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Modified Interval Arithmetic Operations onInterval Valued Neutrosophic Fuzzy Number (IVNFNS) for Solving Neutrosophic Fuzzy Unconstrained Optimization Problem (NFUOP)

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Abstract

Introduction: Neutrosophic quasi-Newton techniques are utilized in many engineering applications, including control theory, electrical engineering, and mechanical engineering, for determining optimal solutions in the face of uncertainty, imprecision, and ambiguity. This paper introduced new modified arithmetic operations on interval valued Neutrosophic fuzzy numbers (IVNFNS).

Objectives: In this paper, New Arithmetic operation for interval-valued neutrosophic fuzzy is introduced. Some fundamental operations are also presented. The need of the interval-valued neutrosophic fuzzy matrix (IVNFM) is explained by an illustration.

Methods: The new Neutrosophic quasi-Newton techniques are optimization algorithms that integrate neutrosophic logic with the quasi-Newton method. Traditional quasi-Newton methods use a deterministic formula to approximate the Hessian matrix, but neutrosophic quasi-Newton methods use neutrosophic logic principles to correct for errors, imprecision, and ambiguity. This technique offers greater flexibility and resilience when dealing with complicated and nonlinear optimization issues, especially when the objective function's parameters are unclear, imprecise, or ambiguous.

Result: To explain the suggested technique, an exemplary numerical example is provided, as well as the Neutrosophic fuzzy output is given using MATLAB programme.

Conclusion: This paper presented how extended interval arithmetic operations and modified interval arithmetic operations can be used to solve unconstrained optimization problems. In comparison, neutrosophic fuzzy numbers. A quasi-Newtonian DFP strategy is used to solve fuzzy and unconstrained optimization problems, and the validity of the proposed approaches is confirmed through numerical examples and MATLAB program results.

Keywords: Neutrosophic Sets, Neutrosophic Interval Arithmetic, Quasi Newton method..

1. Introduction

The optimization of linear and nonlinear systems is covered in this work. When using real-world Difficulties, inexactness and ambiguity are unavoidable when dealing with problems containing just crisp values. In 1965, Zadeh invented the "Fuzzy sets and systems", which play a significant role in dealing with ambiguity and differences. In 1970, Bellman and Zadeh developed "a method for making decisions in a fuzzy environment". ". In 1983, Atanassov introduced his intuitionistic fuzzy set. which is an extension of fuzzy sets that encompassed both situations. Because it contains information that belongs to the set as well as information that does not belong to the set, intuitionistic fuzzy sets are regarded as an extension of fuzzy sets. There is also the chance of a different condition, known as indeterminacy, in real-life uncertainty. Indeterminacy occurs when information of which items belong to and do not belong to the collection is lacking.Neutrosophic sets had three associated essential functions, namely the membership function (T), the nonmembership (F) function and the indeterminacy function (I) defined on the universe of discourse X, the three functions are entirely independent.. Illumination of some of the operators are given with the help of the example. Many methods for solving general nonlinear unconstrained optimisation issues include iteratively minimising a model function which satisfies specific interpolation restrictions. These requirements yield a model that acts similarly to the objective function in the vicinity of the present iteration. Model functions frequently involve second-order derivatives of the objective function, which can be difficult to compute. The fundamental concept underlying quasi-Newton methods is to keep a close approximation to the Hessian matrix. The practical success of quasi-Newton methods has spurred a great deal of interest and research that has resulted in a considerable number of variations of this idea. Because of the analytical difficulties involved in characterising the performance of these algorithms, there is a great need for practical testing to support up theoretical claims. The paper is organized as follows: Section 2 introduces the preliminaries of Triangular Neutrosophic fuzzy numbers (HIFNS), interval valued Neutrosophic fuzzy numbers and the modified arithmetical operations for interval valued Neutrosophic fuzzy numbers (IVIFNS). Section 3 deals with the formulation of Neutrosophic fuzzy Quasi Newton(DFP) Method, the algorithm for solving Neutrosophic fuzzy Quasi Newton DavidonFletcher Powell method. In section 4, an application of these new operations is discussed by a numerical illustration and some concluding remarks are given.

2. Objectives

DEFINITION:1

Let G be a universe. A Neutrosophic Set(NS) A in G is characterised by a truth-membership function $\mathrm{T_A}$, a indeterminacy-membership function $\mathrm{I_A}$, and a falsity membership function $\mathrm{F_A}$. $\mathrm{T_A(x); I_A(x)}$ and $\mathrm{F_A(x)}$ are real standard elements of [0,1]. It can be written as $\mathrm{A} = \{x, (T_A(x); I_A(x) F_A(x)) >: x \in G, T_A(x), I_A(x), F_A(x) \in]^{-}0,1[^{+}\}$. There is no restriction on the sum of $T_A(x): I_A(x) and F_A$(x) , so on $0^{-} \leq supT_A(x) + supI_A(x) + supF_A(x) \leq 3^{+}$.

DEFINITION:2

Let G be a universe. A single valued Neutrosophic Set (SVNS) A, which can be used I a real scientific and engineering applications, in G is characterised by a truth membership function T_A, an indeterminacy-membership function I_A and a falsity membership function F_A. $T_A(x); I_A(x) and F_A(x)$, are real standard elements of [0,1] . It can be written as $\mathrm{A} = \{< x, (T_A(x), I_A(x), F_A(x)) >: x \in E, T_A(x), I_A(x) and F_A \in [0,1]$. Thereis no restriction on the sum of $T_A(x), F_A(x) and I_A(x)$, so $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

DEFINITION:3

Let $j_{\tilde{a}}, k_{\tilde{a}}, q_{\tilde{a}} \in [0,1]$ be any real numbers. A single valued neutrosophic number $\tilde{a} = (s_1, d_1, f_1, g_1); j_{\tilde{a}}), (s_2, d_2, f_2, g_2); k_{\tilde{a}}), (s_3, d_3, f_3, g_3); q_{\tilde{a}}))$, is defined as a special single valued neutrosophic set on the set of real numbers R, whose truth-membership function $\mu_{\tilde{a}}: R \to [0, j_{\tilde{a}}]$, a determinancy-membership function $\nu_{\tilde{a}}: R \to [0, k_{\tilde{a}}]$ and a falsity-membership function $\lambda_{\tilde{a}}: R \to [0, q_{\tilde{a}}]$ as given by

$$\mu_{\tilde{a}}(x) = \begin{cases} f_{\mu l}(x)(s_1 \leq x \leq d_1) \\ j_{\tilde{a}}(d_1 \leq x \leq f_1) \\ f_{\mu r}(x)\ (f_1 \leq x \leq g_1) \\ 0\ otherwise \end{cases}$$

$$\nu_{\tilde{a}}(x) = \begin{cases} f_{\lambda l}(x)\quad (s_3 \leq x \leq d_3) \\ k_{\tilde{a}}(d_3 \leq x \leq f_3) \\ f_{\lambda r}(x)\ (f_3 \leq x \leq g_3) \\ 1\ \ otherwise \end{cases}$$

$$\lambda_{\tilde{a}}(x) = \begin{cases} f_{\nu l}(x)\quad (s_2 \leq x \leq d_2) \\ q_{\tilde{a}}(d_2 \leq x \leq f_2) \\ f_{\nu r}(x)\ (f_2 \leq x \leq g_2) \\ 1\ otherwise \end{cases}$$

DEFINITION [4]

Neutrosophic Number

A neutrosophic set N defined over the universal single valued set of real numbers R is said to be Neutrosophic number if it has the following properties

(i)N is normal, if there exist $x_0 \in R$, such that $T_N(x_0) = 1, (I_N(x_0) = F_N(x_0) = 0)$

(ii)N is covex set for the truth function $T_N(x)$, i.e., $T_N(\mu_{x_1} + (1-\mu)x_2) \geq \min(T_N(x_1), T_N(x_2)), \forall x_1, x_2 \in R, \mu \in [1,0]$

(iii) N is concave set for the indeterminancy function $I_N(x)$ and false function $((F_N(x))$ i.e., $I_N(\mu x_1 + (1-\mu)x_2) \geq \max(I_N(x_1), I_N(x_2)), \forall x_1, x_2 \in R, \mu \in [0,1], F_N(\mu x_1 + (1-\mu)x_2) \geq \max((F_N(x_1)), (F_N(x_2)), \forall x_1, x_2 \in R, \mu \in [0,1]$.

DEFINITION [5]

$(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma}) - \mathbf{levelofNeutrosophicset}$.

A Neutrosophic set with (α, β, γ) −level of X is denoted by $G(\alpha, \beta, \gamma)$ where $\alpha, \beta, \gamma \in [0,1]$ and is defined as $G(\alpha, \beta, \gamma) = \{T_N(x), I_N(x): \forall x \in$

$X, T_N(x) \geq \alpha, I_N(x) \leq \beta,\ F_N(x) \leq \gamma\}$, where $0 \leq \alpha + \beta + \gamma \leq 3$.

2.1 TRIANGULAR NEUTROSOPHIC NUMBER

Let N be a single valued Neutrosophic set (SVNS) having truth ($T_N(x)$), Indeterminacy $I_N(x)$ and false $F_N(x)$ membership function over universal set X, then the triangular number is defined defined as

$$T_N(x) = \begin{cases} \left(\dfrac{(x-a)}{(b-a)}\right) for a \leq x \leq b \\ 1 \quad for x = b \\ \left(\dfrac{(c-x)}{(c-b)}\right) for b < x \leq c \\ 0 \quad Otherwise \end{cases}$$

$$I_N(x) = \begin{cases} \left(\dfrac{(b-x)}{(b-a)}\right) for a \leq x < b \\ 0 \quad for x = b \\ \left(\dfrac{(x-b)}{(c-b)}\right) for b < x \leq c \\ 1 \quad Otherwise \end{cases}$$

$$F_N(x) = \begin{cases} \left(\dfrac{(b-x)}{(b-a)}\right) for a \leq x \leq b \\ 0 \quad for x = b \\ \left(\dfrac{(x-c)}{(c-b)}\right) for b < x \leq c \\ 1 \quad Otherwise \end{cases}$$

where $a \leq b \leq c$ and $a, b, c \in R$. Triangular neutrosophic number are denoted as $N_T(a, b, c)$ where the truth membership function ($T_N(x)$) increase in a linear way for $x \in [a, b]$ and decrease in a linear way for $x \in [b, c]$ for $I_N(x)$ and $F_N(x)$ inverse behaviour is seen from the truth membership for $x \in [a, b]$ and for $x \in [b, c]$

DEFINITION[6]

2.2$(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma})$CUT OF ATRIANGULAR NEUTROSOPHIC NUMBER

A triangular neutrosophic set with $(\alpha, \beta, \gamma) \in [0,1] - cut$is denoted by $A_{TN}(\alpha, \beta, \gamma)$ where $\alpha, \beta, \gamma \in [0,1]$ and is defined as $A_{TN}(\alpha, \beta, \gamma) = \{T_N(x), I_N(x), F_N(x): T_N(x) \geq \alpha, I_N(x) \leq \beta, F_N(x) \leq \gamma, x \in X\}$. Here $0 \leq \alpha + \beta + \gamma \leq 3$ and

$$A_{TN}(\alpha, \beta, \gamma) = \begin{matrix} [a + \alpha(b-a)), (c - \alpha(c-b))] \\ [b - \beta(b-a)), (b + \beta(c-b)] \\ [b - \gamma(b-a)), (b + \gamma(c-b)] \end{matrix}$$

2.3 MODIFIED OPERATIONS OF INTERVAL VALUED NEUTROSOPHIC FUZZY NUMBERS USING FUNCTION PRINCIPAL

The following are the modified operations that can be performed on interval valued Neutrosophic fuzzy numbers(IVNFNS).

Let $\tilde{A}^{Neu} = \{[a_{t_1}, a_{t_2}]: [a_{f_1}, a_{f_2}]; [a_{i_1}, a_{i_2}]\}$

$\tilde{B}^{Neu} = \{[b_{t_1}, b_{t_2}]: [b_{f_1}, b_{f_2}]; [b_{i_1}, b_{i_2}]\}$

Then (i) Addition

$$\tilde{A}^{Neu} + \tilde{B}^{Neu} = \{[a_{t_1} + b_{t_1}, a_{t_2} + b_{t_2}]; [a_{f_1} + b_{f_1}, a_{f_2} + b_{f_2}]; [a_{i_1} + b_{i_1}, a_{i_2} + b_{i_2}]\}$$

(ii)Subtraction

$$\tilde{A}^{Neu} - \tilde{B}^{Neu} = \{[a_{t_1} - b_{t_1}, a_{t_2} - b_{t_2}]; [a_{f_1} - b_{f_1}, a_{f_2} - b_{f_2}]; [a_{i_1} - b_{i_1}, a_{i_2} - b_{i_2}]\}$$

(iii) Multiplication

$$\tilde{A}^{Neu} X \tilde{B}^{Neu} = \left\{ \begin{bmatrix} \min[a_{t_1}b_{t_1}, a_{t_1}b_{t_2}, a_{t_2}b_{t_1}, a_{t_2}b_{t_2}], \\ \max[a_{t_1}b_{t_1}, a_{t_1}b_{t_2}, a_{t_2}b_{t_1}, a_{t_2}b_{t_2}]]; \\ \min[a_{f_1}b_{f_1}, a_{f_1}b_{f_2}, a_{f_2}b_{f_1}, a_{f_2}b_{f_2}], \\ \max[a_{f_1}b_{f_1}, a_{f_1}b_{f_2}, a_{f_2}b_{f_1}, a_{f_2}b_{f_2}]] \\ \min[a_{i_1}b_{i_1}, a_{i_1}b_{i_2}, a_{i_2}b_{i_1}, a_{i_2}b_{i_2}], \\ \max[a_{i_1}b_{i_1}, a_{i_1}b_{i_2}, a_{i_2}b_{i_1}, a_{i_2}b_{i_2}]] \end{bmatrix} \right\};$$

2.4 SECOND TYPE OF MODIFIED OPERATION ON INTERVAL VALUED NEUTROSOPHIC FUZZY NUMBER

According to Irene Hepzibah et al., we developed the modified operations on intervalvaluedNeutrosophic fuzzy numbers here.

Let $\tilde{A}^{Neu} = \{[a_{t_1,}a_{t_2}]; [a_{f_1}, a_{f_2}]; [a_{i_1}, a_{i_2}]\}$

And $\tilde{B}^{Neu} = \{[b_{t_1,}b_{t_2}]; [b_{f_1}, b_{f_2}]; [b_{i_1}, b_{i_2}]\}$

Then we define

$$m_{t_1} = \frac{a_{t_1} + a_{t_2}}{2}$$

$$m_{f_1} = \frac{a_{f_1} + a_{f_2}}{2}$$

$$m_{t_1} = \frac{a_{i_1} + a_{i_2}}{2}$$

$$m_{t_2} = \frac{b_{t_1} + b_{t_2}}{2}$$

$$m_{f_2} = \frac{b_{f_1} + b_{f_2}}{2}$$

$$m_{t_2} = \frac{b_{i_1} + b_{i_2}}{2}$$

(i) Addition

$$\tilde{A}^{Neu} + \tilde{B}^{Neu} = \{[(m_{t_1} + m_{t_2} - k_t); (m_{t_1} + m_{t_2} + k_t)]; [(m_{f_1} + m_{k_2} - k_f); (m_{t_1} + m_{t_2} + k_t)]; [(m_{t_1} + m_{t_2} - k_t); (m_{t_1} + m_{t_2} + k_t)]\}$$

Where

$$k_t = \frac{[b_{t_2} + a_{t_2}] - [b_{t_1} + a_{t_1}]}{2}$$

$$k_f = \frac{[b_{f_1} + a_{f_2}] - [b_{f_1} + a_{f_1}]}{2}$$

$$k_i = \frac{[b_{i_1} + a_{i_2}] - [b_{i_1} + a_{i_1}]}{2}$$

(ii) Subtraction

$$\tilde{A}^{Neu} - \tilde{B}^{Neu} = \{[(m_{t_1} - m_{t_2} - k_t); (m_{t_1} - m_{t_2} + k_t)]; [(m_{f_1} - m_f - k_f); (m_{f_1} - m_{f_2} + k_f)]; [(m_{i_1} - m_{i_2} - k_i); (m_{i_1} - m_{i_2} + k_i)]\}$$

Where

$$k_t = \frac{[b_{t_2} + a_{t_2}] - [b_{t_1} + a_{t_1}]}{2}$$

$$k_f = \frac{[b_{f_1} + a_{f_2}] - [b_{f_1} + a_{f_1}]}{2}$$

$$k_i = \frac{[b_{i_1} + a_{i_2}] - [b_{i_1} + a_{i_1}]}{2}$$

(iii) Multiplication

$$\tilde{A}^{Neu} X \tilde{B}^{Neu} = \{[(m_{t_1} m_{t_2} - k_t); ((m_{t_1} m_{t_2} + k_t)]; [(m_{f_1} m_{f_2} - k_f); ((m_{f_1} m_{f_2} + k_f)]; [(m_{i_1} m_{i_2} - k_i); ((m_{i_1} m_{i_2} + k_i)]\}$$

where

$$k_t = \min(m_{t_1} m_{t_2} - \alpha_t, \beta_t - m_{t_1} m_{t_2})$$
$$\alpha_t = min(a_{t_1} b_{t_1}, a_{t_1} b_{t_2}, a_{t_2} b_{t_1}, a_{t_2} b_{t_2})$$
$$\beta_t = max(a_{t_1} b_{t_1}, a_{t_1} b_{t_2}, a_{t_2} b_{t_1}, a_{t_2} b_{t_2})$$
$$k_f = \min(m_{f_1} m_{f_2} - \alpha_f, \beta_f - m_{f_1} m_{f_2})$$
$$\alpha_f = min(a_{f_1} b_{f_1}, a_{f_1} b_{f_2}, a_{f_2} b_{f_1}, a_{f_2} b_{f_2})$$
$$\beta_f = max(a_{f_1} b_{f_1}, a_{f_1} b_{f_2}, a_{f_2} b_{f_1}, a_{f_2} b_{f_2})$$
$$k_i = \min(m_{i_1} m_{i_2} - \alpha_i, \beta_i - m_{i_1} m_{i_2})$$
$$\alpha_i = min(a_{i_1} b_{i_1}, a_{i_1} b_{i_2}, a_{i_2} b_{i_1}, a_{i_2} b_{i_2})$$
$$\beta_i = max(a_{i_1} b_{i_1}, a_{i_1} b_{i_2}, a_{i_2} b_{i_1}, a_{i_2} b_{i_2})$$

(iv) Inverse

$$\frac{1}{\tilde{A}^{Neu}} = \{[a_{t_1}, a_{t_2}]; [a_{f_1}.a_{f_2}]; [a_{i_1}, a_{i_2}]\} = [\left(\frac{1}{m_{t_1}} - k_t, \frac{1}{m_{t_1}} + k_t\right); \left(\frac{1}{m_{f_1}} - k_f, \frac{1}{m_{f_1}} + k_f\right); \left(\frac{1}{m_{i_1}} - k_i, \frac{1}{m_{i_1}} + k_i\right)$$

where

$$k_t = \min [(\frac{1}{a_{t_2}}\left(\frac{a_{t_2} - a_{t_1}}{a_{t_1} + a_{t_2}}\right), (\frac{1}{a_{t_1}}\left(\frac{a_{t_2} - a_{t_1}}{a_{t_1} + a_{t_2}}\right)]$$

$$k_f = \min [(\frac{1}{a_{f_2}}\left(\frac{a_{f_2} - a_{f_1}}{a_{f_1} + a_{f_2}}\right), (\frac{1}{a_{f_1}}\left(\frac{a_{f_2} - a_{f_1}}{a_{f_1} + a_{f_2}}\right)]$$

$$k_i = \min [(\frac{1}{a_{i_2}}\left(\frac{a_{i_2} - a_{i_1}}{a_{i_1} + a_{i_2}}\right), (\frac{1}{a_{i_1}}\left(\frac{a_{i_2} - a_{i_1}}{a_{i_1} + a_{i_2}}\right)]$$

For all positive real numbers $a_{t_1}, a_{t_2}, a_{f_1}, a_{f_2}, a_{i_1}, a_{i_2}$ and $0 \in \{[a_{t_1}, a_{t_2}]; [a_{f_1}, a_{f_2}]; [a_{i_1,} a_{i_2}]\}$

(v) Scalar Multiplication

Let $\lambda \in R, then$

$$\lambda \tilde{A}^{Neu} = \{[\lambda a_{t_1}, \lambda a_{t_2}]; [\lambda a_{f_1}, \lambda a_{f_2}]; [\lambda a_{i_1}, \lambda a_{i_2}]\}$$

for $\lambda \geq 0$

$$\lambda \tilde{A}^{Neu} = \{[\lambda a_{t_1}, \lambda a_{t_2}]; [\lambda a_{f_1}, \lambda a_{f_2}]; [\lambda a_{i_1}, \lambda a_{i_2}]\}$$

for $\lambda \leq 0$

Midpoint

Let $\tilde{A}^{Neu} = \{[a_{t_1}, a_{t_2}]; [a_{f_1}, a_{f_2}]; [a_{i_1}, a_{i_2}]\}$ and $\tilde{B}^{Neu} = \{[b_{t_1}, b_{t_2}]; [b_{f_1}, b_{f_2}]; [b_{i_1}, b_{i_2}]\}$

Then

$$M(\tilde{A}^{Neu}) = \frac{a_{t_1} + a_{t_2}}{2}, M(\tilde{B}^{Neu}) = \frac{b_{t_1} + b_{t_2}}{2}$$

$$M\left(\tilde{A}^{Neu'}\right) = \frac{a_{f_1} + a_{f_2}}{2}, M\left(\tilde{B}^{Neu'}\right) = \frac{b_{f_1} + b_{f_2}}{2}$$

2.5 CONDITION ON SUBTRACTION OPERATOR

Let $\tilde{A}^{Neu} = \{[a_{t_1}, a_{t_2}]; [a_{f_1}, a_{f_2}]; [a_{i_1}, a_{i_2}]\}$ and $\tilde{B}^{Neu} = \{[b_{t_1}, b_{t_2}]; [b_{f_1}, b_{f_2}]; [b_{i_1}, b_{i_2}]\}$

Then $\tilde{A}^{Neu} - \tilde{B}^{Neu}$ $=\{[a_{t_1} - b_{t_1}, a_{t_2} - b_{t_2}]; [a_{f_1} - b_{f_1}, a_{f_2} - b_{tf_2}]; [a_{i_1} - b_{i_1}, a_{i_2} - b_{i_2}]\}$

The new subtraction operation exists if the following conditions are satisfied $D(\tilde{A}^{Neu}) \geq D(\tilde{B}^{Neu}), D\left(\tilde{A}^{Neu'}\right) \geq D\left(\tilde{B}^{Neu'}\right)$and $D\left(\tilde{A}^{Neu''}\right) \geq D\left(\tilde{B}^{Neu''}\right)$, where $D(\tilde{A}^{Neu}) = \frac{a_{t_1}-a_{t_2}}{2}$, $D(\tilde{B}^{Neu}) = \frac{b_{t_2}-b_{t_1}}{2}$, $D\left(\tilde{A}^{Neu'}\right) = \frac{a_{f_1}-a_{f_2}}{2}, D\left(\tilde{B}^{Neu'}\right) = \frac{b_{f_2}-b_{f_1}}{2}$, $D\left(\tilde{A}^{Neu''}\right) = \frac{a_{i_1}-a_{i_2}}{2}$, $D\left(\tilde{B}^{Neu''}\right) = \frac{b_{i_2}-b_{i_1}}{2}$.

Here D denotes difference point of an interval values Neutrosophic function.

2.6 PROPERTIES OF SUBTRACTION OPERATOR

(i) Inverse Property of +:
$$\tilde{B}^{Neu} + (\tilde{A}^{Neu} - \tilde{B}^{Neu}) = (\tilde{A}^{Neu} - \tilde{B}^{Neu}) + \tilde{B}^{Neu}$$

(ii) Multiplication by a Scalar:
$$\lambda(\tilde{A}^{Neu} - \tilde{B}^{Neu}) = \lambda\tilde{A}^{Neu} - \lambda\tilde{B}^{Neu}$$

(iii) Neutral Element:
$$\tilde{A}^{Neu} - \tilde{0}^{Neu} = \tilde{A}^{Neu}$$

(iv) Associativity:
$$\tilde{A}^{Neu} - (\tilde{B}^{Neu} - \tilde{C}^{Neu}) = (\tilde{A}^{Neu} - \tilde{B}^{Neu}) - \tilde{C}^{Neu}$$

(v) Inverse Element:
Any Interval valued Neutrosophic fuzzy number is its own inverse under the modified subtraction .e., $\tilde{A}^{Neu}$-$\tilde{A}^{Neu}$=$\tilde{0}^{Neu}$.

(vi) Regularity : $\tilde{A}^{Neu} - \tilde{B}^{Neu} = \tilde{A}^{Neu} - \tilde{C}^{Neu} \Longrightarrow \tilde{B}^{Neu} = \tilde{C}^{Neu}$

(vii) Pseudo – Distributivity with respect to +:
$(\tilde{A}^{Neu} + \tilde{B}^{Neu}) - (\tilde{C}^{Neu} + \tilde{D}^{Neu})$=($\tilde{A}^{Neu} - \tilde{C}^{Neu}) + (\tilde{B}^{Neu} - \tilde{D}^{Neu})$

2.7 A NEW DIVISION ON INTERVAL VALUED NEUTROSOPHIC FUZZY NUMBER

Condition on Division Operator

Let $\tilde{A}^{Neu} = \{[a_{t_1}, a_{t_2}]; [a_f, a_{f_2}]; [a_{i_1}, a_{i_2}]\}$ and $\tilde{B}^{Neu} = \{[b_{t_1}, b_{t_2}]; [b_f, b_{f_2}]; [b_{i_1}, b_{i_2}]\}$

$$\frac{\tilde{A}^{Neu}}{\tilde{B}^{Neu}} = \left\{\left[\frac{a_{t_1}}{b_{t_1}}, \frac{a_{t_2}}{b_{t_2}}\right]; \left[\frac{a_{f_1}}{b_{f_1}}, \frac{a_{f_2}}{b_{f_2}}\right]; \left[\frac{a_{i_1}}{b_{i_1}}, \frac{a_{i_2}}{b_{i_2}}\right]\right\}$$

The New division operator exist onl if the following conditions are sstisfied

$$\left|\frac{D(\tilde{A}^{Neu})}{M(\tilde{A}^{Neu})}\right| \geq \left|\frac{D(\tilde{B}^{Neu})}{M(\tilde{B}^{Neu})}\right|$$

$$\left|\frac{D(\tilde{A}^{Neu})'}{M(\tilde{A}^{Neu})'}\right| \geq \left|\frac{D(\tilde{B}^{Neu})'}{M(\tilde{B}^{Neu})'}\right|$$

$$\left|\frac{D(\tilde{A}^{Neu})''}{M(\tilde{A}^{Neu})''}\right| \geq \left|\frac{D(\tilde{B}^{Neu})''}{M(\tilde{B}^{Neu})''}\right|$$

And the negative interval valued Neutrosophic fuzzy number should be changed into negative multiplication of positive interval valued Neutrosophic fuzzy number.

3. Fuzzy quasi Newton Davidon Fletcher-Powell
4. Method

The newton ,type methods call for the evaluation of the positive definite hessian matrix as well as solving a linear system at each step. In Fuzzy quasi newton method, we have

$$\tilde{g}^{(k+1)} - \tilde{g}^{(k)} = Q\tilde{x}^{(k+1)} - \tilde{x}^{(k)}$$

Using the notation $\Delta\tilde{g}^{(k)} \ = \tilde{g}^{(k+1)} - \tilde{g}^{(k)}$ and $\Delta\tilde{x}^{(k)}$=$\tilde{x}^{(k+1)} - \tilde{x}^{(k)}$, we get $\Delta\tilde{g}^{(k)} \ = \tilde{Q}\Delta\tilde{x}^{(k)}$ where $\tilde{Q}$ is the hessian matrix. The matrix $\tilde{Q} satisfies \tilde{Q}^{-1}\Delta\tilde{g}^{(i)} \ = \Delta\tilde{x}^{(i)}, 0 \leq i \leq k.$

The above equation gives the motivation to obtain a condition for computing approximation $\widetilde{H_0}, \widetilde{H_1}, \widetilde{H_2} \ldots., \widetilde{H_n}$to the inverse hessian. It satisfies the equation

$$\widetilde{H_{k+1}}\Delta\tilde{g}^{(i)} \ = \Delta\tilde{x}^{(i)}, 0 \leq i \leq k \qquad 3.1.1$$

Hence, the fuzzy Quasi-Newton iteration is described as follows:

1.Beginning with stage $k = 0$, we calculate positive matrix $\widetilde{H_k}(\widetilde{H_0} = \tilde{I})$ through a process to be described later and iterates $\tilde{x}^{(k)}(\tilde{x}^{(0)} istheinitialone)$

2.Set the direction search $\tilde{d}^{(k)}$=-$\widetilde{H_k}\Delta\tilde{g}^{(k)}$.

3.Let $\tilde{\alpha}^k = min_\alpha f(\tilde{x}^{(k)} + \tilde{\alpha}\tilde{d}^{(k)}$ and define the next iterate $\tilde{x}^{(k+1)}$ as $\tilde{x}^{(k+1)} = \tilde{x}^{(k)} + \tilde{\alpha}^k\tilde{d}^{(k)}$.

For a quadratic function $\tilde{f}, \tilde{\alpha}^k$ are computed by the following formula

$$\tilde{\alpha}^k = -\frac{\langle\tilde{g}^k, \tilde{d}^k\rangle}{\langle\widetilde{Qd}^k, \tilde{d}^k\rangle}$$

3.1 Algorithm for Fuzzy Neutrosophic Quasi Newton Davidon-Fletcher-Powell Method

Step 1: Consider the unconstrained optimization problem with Intuitionistic fuzzy triangular coefficient $\tilde{g}(\tilde{x}^{(k)})$ Input $\tilde{x}^{(0)^{Neu}}, \in$.

Step 2:Calculate $\widetilde{H_0}^{Neu} = I, \tilde{g}^{(0)^{Neu}} = \nabla\tilde{f}^{Neu} k = 0$

Step 3:Repeat the process.

Step 4:Calculate $\tilde{d}^{(k)^{Neu}} = -\widetilde{H_k}^{Neu} \Delta\tilde{g}^{(k)^{Neu}}$

Step 5: Using the one dimensional search to minimize the $\tilde{f}\left(\tilde{x}^{k^{Neu}} + \tilde{\alpha}^{Neu}\left(\tilde{d}^{k}\right)^{Neu}\right)$, to get $\tilde{\alpha}^{k^{Neu}}$.

Step 6: $\tilde{x}^{k+1^{Neu}} = \tilde{x}^{k^{Neu}} + \tilde{\alpha}^{k^{Neu}}\left(\tilde{d}^{k}\right)^{Neu}$.

Step 7: Find the difference, $\Delta\tilde{x}^{k+1^{Neu}} = \tilde{x}^{k+1^{Neu}} - \tilde{x}^{k^{Neu}}$

Step 8: Calculate $\Delta\tilde{g}^{k+1^{Neu}} = \tilde{g}^{k+1^{Neu}} - \tilde{g}^{k^{Neu}}$.

Step9: Then calculate

$$\widetilde{H_{k+1}}^{Neu} = \widetilde{H_k}^{Neu} + \frac{\Delta\tilde{x}^{(k)^{Neu}}\left(\Delta\tilde{x}^{(k)^{Neu}}\right)^{Neu}}{\left\langle \tilde{g}^{k^{Neu}}, \Delta\tilde{x}^{(k)^{Neu}} \right\rangle} - \frac{\widetilde{H_k}^{Neu} \Delta\tilde{g}^{(k)^{Neu}}\left(\Delta\tilde{g}^{(k)^{Neu}}\right)^{Neu} \widetilde{H_k}^{Neu}}{\left\langle \widetilde{H_k}^{Neu} \Delta\tilde{g}^{(k)^{Neu}}, \Delta\tilde{g}^{(k)^{Neu}} \right\rangle}. k \leftarrow k + 1.$$

Step 10: Until $\left\|\Delta\tilde{x}^{(k)^{Neu}}\right\| <\in or \left\|\Delta\tilde{g}^{(k)^{Neu}}\right\| <\in$.

Step 11: Find the optimal solution $\tilde{x}^{(*)} \leftarrow \tilde{x}^{(k)^{I}}$.

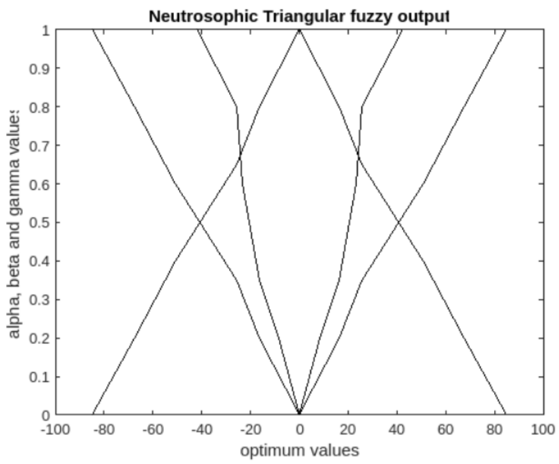
5. Illustrative Example

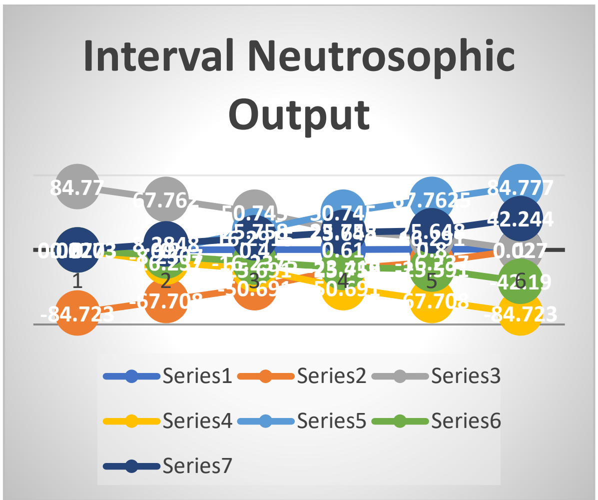
Let us consider the Neutrosophic Triangular fuzzy unconstrained optimization problem (0.5,1,1.5);(0.5,1,1.5);(0.75,1,1.25)x_1^2+ (0.5,1,1.5); (0.5,1,1.5); (0.75,1,1.25)$x_1 x_2$ + (1.5,2,2.5);(1.5,2,2.5);(1.75,2,2.25) x_2^2.

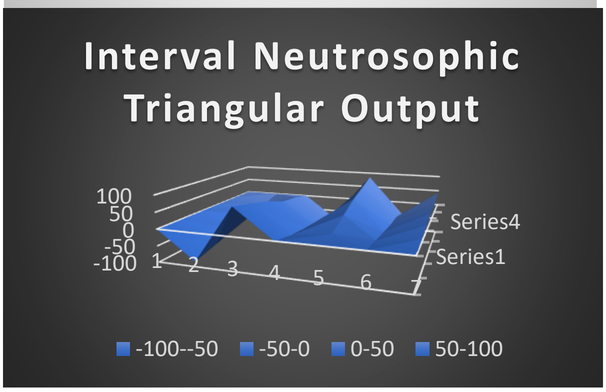
(α) – *cut*	**Lower bound of Truth value at Optimum value**	**Upper bound of Truth value at Optimum value**
0	-84.723	84.77
0.20	-67.708	67.762
0.40	-50.691	50.745
0.61	-25.759	25.755
0.80	-16.237	16.291
1.00	0.0270	0.0270

(β) – *cut*	**Lower bound of Indeterminacy value at Optimum value**	**Upper bound of Indeterminacy value at Optimum value**
0	0.0270	0.0270
0.20	-16.237	16..291
0.40	-25.691	25.758
0.61	-50.691	50.745
0.80	-67.708	67.7625
1.00	-84.723	-84.777

(γ) – *cut*	**Lower bound of Falsity value at Optimum value**	**Upper bound of Falsity value at Optimum value**
0	0.02703	0.02703
0.20	-8.2307	8.2848
0.40	-16.2375	16.2915
0.61	-23.418	23.648
0.80	-25.591	25.648
1.00	-42.190	42.244







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