***Journal of Harbin Engineering University***
***ISSN: 1006-7043***

Vol 45 No. 2
February 2024

# Multi-Objective Optimization of a Two-Stage Helical Gearbox Sets to Reduce Gearbox Volume and Improve Gearbox Efficiency

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**Abstract**

The goal of this research is to look into multi-target optimization of a two-stage helical gearbox with second-stage double gear sets (SSDGS) in order to find the best critical design parameters for reducing gearbox volume and increasing gearbox efficiency. In two steps, the Taguchi technique and gray-relation analysis (GRA) were used to solve the problem. The single-objective optimization problem was faced first to narrow the gap between variable levels, and then the multi-objective optimization problem was solved to determine the optimal primary design variables. The first and second stage wheel face width coefficients (CWFW), permissible contact stresses (ACS), and first stage gear ratio were also calculated. The study's findings were utilized to identify the best values for five critical design parameters of a two-stage helical gearbox with SSDGS.

**Keywords**: Helical gearbox, Double gear sets, Optimization, Multi-objective, Gear ratio, Gearbox volume, Gearbox efficiency.

**Introduction**

A mechanical drive system's gearbox is a vital component. Its purpose is to reduce speed while boosting torque from the motor shaft to the working shaft. As a result, the optimal design of the gearbox has emerged as a significant issue that has piqued the interest of many scientists.A number of studies on the optimal configuration of helical gearboxes have been undertaken so far. For single optimization issues, many alternative objective functions have been chosen. Minimal gearbox length [1-3], minimal gear mass [4], minimum gearbox cross-section [5], minimum gearbox expenses [6-9], or reasonable gearbox structure [10, 11] are some examples.

Furthermore, several types of gearboxes have been explored, including helical gearboxes [2, 10, 12–15], bevel gearboxes [6, 16–19], and worm gearboxes [10, 11]. Several studies on determining the major design parameters of gearboxes when developing multi-objective optimization have recently been published [20, 21]. In the preceding research, the single aim function was either maximal efficiency and minimal total mass of a two-stage helical gearbox [20] or maximal efficiency and minimal volume of a bevel-helical gearbox [21].

Numerous studies on the best design of gearboxes have been conducted, according to the previously described study. However, no studies on multi-objective optimization for a two-stage helical gearbox with SSDGS with the single objective functions of greatest gearbox efficiency and smallest gearbox volume have been done.The purpose of this research is to use SSDGS to investigate multi-target optimization learning for a two-stage helical gearbox. In this effort, two distinct goals were pursued: reducing gearbox volume and increasing gearbox efficiency. Furthermore, the CWFW for both stages, the ACS for both stages, and the first stage gear ratio were all examined. Furthermore, by combining the Taguchi approach with the GRA, the multi-objective optimization problem in gearbox design was solved in two stages. The best values for five critical design variables for developing a two-stage helical gearbox with SSDGS were also provided.

**Optimization Problem**

**Gearbox volume determination**

The volume of the gearbox $V_{gb}$ is determined by (Fig. 1):

$$V_{gb} = L \cdot B \cdot H \tag{1}$$

Where, L, B, and H can be calculated by (Fig. 1):

$$L = \frac{d_{w11}}{2} + a_{w1} + a_{w2} + \frac{d_{w22}}{2} + 2 \cdot S_G + 2 \cdot k \tag{2}$$

$$B = b_{w1} + b_{w2} + 7 \cdot S_G \tag{3}$$

$$H = \max(d_{w21}, d_{w22}) + 8.5 \cdot S_G \tag{4}$$

In (1), $k = 8 \div 12$[22]; $d_{w11}, d_{w21}, d_{w22}$ are gear pitch diameters of the first and second stages which can be calculated by [22]:

$$d_{w11} = 2a_{w1}/(u_1 + 1) \tag{5}$$

$$d_{w21} = 2 \cdot\ a_{w1} \cdot u_1/(u_1 + 1) \tag{6}$$

$$d_{w22} = 2\ a_{w2} u_2\ /(u_2 + 1) \tag{7}$$

In the above equations, $a_{w1}$ and $a_{w2}$ are the center distances of stages 1 and 2; $b_{w1}$ and $b_{w2}$ are the gear width of stage 1 and 2. These elements are found by [22]:

$$a_{w1} = k_a(u_1 + 1)\sqrt[3]{T_{11} k_{H\beta 1}/([\sigma_{H1}]^2 u_1 X_{ba1})} \tag{8}$$

$$a_{w2} = k_a(u_2 + 1)\sqrt[3]{T_{12} k_{H\beta 2}/([\sigma_{H2}]^2 u_2 X_{ba2})} \tag{9}$$

$$b_{w1} = X_{ba1} \cdot a_{w1} \tag{10}$$

$$b_{w2} = X_{ba2} \cdot a_{w1} \tag{11}$$

In (8) and (9), $k_a = 43$ is the material coefficient [22]; $k_{H\beta 1}$ and $k_{H\beta 2}$are the contacting load ratio for pitting resistance of stages 1 and 2; $k_{H\beta 1} = 1.0 \div 1.06$ and $k_{H\beta 2} = 1.02 \div 1.28$[22]. $[\sigma_{H1}]$and $[\sigma_{H2}]$ are ACS of stages 1 and 2 (MPa); $u_1$ and $u_2$ are the gear ratios of stages 1 and 2. $X_{ba1}$ and $X_{ba2}$ are CWFW and $T_{11}$ and $T_{12}$ are the torque on the pinion (Nmm) of stages 1 and 2:

$$T_{11} = T_{out}/\left(u_g \cdot\ \eta_{hg}^2 \cdot \eta_b^3\right)$$

$$T_{12} = T_{out}/\left(2 \cdot u_2 \cdot \eta_{hg} \cdot \eta_{be}^2\right)$$

Where, $T_{out}$ is the output torque (N.mm); $\eta_{hg}$ is the helical gear efficiency ($\eta_{hg} = 0.96 \div 0.98$); $\eta_b$ is the rolling bearing efficiency ($\eta_h$=0.99÷0.995) [22].

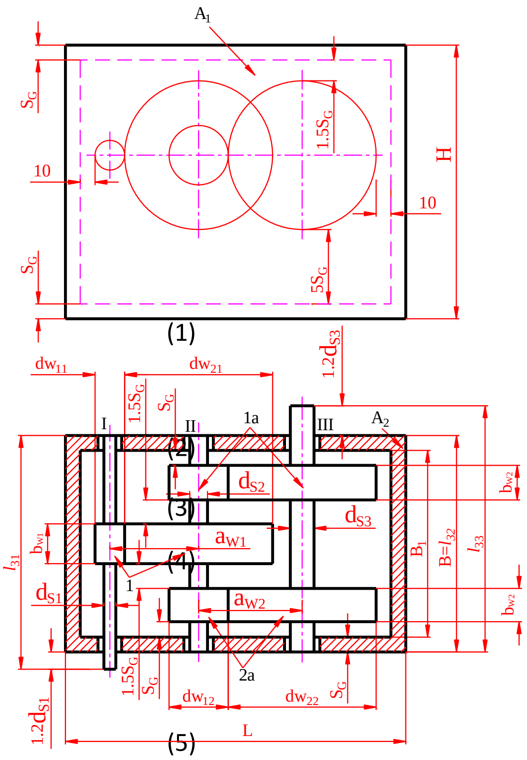


**Figure 1. Calculated schema**

**Gearbox efficiency determination**

The efficiency of the gearbox is calculated by:

$$\eta_{gb} = \frac{100 \cdot P_l}{P_{in}}$$

In which, $P_l$ is the overall gearbox power loss[23]:

$$P_l = P_{lg} + P_{lb} + P_{ls}$$

Wherein, $P_{lg}$ is the overall gear power loss; $P_{lb}$ is the bearing power loss; $P_{ls}$ is the seal power loss. These parts are determined by:

+) The power loss in gears:

$$P_{lg} = \sum_{i=1}^{2} P_{lgi}$$

In which, $P_{lgi}$ is the power losses in gears of i stage:

$$P_{lgi} = P_{gi} \cdot \left(1 - \eta_{gi}\right)$$

Where,$\eta_{gi}$ is the anticipated gear efficiency of the i stage [24]:

$$\eta_{gi} = 1 - \left(\frac{1+1/u_i}{\beta_{ai}+\beta_{ri}}\right) \cdot \frac{f_i}{2} \cdot \left(\beta_{ai}^2 + \beta_{ri}^2\right)$$

In which, $u_i$ is the gear ratio of i stage; f is the friction coefficient; $\beta_{ai}$ and $\beta_{ri}$ are the arcs of approach and retreat on the i stage which are found by [24]:

$$\beta_{ai} = \frac{\left(R_{e2i}^2 - R_{02i}^2\right)^{1/2} - R_{2i} \cdot \sin\alpha}{R_{01i}}$$

$$\beta_{ri} = \frac{\left(R_{e1i}^2 - R_{01i}^2\right)^{1/2} - R_{1i} \cdot \sin\alpha}{R_{01i}}$$

In which, α is the pressure angle; $R_{e1i}$ and $R_{e2i}$are the outer radiuses of the pinion and gear; $R_{1i}$ and $R_{2i}$ are the pitch radiuses of the pinion and gear; $R_{01i}$ and $R_{01i}$ are the base-circle radiuses of the pinion and gear.

In (18), f is friction coefficient which is determined by [21]:

- When the sliding velocity v ≤ 0.424 (m/s):

$$f = -0.0877 \cdot v + 0.0525$$

- When the sliding velocity v > 0.424 (m/s) :

$$f = 0.0028 \cdot v + 0.0104$$

+) The bearing power loss is found by [23]:

$$P_{lb} = \sum_{i=1}^{6} f_b \cdot F_i \cdot v_i \quad (21)$$

Where, $f_b = 0.0011$ is the bearing friction coefficient (for radical ball bearings with angular contact) [23]; F is the bearing load (N); v is the peripheral speed; i is the bearing ordinal number (i = 1÷6).

+) The overall power loss in seals can be determined by [23]:

$$P_s = \sum_{i=1}^{2} P_{si} \quad (22)$$

Where $P_{si}$ is the power loss in a single seal (w):

$$P_{si} = [145 - 1.6 \cdot t_{oil} + 350 \cdot log\, log(VG_{40} + 0.8)] \cdot d_s^2 \cdot n \cdot 10^{-7} \quad (23)$$

With $VG_{40}$ is the ISO Viscosity Grade number.

**Objective functions and constrains**

**Objective function**

In this study, the multi-objective optimization problem has two separate objectives:

Minimizing the gearbox volume:

$$\min f_2(X) = V_{gb} \quad (17)$$

Maximizing the gearbox efficiency:

$$\min f_1(X) = \eta_{gb} \quad (18)$$

Where, X is design variable vector. In this task, five main design factors including $u_1$, $Xba_1$, $Xba_2$, $AS_1$, and $AS_2$ are chosen as variables. Therefore, we have:

$$X = \{u_1, Xba_1, Xba_2, AS_1, AS_2\}$$

**Constrains**

The multi-objective function consists of the following constraints:

$$1 \le u_1 \le 9 \text{ and } 1 \le u_2 \le 9 \quad (19)$$

$$0.25 \le k_{be} \le 0.3 \text{ and } 0.25 \le X_{ba} \le 0.4 \quad (20)$$

$$350 \le AS_1 \le 420 \text{ and } 350 \le AS_2 \le 420$$

**Methods**

Five major design criteria have been selected for consideration in this study. Table 1 displays the minimum and maximum values for various variables. To solve the optimization problem, the Taguchi technique and gray relation analysis were applied. The L25 ($5^5$) design was used to optimize the number of levels for each variable. However, among the variables considered, $u_1$ has a somewhat wide range (in this example, $u_1$ = 1÷9, see Table 1). The difference in the values of these attributes remained substantial even with five levels (in this example, the difference is (9-1)/4 = 2).

To help close the gap between variable values spread throughout a wide range, the 2-stage multi-objective optimization problem solution approach [20] was applied. The first stage of this technique addresses a single-objective optimization problem, while the second addresses a multi-objective optimization problem to discover the optimal core design features.

**Table 1. Main design factors and their maximum and minimum limits**

| Factor | Notation | Minimum limit | Maximum limit |
|---|---|---|---|
| Gear ratio of stage 1 | $u_1$ | 1 | 9 |
| CWFW of stage 1 | $X_{ba1}$ | 0.25 | 0.3 |
| CWFW of stage 2 | $X_{ba2}$ | 0.25 | 0.4 |
| ACS of stage 1 (MPa) | $AS_1$ | 350 | 420 |
| ACS of stage 2 (MPa) | $AS_2$ | 350 | 420 |

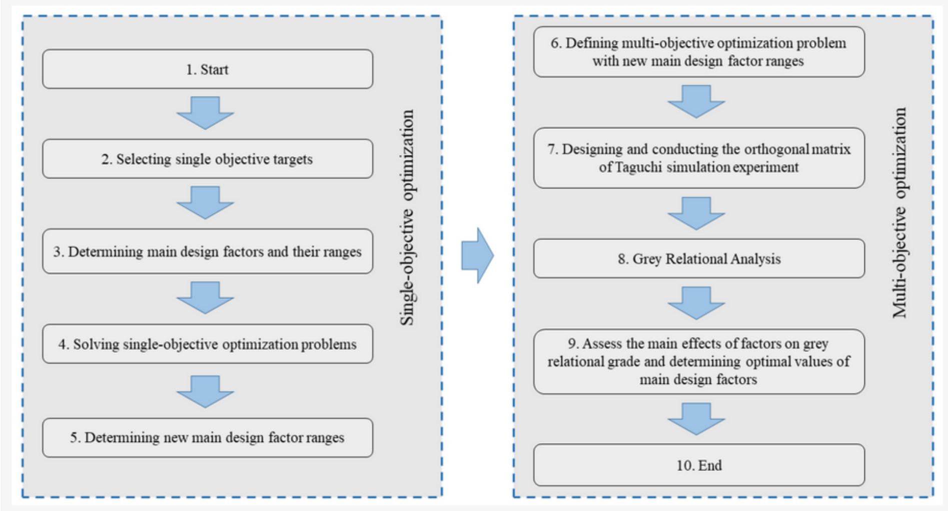


**Figure. 1. Method to solve multi-objective problem [21]**

**Results**

**Single-Objective Optimization**

The direct search strategy is used in this study to address the single-objective optimization problem. A Matlab-based computer program was also created to address two single-objective problems: reducing gearbox bottom area and optimizing gearbox efficiency. Figure 3 depicts a relationship between the ideal gear ratio of the first stage $u_1$ and the total gearbox ratio ut based on the program's findings. In addition, as shown in Table 2, additional constraints for the variable u1 have been devised.

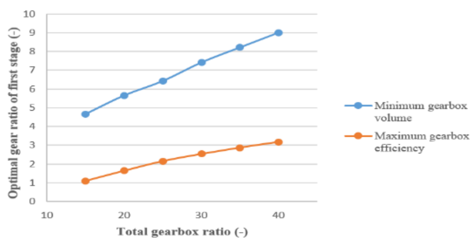


**Figure. 2. Optimal gear ratio of stage $u_1$ versus total gearbox ratio**

**Table 2.New constraints of $u_1$**

| $u_t$ | $u_1$ | |
|---|---|---|
| | Minimum limit | Maximum limit |
| 15 | 1.09 | 4.67 |
| 20 | 1.63 | 5.66 |
| 25 | 2.14 | 6.43 |
| 30 | 2.52 | 7.42 |
| 35 | 2.86 | 8.22 |
| 40 | 3.16 | 9.00 |

**Multi-Objective Optimization**

The aim of the multi-objective optimization problem for a two-stage helical gearbox with SSDGS in this work is to discover the optimal primary design variables with a given total gear-box ratio that satisfies two single-objective functions: decreasing gearbox bottom area and optimizing gearbox efficiency. A computer experiment was conducted to accomplish this. The primary design features and their values are shown in Table 3 for $u_t$ = 20.

The Taguchi technique and the L25 ($5^5$) design were used to create the experimental design, and the data was analyzed using Minitab R18 software.

Table 4 displays the experimental matrix and results for $u_t$ = 20.

**Table 3. Main design factors and their levels for $u_t$ = 20**

| Factor | Notation | Level | | | | |
|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 |
| Gear ratio of stage 1 | $u_1$ | 1.63 | 2.6375 | 3.645 | 3.6525 | 5.66 |
| CWFW of stage 1 | Xba1 | 0.25 | 0.2675 | 0.275 | 0.2875 | 0.3 |
| CWFW of stage 2 | Xba2 | 0.25 | 0.2875 | 0.325 | 0.3625 | 0.4 |
| ACS of stage1 (MPa) | AS1 | 350 | 368 | 386 | 404 | 420 |
| ACS of stage2 (MPa) | AS2 | 350 | 368 | 386 | 404 | 420 |

**Table 4. Experimental matrix and output results for $u_t$ = 20**

| Exp. No. | Input Factors | | | | | $V_{gb}$ | $\eta_{gb}$ |
|---|---|---|---|---|---|---|---|
| | $u_1$ | $X_{ba1}$ | $X_{ba2}$ | $AS_1$ | $AS_2$ | ($dm^3$) | (%) |
| 1 | 1.6300 | 0.2500 | 0.2500 | 350 | 350 | 28.666 | 95.572 |
| 2 | 1.6300 | 0.2625 | 0.2875 | 368 | 368 | 25.557 | 95.540 |
| 3 | 1.6300 | 0.2750 | 0.3250 | 386 | 386 | 22.995 | 95.534 |
| 4 | 1.6300 | 0.2875 | 0.3625 | 404 | 404 | 20.845 | 95.500 |
| 5 | 1.6300 | 0.3000 | 0.4000 | 420 | 420 | 19.177 | 95.498 |
| 6 | 2.6375 | 0.2500 | 0.2875 | 386 | 404 | 17.223 | 95.224 |
| 7 | 2.6375 | 0.2625 | 0.3250 | 404 | 420 | 15.705 | 95.217 |
| 8 | 2.6375 | 0.2750 | 0.3625 | 420 | 350 | 19.688 | 95.275 |
| 9 | 2.6375 | 0.2875 | 0.4000 | 350 | 368 | 19.240 | 95.235 |
| 10 | 2.6375 | 0.3000 | 0.2500 | 368 | 386 | 19.598 | 95.251 |
| 11 | 3.6450 | 0.2500 | 0.3250 | 420 | 368 | 16.340 | 95.016 |
| 12 | 3.6450 | 0.2625 | 0.3625 | 350 | 386 | 16.179 | 95.015 |
| 13 | 3.6450 | 0.2750 | 0.4000 | 368 | 404 | 14.688 | 94.997 |
| 14 | 3.6450 | 0.2875 | 0.2500 | 386 | 420 | 15.217 | 95.035 |
| 15 | 3.6450 | 0.3000 | 0.2875 | 404 | 350 | 18.527 | 95.041 |
| 16 | 4.6525 | 0.2500 | 0.3625 | 368 | 420 | 13.472 | 94.756 |
| 17 | 4.6525 | 0.2625 | 0.4000 | 386 | 350 | 16.250 | 94.794 |
| 18 | 4.6525 | 0.2750 | 0.2500 | 404 | 368 | 16.556 | 94.838 |
| 19 | 4.6525 | 0.2875 | 0.2875 | 420 | 386 | 14.856 | 94.839 |
| 20 | 4.6525 | 0.3000 | 0.3250 | 350 | 404 | 14.858 | 94.785 |
| 21 | 5.6600 | 0.2500 | 0.4000 | 404 | 386 | 15.033 | 94.578 |
| 22 | 5.6600 | 0.2625 | 0.2500 | 420 | 404 | 13.827 | 94.614 |
| 23 | 5.6600 | 0.2750 | 0.2875 | 350 | 420 | 15.827 | 94.585 |
| 24 | 5.6600 | 0.2875 | 0.3250 | 368 | 350 | 16.911 | 94.621 |
| 25 | 5.6600 | 0.3000 | 0.3625 | 386 | 368 | 15.542 | 94.607 |

When dealing with multi-objective optimization problems, the Taguchi and GRA techniques are applied. The major steps in this approach are as follows:

Determine the signal-to-noise ratio (S/N) using the following equations:

The lower the gearbox volume, the higher the S/N:

$$SN = -10log_{10}(\frac{1}{n}\sum_{i=1}^{m} y_i^2) \quad (30)$$

The higher the S/N, the better the gearbox efficient:

$$SN = -10log_{10}(\frac{1}{n}\sum_{i=1}^{m} \frac{1}{y_i^2})$$

Where $y_i$ is the output result and m is the number of experimental repetitions. Because this is a simulation, m = 1; no repeats are required. Thecalculated S/N values for the objectives are shown in Table 5.

**Table 5. S/N values for each experiment when $u_t$=20**

| Exp. No. | Input Factors | | | | | $V_{gb}$ | | $\eta_{gb}$ | |
|---|---|---|---|---|---|---|---|---|---|
| | $u_1$ | $X_{ba1}$ | $X_{ba2}$ | $AS_1$ | $AS_2$ | ($dm^3$) | S/N | (%) | S/N |
| 1 | 1.6300 | 0.2500 | 0.2500 | 350 | 350 | 28.666 | -29.1473 | 95.572 | 39.6066 |
| 2 | 1.6300 | 0.2625 | 0.2875 | 368 | 368 | 25.557 | -28.1502 | 95.540 | 39.6037 |
| 3 | 1.6300 | 0.2750 | 0.3250 | 386 | 386 | 22.995 | -27.2327 | 95.534 | 39.6032 |
| 4 | 1.6300 | 0.2875 | 0.3625 | 404 | 404 | 20.845 | -26.3800 | 95.500 | 39.6001 |
| 5 | 1.6300 | 0.3000 | 0.4000 | 420 | 420 | 19.177 | -25.6556 | 95.498 | 39.5999 |
| 6 | 2.6375 | 0.2500 | 0.2875 | 386 | 404 | 17.223 | -24.7222 | 95.224 | 39.5749 |
| 7 | 2.6375 | 0.2625 | 0.3250 | 404 | 420 | 15.705 | -23.9208 | 95.217 | 39.5743 |
| 8 | 2.6375 | 0.2750 | 0.3625 | 420 | 350 | 19.688 | -25.8840 | 95.275 | 39.5796 |
| 9 | 2.6375 | 0.2875 | 0.4000 | 350 | 368 | 19.240 | -25.6841 | 95.235 | 39.5759 |
| 10 | 2.6375 | 0.3000 | 0.2500 | 368 | 386 | 19.598 | -25.8442 | 95.251 | 39.5774 |
| 11 | 3.6450 | 0.2500 | 0.3250 | 420 | 368 | 16.340 | -24.2650 | 95.016 | 39.5559 |
| 12 | 3.6450 | 0.2625 | 0.3625 | 350 | 386 | 16.179 | -24.1790 | 95.015 | 39.5558 |
| 13 | 3.6450 | 0.2750 | 0.4000 | 368 | 404 | 14.688 | -23.3393 | 94.997 | 39.5542 |
| 14 | 3.6450 | 0.2875 | 0.2500 | 386 | 420 | 15.217 | -23.6466 | 95.035 | 39.5577 |
| 15 | 3.6450 | 0.3000 | 0.2875 | 404 | 350 | 18.527 | -25.3561 | 95.041 | 39.5582 |
| 16 | 4.6525 | 0.2500 | 0.3625 | 368 | 420 | 13.472 | -22.5886 | 94.756 | 39.5321 |
| 17 | 4.6525 | 0.2625 | 0.4000 | 386 | 350 | 16.250 | -24.2171 | 94.794 | 39.5356 |
| 18 | 4.6525 | 0.2750 | 0.2500 | 404 | 368 | 16.556 | -24.3791 | 94.838 | 39.5396 |
| 19 | 4.6525 | 0.2875 | 0.2875 | 420 | 386 | 14.856 | -23.4380 | 94.839 | 39.5397 |
| 20 | 4.6525 | 0.3000 | 0.3250 | 350 | 404 | 14.858 | -23.4392 | 94.785 | 39.5348 |
| 21 | 5.6600 | 0.2500 | 0.4000 | 404 | 386 | 15.033 | -23.5409 | 94.578 | 39.5158 |
| 22 | 5.6600 | 0.2625 | 0.2500 | 420 | 404 | 13.827 | -22.8146 | 94.614 | 39.5191 |
| 23 | 5.6600 | 0.2750 | 0.2875 | 350 | 420 | 15.827 | -23.9880 | 94.585 | 39.5164 |
| 24 | 5.6600 | 0.2875 | 0.3250 | 368 | 350 | 16.911 | -24.5634 | 94.621 | 39.5198 |
| 25 | 5.6600 | 0.3000 | 0.3625 | 386 | 368 | 15.542 | -23.8301 | 94.607 | 39.5185 |

The data quantities are different for the two single-objective functions. The data must be normalized, or brought to a consistent scale, to ensure comparability. To normalize the data, the normalization value $Z_{ij}$, which ranges from 0 to 1, is employed. The following formula is used to calculate this value:

$$Z_i = \frac{SN_i - min\ (SN_i, = 1,2,..n)}{max(SN_i, j = 1,2,..n) - min\ (SN_i, = 1,2,..n)} \quad (32)$$

With n=25 is the experimental number.

The grey relational factor is calculated by:

$$y_i(k) = \frac{\Delta_{min} + \xi . \Delta_{max}(k)}{\Delta_i(k) + \xi . \Delta_{max}(k)} \quad (33)$$

In which, i=1,2,...,n; k=2 is the objective number; $\Delta_j(k) = \left\| Z_0(k) - Z_j(k) \right\|$ with Z0(k) and Zj(k) are the reference and particular comparison sequence; $\Delta_{min}$ and $\Delta_{max}$ are the minimum and maximum values of i(k); ζ=0.5 is the characteristic coefficient.

Finding the coefficient of grey relations by:

$$\overline{y_i} = \frac{1}{k} \sum_{j=0}^{k} y_{ij}\ (k)$$

In which $y_{ij}$ is the grey relation value of the i[th] experiment's j[th] output target. Table 6 shows the projected grey relation number $y_i$ as well as the average grey relation valuefor each test.

To create harmony among the output elements, a higher average gray relation value is advised. As a result, the objective function of a multi-objective problem can be reduced to a single-objective optimization problem, yielding the mean gray relation value.

**Table 6. Values of $\Delta_i(k)$ and $\overline{y_i}$**

| Exp. No | S/N | | Zi | | $\Delta_i$ (k) | | Grey relation value $y_i$ | | $\overline{y_i}$ |
|---|---|---|---|---|---|---|---|---|---|
| | | | $V_{gb}$ | $\eta_{gb}$ | | | | | |
| | $V_{gb}$ | $\eta_{gb}$ | Reference values | | $V_{gb}$ | $\eta_{gb}$ | $V_{gb}$ | $\eta_{gb}$ | |
| | | | 1.000 | 1.000 | | | | | |
| 1 | -29.1473 | 39.6066 | 0.0000 | 1.0000 | 1.000 | 0.000 | 0.333 | 1.000 | 0.667 |
| 2 | -28.1502 | 39.6037 | 0.1520 | 0.9680 | 0.848 | 0.032 | 0.371 | 0.940 | 0.655 |
| 3 | -27.2327 | 39.6032 | 0.2919 | 0.9620 | 0.708 | 0.038 | 0.414 | 0.929 | 0.672 |
| 4 | -26.3800 | 39.6001 | 0.4219 | 0.9279 | 0.578 | 0.072 | 0.464 | 0.874 | 0.669 |
| 5 | -25.6556 | 39.5999 | 0.5324 | 0.9259 | 0.468 | 0.074 | 0.517 | 0.871 | 0.694 |
| 6 | -24.7222 | 39.5749 | 0.6747 | 0.6511 | 0.325 | 0.349 | 0.606 | 0.589 | 0.597 |
| 7 | -23.9208 | 39.5743 | 0.7969 | 0.6441 | 0.203 | 0.356 | 0.711 | 0.584 | 0.648 |
| 8 | -25.8840 | 39.5796 | 0.4976 | 0.7023 | 0.502 | 0.298 | 0.499 | 0.627 | 0.563 |
| 9 | -25.6841 | 39.5759 | 0.5280 | 0.6621 | 0.472 | 0.338 | 0.514 | 0.597 | 0.556 |
| 10 | -25.8442 | 39.5774 | 0.5036 | 0.6782 | 0.496 | 0.322 | 0.502 | 0.608 | 0.555 |
| 11 | -24.2650 | 39.5559 | 0.7444 | 0.4419 | 0.256 | 0.558 | 0.662 | 0.473 | 0.567 |
| 12 | -24.1790 | 39.5558 | 0.7575 | 0.4409 | 0.242 | 0.559 | 0.673 | 0.472 | 0.573 |
| 13 | -23.3393 | 39.5542 | 0.8856 | 0.4228 | 0.114 | 0.577 | 0.814 | 0.464 | 0.639 |
| 14 | -23.6466 | 39.5577 | 0.8387 | 0.4611 | 0.161 | 0.539 | 0.756 | 0.481 | 0.619 |
| 15 | -25.3561 | 39.5582 | 0.5780 | 0.4671 | 0.422 | 0.533 | 0.542 | 0.484 | 0.513 |
| 16 | -22.5886 | 39.5321 | 1.0000 | 0.1798 | 0.000 | 0.820 | 1.000 | 0.379 | 0.689 |
| 17 | -24.2171 | 39.5356 | 0.7517 | 0.2182 | 0.248 | 0.782 | 0.668 | 0.390 | 0.529 |
| 18 | -24.3791 | 39.5396 | 0.7270 | 0.2626 | 0.273 | 0.737 | 0.647 | 0.404 | 0.525 |
| 19 | -23.4380 | 39.5397 | 0.8705 | 0.2636 | 0.130 | 0.736 | 0.794 | 0.404 | 0.599 |
| 20 | -23.4392 | 39.5348 | 0.8703 | 0.2091 | 0.130 | 0.791 | 0.794 | 0.387 | 0.591 |
| 21 | -23.5409 | 39.5158 | 0.8548 | 0.0000 | 0.145 | 1.000 | 0.775 | 0.333 | 0.554 |
| 22 | -22.8146 | 39.5191 | 0.9656 | 0.0364 | 0.034 | 0.964 | 0.936 | 0.342 | 0.639 |
| 23 | -23.9880 | 39.5164 | 0.7866 | 0.0071 | 0.213 | 0.993 | 0.701 | 0.335 | 0.518 |
| 24 | -24.5634 | 39.5198 | 0.6989 | 0.0435 | 0.301 | 0.957 | 0.624 | 0.343 | 0.484 |
| 25 | -23.8301 | 39.5185 | 0.8107 | 0.0293 | 0.189 | 0.971 | 0.725 | 0.340 | 0.533 |

The findings of an ANOVA test run to analyze the effect of the key design parameters on the average grey relation value $\overline{y_i}$ are shown in Table 7.

Table 7 shows that u1 has the most influence on $\overline{y_i}$ (47.34%), followed by AS2 (25.58%), Xba1 (6.2%), AS1 (4.32%), and Xba2 (2.63%). Using ANOVA analysis, Table 8 shows the order of the effect of the primary design factors on $\overline{y_i}$.

**Table 7. Analysis of variance for means**

**Analysis of Variance for Means**

| Source | DF | Seq SS | Adj SS | Adj MS | F | P | C (%) |
|---|---|---|---|---|---|---|---|
| u1 | 4 | 0.043150 | 0.043150 | 0.010788 | 4.33 | 0.092 | 47.34 |
| Xba1 | 4 | 0.005653 | 0.005653 | 0.001413 | 0.57 | 0.702 | 6.20 |
| Xba2 | 4 | 0.002400 | 0.002400 | 0.000600 | 0.24 | 0.902 | 2.63 |
| AS1 | 4 | 0.003934 | 0.003934 | 0.000984 | 0.39 | 0.805 | 4.32 |
| AS2 | 4 | 0.026049 | 0.026049 | 0.006512 | 2.61 | 0.187 | 28.58 |
| Residual Error | 4 | 0.009967 | 0.009967 | 0.002492 | | | 10.93 |
| Total | 24 | 0.091152 | | | | | |

**Model Summary**

| S | R-Sq | R-Sq(adj) |
|---|---|---|
| 0.0499 | 89.07% | 34.39% |

Determining the best major design factors: The best factor set, in theory, would include core design aspects with the highest S/N values. As a result, the impact of the major design elements on the S/N ratio (Fig. 4) was estimated. Furthermore, using the Figure 4 chart, the optimal set of multi-objective parameters (corresponding to the red points) may be easily determined. Table 9 shows the appropriate levels and values for the multi-objective function's essential design variables.

Proposal modeling evaluation: Figure 5 depicts the Anderson-Darling method results, which are used to assess the adequacy of the suggested model. The data points corresponding to the experimental observations (shown as blue dots in the graph) fall within the 95% standard deviation zone defined by the top and bottom bounds. Furthermore, the p-value of 0.267 is much greater than the level of significance of $\alpha = 0.05$. These results demonstrate that the empirical model utilized in this study is suitable for evaluation.

**Table 8. Response table for means**

**Response Table for Means**

| Level | u1 | Xba1 | Xba2 | AS1 | AS2 |
|---|---|---|---|---|---|
| 1 | 0.6713 | 0.6149 | 0.6009 | 0.5807 | 0.5511 |
| 2 | 0.5837 | 0.6087 | 0.5766 | 0.6045 | 0.5672 |
| 3 | 0.5821 | 0.5833 | 0.5922 | 0.5899 | 0.5906 |
| 4 | 0.5868 | 0.5852 | 0.6053 | 0.5819 | 0.6269 |
| 5 | 0.5454 | 0.5771 | 0.5943 | 0.6123 | 0.6335 |
| Delta | 0.1259 | 0.0378 | 0.0287 | 0.0316 | 0.0824 |
| Rank | 1 | 3 | 5 | 4 | 2 |

Average of grey analysis value: 0.594

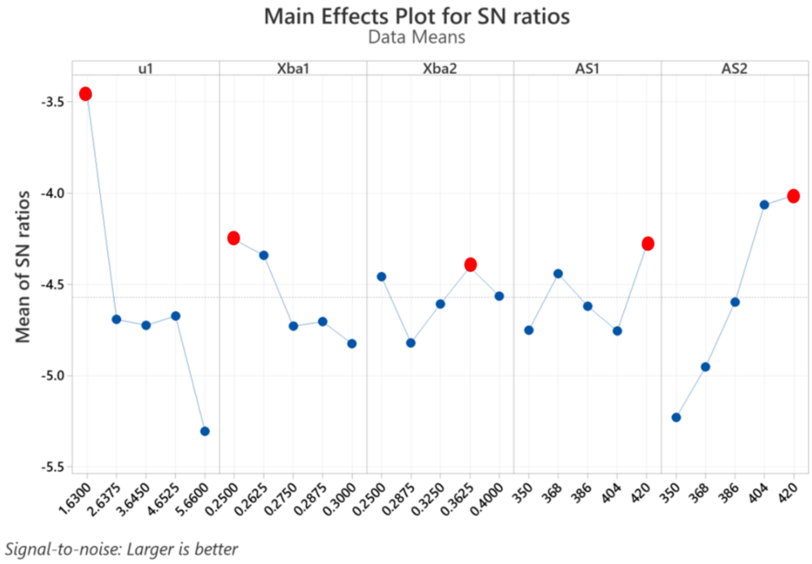


***Figure. 3*. Main effects plot for S/N ratios**

**Table 9. Optimum values of main design factors**

| No. | Input Parameters | Code | Optimum Level | Optimum Value |
|---|---|---|---|---|
| 1 | Gear ratio of first stage | $u_1$ | 1 | 1.63 |
| 2 | CWFW of stage 1 | $X_{ba1}$ | 1 | 0.25 |
| 3 | CWFW of stage 2 | $X_{ba2}$ | 4 | 0.3625 |
| 4 | ACS of stage 1 (MPa) | $AS_1$ | 5 | 420 |
| 5 | ACS of stage 2 (MPa) | $AS_2$ | 5 | 420 |

Proceed in the same way as with $u_t$=20, but with $u_t$ values 15, 25, 30, 35, and 40. Table 10 illustrates the optimum values of the five key design parameters at various $u_t$ for each of the five primary design parameters. The link between the ideal first-stage gear ratio and the overall gearbox ratio is depicted in Figure 6. Furthermore, the following regression formula (with $R^2$=0.999) was presented to obtain the ideal values of $u_1$:

$$u_1 = 2.1241 \cdot \ln(u_t) - 4.694 \quad (35)$$

After obtaining $u_1$, the optimal gear ratio of second stage $u_2$ can be found by $u_2=u_t/u_1$.

**Table 10. Optimal values of main design factors**

| No. | ut | | | | | |
|---|---|---|---|---|---|---|
| | 15 | 20 | 25 | 30 | 35 | 40 |

| | | | | | | |
|---|---|---|---|---|---|---|
| **u1** | **1.09** | **1.63** | **2.14** | **2.52** | **2.86** | **3.16** |
| **Xba1** | **0.25** | **0.25** | **0.25** | **0.25** | **0.25** | **0.25** |
| **Xba2** | **0.3625** | **0.3625** | **0.3625** | **0.3625** | **0.3625** | **0.3625** |
| **AS1** | **420** | **420** | **420** | **420** | **420** | **420** |
| **AS2** | **420** | **420** | **420** | **420** | **420** | **420** |

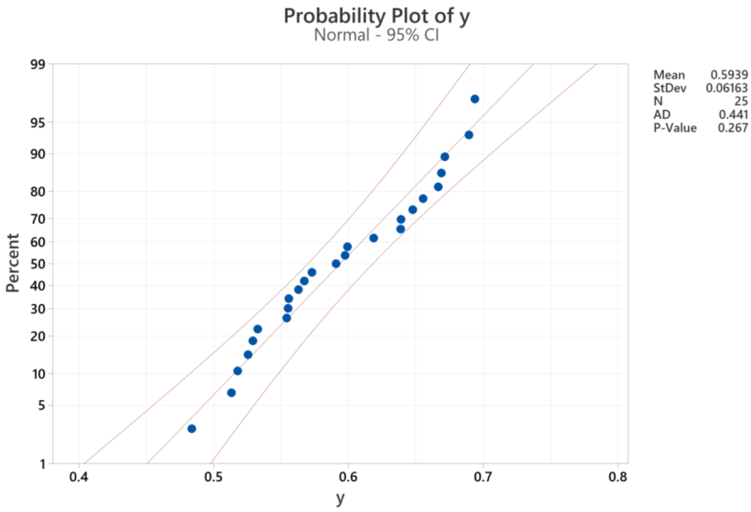


**Figure. 4. Probability plot of $\bar{y}$**

**Conclusion**

The results of a multi-objective optimization study on optimizing a two-stage helical gearbox with SSDGS to reduce gearbox volume and improve gearbox efficiency are discussed in this paper. The first stage of this research enhanced the gear ratio, the efficiency of wheel face width in stages 1 and 2, and the permitted contact stress in stages 1 and 2. A simulation experiment based on the Taguchi L25 type was devised and carried out to address this issue. The impact of key design features on the multi-objective goal was also investigated. The gear ratio u1 was discovered to have the biggest influence (47.34%), followed by AS2 (25.58%), $X_{ba1}$ (6.2%), $AS_1$ (4.32%), and $X_{ba2}$ (2.63%). Furthermore, the optimal values for the critical gearbox characteristics have been advised. A regression technique (Equation 37) was also introduced to discover the optimal first-stage $u_1$ gear ratio..

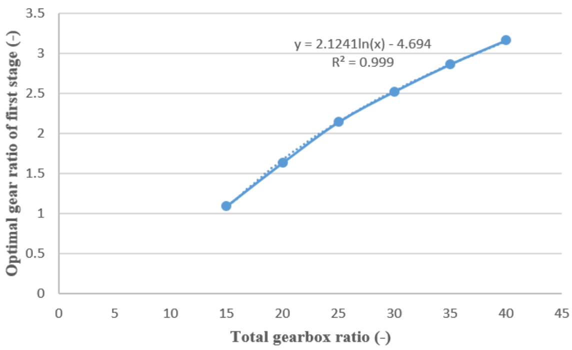


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**Figure. 5. Optimal gear ratio of first stage versus total gearbox ratio.**

**Acknowledgement**

This work was supported by Thai Nguyen University of Technology.

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