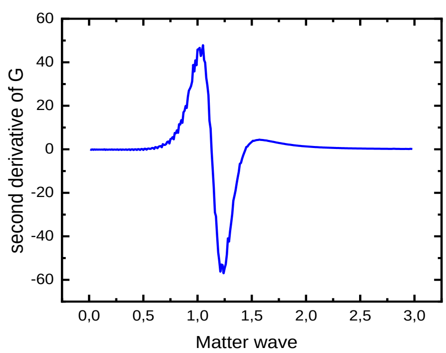

Figure 2 the second derivative of the Green function according matter wave for $\varepsilon = -0.447$.

The solution of the Bethe-Salpeter $F(p, \varepsilon)$ equation in the coherent transport regime of matter waves (i.e. in the presence of quantum corrections) is shown in Figure 3.

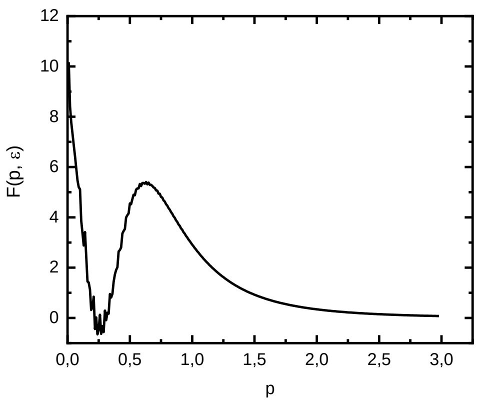

Figure 3. Solution of Bethe-Salpeter equation with $\boldsymbol{\varepsilon = -0.447}$.

In the presence of quantum corrections, the energetic atoms are displaced by the interference giving rise to an increase in the intensity of the function $F(p, \varepsilon)$, which results in a large anisotropic moment due to their deviations by disorder effect. We clearly observe a jump at low momentum, particularly near p ≈ 0.01. This jump is a result of the correlations induced by optical disorder.

Numerical calculations reveal that $F(p, \varepsilon)$ converges to 0, when the momentum becomes very large ($p > 1$), In such a regime, weak localization corrections disappear and classical diffusion takes place.

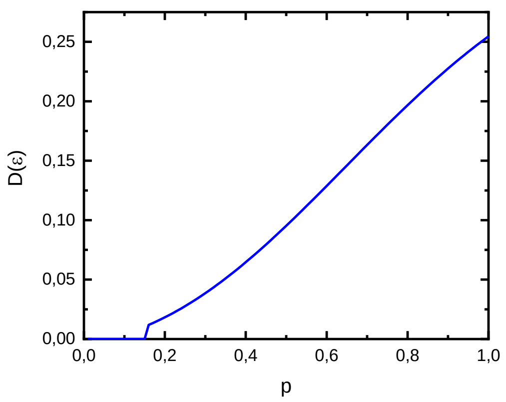

Figure 4 Diffusion coefficient as function of momentum for ε=-0.447

Let's delve deeper into transport properties by examining the diffusion coefficient. To achieve this, We plot the curve of $D(\varepsilon)$versus p to identify the transition point. The figure clearly indicates that D equals zero in the range of [0, 0.14]. Within this interval, atoms are trapped due to the quantum corrections induced by disorder. In this scenario, atom transport is influenced by interference. However, for p > 0.15, $D(\varepsilon)$ increases with momentum, and at this point, the regime shifts to a diffusive phase. At $p = 0.15$, the atoms undergo a transition from a localized phase to a diffusive phase.

4. Conclusion

In this paper, we have studies the propagation of a matter wave in a random optical disorder at long times and large distance. Starting from the Bethe - Salpeter equation to calculate numerically the current density $j(p)$. the results show that the current density becomes important when quantum corrections are included. However, at higher pulses, the current density drops to 1, indicating a phase shift toward the classical regime.

We have focused on quantum properties, specifically considering quantum corrections resulting from multiple scattering. By analyzing the behavior of the diffusion constant in relation to the momentum of the atoms, we observe its disappearance from $p = 0$ to $p = 0.01$, marking the absence of diffusion as initially anticipated by Anderson. Finally, at $p = 0.15$, a transition takes place from a localized regime to a diffusive regime.

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