

Exploring Second Order Bipolar Fuzzy Topological Spaces for Enhanced Decision-Making in Medical Diagnosis

Muthamizhselvi. S^{*}, Vijayalakshmi.V.M¹

Department of Mathematics, Avinashilingam Institute for Home Science and Higher Education for Women,
Coimbatore, Tamil Nadu-601 043.

Department of Mathematics, Avinashilingam Institute for Home Science and Higher Education for Women,
Coimbatore, Tamil Nadu-601 043.

Abstract

Aim of this article is to define the new concept of second order bipolar fuzzy topological spaces. Basic properties and related theorems are proved with suitable examples. An Application of decision making in Medical Diagnosis using second order bipolar fuzzy topology has been proposed and illustrated using a real life problem as an example.

Keywords: second order bipolar fuzzy set, second order bipolar fuzzy topological spaces.

1. Introduction

A second order bipolar fuzzy topological space (SOBFTS) is a mathematical framework that extends traditional topological space to accommodate second-order bipolar fuzzy sets. SOBFTS is a collection of second order bipolar fuzzy sets which are the ordered pairs of Second order fuzzy sets whose degree of memberships are described using bipolar values, allowing for a more nuanced representation of uncertainty and ambiguity. This concept plays a vital role in dealing with imprecise and uncertain data in various fields, including decision-making, pattern recognition, and artificial intelligence, providing a versatile and powerful tool for modeling complex real-world scenarios. The notion of fuzzy set theory was introduced by Zadeh [7] which is an universality of classical set theory. Fuzzy set (first order fuzzy set) is a mapping on a set X with values in the closed unit interval $I=[0,1]$. The concept of fuzzy topology was first introduced by Chang [1] and then modified by Lowen [5].

Zadeh [8] proposed the concept of second order fuzzy set which is defined as a map from a set X to $I^!$. Definition of fuzzy point was introduced by Wong [6]. The second order fuzzy sets are characterized by a membership function, the grade

(fuzzy grade) of which is a fuzzy set in the unit interval $[0,1]$ rather than a point in $[0,1]$.

Zhang [9] introduced the notion of a bipolar fuzzy set. J. Kim, S. K. Samata, P. K. Lim, J. G. Lee and K. Hur [4] introduced bipolar fuzzy topology by following Lowen's fuzzy topology. Using second order fuzzy sets Kalaichelvi [2,3] extended first order fuzzy topology of Chang and Lowen to second order fuzzy topology.

Medical diagnosis involves complex decision-making processes, where the use of second-order bipolar fuzzy sets provide a nuanced approach. These sets extend traditional fuzzy logic by incorporating both membership and non-membership degrees, allowing for a more comprehensive representation of uncertainty in diagnostic information. In this context, the topology of second-order bipolar fuzzy sets plays a crucial role. It defines the relationships and interactions between different elements within the diagnostic framework. The topology provides a structured way to analyze the connections among test results and patient history, allowing healthcare professionals to make decisions

In this paper, the concept of second order bipolar fuzzy set and second order bipolar fuzzy topological spaces are discussed and some results are proved with examples under the concepts

which lead us to the new approach of rel world judgemental problems. An Application of decision making in Medical Diagnosis using second order bipolar fuzzy topology has been proposed. The case study of a patient is analysed and the result has been concluded.

2. Preliminary definitions

Definition 2.1

Let X be an arbitrary nonempty set. Let $I = [0,1]$. A **fuzzy set** in X is a mapping from X into I that is a fuzzy set is an element of I^X .

Definition 2.2

A **fuzzy point** x_t in a set X , with x in X and t in $(0,1]$ is a fuzzy set in X defined by $x_t(y) = t$ for $y = x$ and $x_t(y) = 0$ for $y \neq x$. A fuzzy point x_t in X said to **belong** to a fuzzy set f , written as $x_t \in f$ iff $x_t(x) \leq f(x)$ that is $t \leq f(x)$. Hence $x_t \notin f \Leftrightarrow t > f(x)$. Two fuzzy points are said to be **distinct** iff their supports are distinct.

Definition 2.3

Let X be a nonempty set. A subset $\tau \subset I^X$ is called a **Chang fuzzy topology** on X iff τ satisfies the following requirements:

- (i) The constant fuzzy sets **0** and **1** belong to τ
- (ii) $f_\lambda \in \tau$ for each $\lambda \in \Lambda$
implies $\bigvee_{\lambda \in \Lambda} f_\lambda \in \tau$
- (iii) $f, g \in \tau$ implies $f \wedge g \in \tau$

The pair (X, τ) is called a **Chang fuzzy topological space**.

Definition 2.4

Let X be a nonempty set. A subset $\tau \subset I^X$ is called a **Lowen fuzzy topology** on X iff the following conditions are satisfied:

- (i) All constant fuzzy sets belong to τ
- (ii) $f_\lambda \in \tau$ for each $\lambda \in \Lambda$
implies $\bigvee_{\lambda \in \Lambda} f_\lambda \in \tau$

- (iii) $f, g \in \tau$ implies $f \wedge g \in \tau$

The pair (X, τ) is called a **Lowen fuzzy topological space**. The elements in τ are called **open fuzzy sets** of the fuzzy topological space (X, τ) . Complements of open sets are called **closed fuzzy sets** of (X, τ) .

Definition 2.5

Let X be a nonempty set. Then a pair $A_{bp} = (A_{bp}^+, A_{bp}^-)$ is called **bipolar-valued fuzzy set or bipolar fuzzy set** in X , where $A_{bp}^+ : X \rightarrow [0,1]$ and $A_{bp}^- : X \rightarrow [-1,0]$. The set of all bipolar fuzzy set in X is denoted as $BPF(X)$.

Definition 2.6

Let X be a nonempty set. A subset $\tau_{\mathfrak{B}} \subset BPF(X)$ is called a **bipolar fuzzy topology on X** , iff $\tau_{\mathfrak{B}}$ satisfies the following axioms:

- (i) $0_{bp}, 1_{bp} \in \tau_{\mathfrak{B}}$
- (ii) $A_{bp} \cap B_{bp} \in \tau_{\mathfrak{B}}$, for any $A_{bp}, B_{bp} \in \tau_{\mathfrak{B}}$
- (iii) $\bigcup_{\lambda \in \Lambda} (A_{bp})_\lambda \in \tau_{\mathfrak{B}}$, for any $\left((A_{bp})_\lambda \right)_{\lambda \in \Lambda} \in \tau_{\mathfrak{B}}$

The pair $(X, \tau_{\mathfrak{B}})$ is called a bipolar fuzzy topological space.

Definition 2.7

Let X be a nonempty set and let $\tau_{\mathfrak{B}} \subset BPF(X)$. Then $\tau_{\mathfrak{B}}$ is called a **bipolar fuzzy topology on X** , if it satisfies the following axioms:

- (i) All constant bipolar fuzzy sets belong to $\tau_{\mathfrak{B}}$
- (ii) $A_{bp}, B_{bp} \in \tau_{\mathfrak{B}}$

implies $A_{bp} \cap B_{bp} \in \tau_{\mathfrak{B}}$

$\left((A_{bp})_\lambda \right)_{\lambda \in \Lambda} \in \tau_{\mathfrak{B}}$

for each $\lambda \in \Lambda$

implies $\bigcup_{\lambda \in \Lambda} (A_{bp})_\lambda \in \tau_{\mathfrak{B}}$

The pair $(X, \tau_{\mathfrak{B}})$ is called a bipolar fuzzy topological space and each member of $\tau_{\mathfrak{B}}$ is called a bipolar fuzzy open set (BPFOS) in X . Complements of bipolar fuzzy open sets are called bipolar fuzzy closed sets of $(X, \tau_{\mathfrak{B}})$.

Definition 2.8

Let $x \in X, (\alpha, \beta) \in (0,1] \times [-1,0)$ and let $A_{bp} \in BPF(X)$. Then

- (i) $x_{(\alpha,\beta)}$ is called a **bipolar fuzzy point** in X with the value (α, β) and the support x , if for each $y \in X$,

$$[x_{(\alpha,\beta)}](y) = \begin{cases} (\alpha, \beta), & \text{if } y = x \\ (0,0), & \text{otherwise} \end{cases}$$

- (ii) $x_{(\alpha,\beta)}$ is said to belong to A_{bp} , denoted by $x_{(\alpha,\beta)} \in A_{bp}$, if

$$A_{bp}^+(x) \geq \alpha \text{ and } A_{bp}^-(x) \leq \beta$$

The set of all bipolar fuzzy points in X is denoted as $BPF_p(X)$. It is clear that

$$A_{bp} = \cup \{x_{(\alpha,\beta)} \in BPF_p(X) : x_{(\alpha,\beta)} \in A_{bp}\}, \text{ for each } A_{bp} \in BPF(X).$$

Definition 2.9

Let X be an arbitrary nonempty set. A **second order fuzzy set** on X is a map

$$\hat{f} : X \rightarrow I^I \text{ where } I \text{ is the closed unit interval } [0,1].$$

The family of second order fuzzy sets is denoted by $(I^I)^X$. (i)

Definition 2.10

Let X be a nonempty set. A collection $\hat{\tau}$ of second order fuzzy sets on X defines a **second order fuzzy topology** on X if the following conditions are (ii) satisfied:

- (i) $\hat{0}, \hat{1} \in \hat{\tau}$
 (ii) $\hat{f}_\lambda \in \hat{\tau}$ for each $\lambda \in \Lambda$ implies $(\bigvee_{\lambda \in \Lambda} \hat{f}_\lambda) \in \hat{\tau}$
 (iii) $\hat{f}_i \in \hat{\tau}$ for $i=1$ to n implies $\bigwedge_{i=1}^n \hat{f}_i \in \hat{\tau}$ (iii)

The pair $(X, \hat{\tau})$ is called a **second order Chang fuzzy topological space**.

A **Second order Lowen fuzzy topology** $\hat{\tau}$ on (iv) X is defined by replacing axiom (i), in the above definition by axiom (i)'.

- (i)' All constant second order fuzzy sets $\in \hat{\tau}$

The pair $(X, \hat{\tau})$ is called a **second order Lowen fuzzy topological space**.

The elements of $\hat{\tau}$ are called **second order fuzzy open sets**.

3. Second Order Bipolar Fuzzy Topological Spaces

Definition 3.1

Let X be a nonempty set. Then a pair $\hat{A}_{bp} = (\hat{A}_{bp}^+, \hat{A}_{bp}^-)$ is called a **second-order bipolar fuzzy set** in X where $\hat{A}_{bp}^+ : X \rightarrow [0,1]^{[0,1]}$ such that $\hat{A}_{bp}^+(x)(\alpha) \in [0,1]$ & $\hat{A}_{bp}^- : X \rightarrow [-1,0]^{[0,1]}$ such that $\hat{A}_{bp}^-(x)(\alpha) \in [-1,0]$, where $\alpha \in I$ & $x \in X$. The set of all second-order bipolar fuzzy sets in X is denoted as $SBPF(X)$.

Example

Let X be a nonempty set, let $\hat{A}_{bp} = (\hat{A}_{bp}^+, \hat{A}_{bp}^-)$
 $\hat{A}_{bp}^+(x_1)(\alpha) = 0.2, \hat{A}_{bp}^-(x_1)(\alpha) = -0.5$
 $\hat{A}_{bp}^+(x_2)(\alpha) = 0.6, \hat{A}_{bp}^-(x_2)(\alpha) = -0.4$

Definition 3.2

The **second-order bipolar fuzzy null set** denoted by $\hat{0}_{bp} = (\hat{0}_{bp}^+, \hat{0}_{bp}^-)$ is a second-order bipolar fuzzy set in X defined as $\hat{0}_{bp}^+(x)(\alpha) = 0$ and $\hat{0}_{bp}^-(x)(\alpha) = 0$, for every $x \in X$ and for every $\alpha \in I$.

The **second-order bipolar fuzzy whole set** denoted by $\hat{1}_{bp} = (\hat{1}_{bp}^+, \hat{1}_{bp}^-)$ is a second-order bipolar fuzzy set in X defined as $\hat{1}_{bp}^+(x)(\alpha) = 1$ and $\hat{1}_{bp}^-(x)(\alpha) = -1$, for every $x \in X$ and for every $\alpha \in I$.

The **second-order bipolar fuzzy constant set** denoted by $\hat{\alpha}_{bp} = (\hat{\alpha}_{bp}^+, \hat{\alpha}_{bp}^-)$ is a second-order bipolar fuzzy set in X defined as $\hat{\alpha}_{bp}^+(x)(\alpha) = \alpha$ and $\hat{\alpha}_{bp}^-(x)(\alpha) = -\alpha$, for every $x \in X$ and for every $\alpha \in I$.

Let $\hat{A}_{bp}, \hat{B}_{bp} \in SBPF(X)$. \hat{A}_{bp} is called a **second-order bipolar fuzzy subset** of \hat{B}_{bp} denoted as $\hat{A}_{bp} \subseteq \hat{B}_{bp}$, if for every $x \in X$ and for every $\alpha \in I$,

$$\widehat{A}_{bp}^+(x)(\alpha) \leq \widehat{B}_{bp}^+(x)(\alpha) \quad \text{and} \quad \widehat{A}_{bp}^-(x)(\alpha) \geq \widehat{B}_{bp}^-(x)(\alpha)$$

- (v) Let $\widehat{A}_{bp}, \widehat{B}_{bp} \in \text{SBPF}(X)$. The **union** of \widehat{A}_{bp} and \widehat{B}_{bp} is denoted as $(\widehat{A}_{bp} \cup \widehat{B}_{bp})$ is also a second order bipolar fuzzy set in X and is defined as

$$\begin{aligned} & (\widehat{A}_{bp} \cup \widehat{B}_{bp}) \\ &= \left((\widehat{A}_{bp}^+ \vee \widehat{B}_{bp}^+), (\widehat{A}_{bp}^- \wedge \widehat{B}_{bp}^-) \right), \text{ where} \quad (ix) \\ & (\widehat{A}_{bp}^+ \vee \widehat{B}_{bp}^+): X \rightarrow [0,1]^{[0,1]} \text{ and } (\widehat{A}_{bp}^- \wedge \widehat{B}_{bp}^-): X \rightarrow [-1,0]^{[0,1]} \text{ such that} \\ & \quad (\widehat{A}_{bp}^+ \vee \widehat{B}_{bp}^+)(x)(\alpha) = \max \{ \widehat{A}_{bp}^+(x)(\alpha), \widehat{B}_{bp}^+(x)(\alpha) \} \text{ and} \\ & \quad (\widehat{A}_{bp}^- \wedge \widehat{B}_{bp}^-)(x)(\alpha) = \min \{ \widehat{A}_{bp}^-(x)(\alpha), \widehat{B}_{bp}^-(x)(\alpha) \} \quad \text{for every} \\ & x \in X \text{ \& for every } \alpha \in I. \end{aligned}$$

- (vi) Let $\widehat{A}_{bp}, \widehat{B}_{bp} \in \text{SBPF}(X)$. The **intersection** of \widehat{A}_{bp} and \widehat{B}_{bp} is denoted as $(\widehat{A}_{bp} \cap \widehat{B}_{bp})$ is also a second order bipolar fuzzy set in X and is defined as

$$\begin{aligned} & (\widehat{A}_{bp} \cap \widehat{B}_{bp}) \\ &= \left((\widehat{A}_{bp}^+ \wedge \widehat{B}_{bp}^+), (\widehat{A}_{bp}^- \vee \widehat{B}_{bp}^-) \right), \text{ where} \\ & (\widehat{A}_{bp}^+ \wedge \widehat{B}_{bp}^+): X \rightarrow [0,1]^{[0,1]} \text{ and} \\ & (\widehat{A}_{bp}^- \vee \widehat{B}_{bp}^-): X \rightarrow [-1,0]^{[0,1]} \text{ such that} \\ & \quad (\widehat{A}_{bp}^+ \wedge \widehat{B}_{bp}^+)(x)(\alpha) = \min \{ \widehat{A}_{bp}^+(x)(\alpha), \widehat{B}_{bp}^+(x)(\alpha) \} \text{ and} \\ & \quad (\widehat{A}_{bp}^- \vee \widehat{B}_{bp}^-)(x)(\alpha) = \max \{ \widehat{A}_{bp}^-(x)(\alpha), \widehat{B}_{bp}^-(x)(\alpha) \} \text{ for every } x \in X \\ & \text{and for every } \alpha \in I. \end{aligned}$$

- (vii) For a family $\{(\widehat{A}_{bp})_\lambda / \lambda \in \Lambda\}$ of second order bipolar fuzzy sets the intersection $\bigcap_{\lambda \in \Lambda} (\widehat{A}_{bp})_\lambda$ is defined by,

$$\begin{aligned} & \bigcap_{\lambda \in \Lambda} (\widehat{A}_{bp})_\lambda = \left(\bigwedge_{\lambda \in \Lambda} (\widehat{A}_{bp}^+)_\lambda, \bigvee_{\lambda \in \Lambda} (\widehat{A}_{bp}^-)_\lambda \right), \\ & \text{where} \\ & \left(\bigwedge_{\lambda \in \Lambda} (\widehat{A}_{bp}^+)_\lambda \right)(x)(\alpha) = \min \{ (\widehat{A}_{bp}^+)_\lambda(x)(\alpha) \} \\ & \left(\bigvee_{\lambda \in \Lambda} (\widehat{A}_{bp}^-)_\lambda \right)(x)(\alpha) = \max \{ (\widehat{A}_{bp}^-)_\lambda(x)(\alpha) \}, \\ & \text{for every } x \in X \text{ and for every } \alpha \in I. \end{aligned}$$

- (viii) For a family $\{(\widehat{A}_{bp})_\lambda / \lambda \in \Lambda\}$ of second order bipolar fuzzy sets, the union $\bigcup_{\lambda \in \Lambda} (\widehat{A}_{bp})_\lambda$ is defined by

$$\bigcup_{\lambda \in \Lambda} (\widehat{A}_{bp})_\lambda = \left(\bigvee_{\lambda \in \Lambda} (\widehat{A}_{bp}^+)_\lambda, \bigwedge_{\lambda \in \Lambda} (\widehat{A}_{bp}^-)_\lambda \right),$$

where

$$\left(\bigvee_{\lambda \in \Lambda} (\widehat{A}_{bp}^+)_\lambda \right)(x)(\alpha) = \max \{ (\widehat{A}_{bp}^+)_\lambda(x)(\alpha) \}$$

and

$$\left(\bigwedge_{\lambda \in \Lambda} (\widehat{A}_{bp}^-)_\lambda \right)(x)(\alpha) = \min \{ (\widehat{A}_{bp}^-)_\lambda(x)(\alpha) \},$$

for every $x \in X$ and for every $\alpha \in I$.

Let $x \in X, (r, s) \in (0,1) \times [-1,0]$. A

second-order bipolar fuzzy point

$\widehat{x}_{(r,s)} = (\widehat{x}_{(r,s)}^+, \widehat{x}_{(r,s)}^-)$ is a second-order bipolar fuzzy set in X defined as follows:

$\widehat{x}_{(r,s)}^+(x)$ = the constant bipolar fuzzy set r and

$\widehat{x}_{(r,s)}^-(y)$ = the constant bipolar fuzzy set 0 for $y \neq x$

$\widehat{x}_{(r,s)}^+(x)$ = the constant bipolar fuzzy set s and

$\widehat{x}_{(r,s)}^-(y)$ = the constant bipolar fuzzy set 0 for $y \neq x$

A second-order bipolar fuzzy point $\widehat{x}_{(r,s)}$ is said to **belong** to a second-order bipolar fuzzy set $\widehat{A}_{bp} = (\widehat{A}_{bp}^+, \widehat{A}_{bp}^-)$ iff $\widehat{A}_{bp}^+(x)(\alpha) \geq r$, for every $x \in X$ and for every $\alpha \in (0,1]$ and $\widehat{A}_{bp}^-(x)(\alpha) \leq s$, for every $x \in X$ and for every $\alpha \in [-1,0]$. Two second-order bipolar fuzzy point $\widehat{x}_{(r,s)}, \widehat{y}_{(m,n)}$ are said to be **distinct** iff $x \neq y$.

Definition 3.3

For a second-order bipolar fuzzy set \widehat{A}_{bp} on X , the **support** of \widehat{A}_{bp} is defined in two different ways as follows:

$$\begin{aligned} \mathbb{S}_1(\widehat{A}_{bp}) &= \{x \in X / \widehat{A}_{bp}^+(x)(\alpha) > 0, \\ & \quad \widehat{A}_{bp}^-(x)(\alpha) < 0 \text{ for some } \alpha \in I\} \\ \mathbb{S}_2(\widehat{A}_{bp}) &= \{x \in X / \widehat{A}_{bp}^+(x)(\alpha) > 0, \\ & \quad \widehat{A}_{bp}^-(x)(\alpha) < 0 \text{ for every } \alpha \in I\}. \end{aligned}$$

Remark 3.4

$$\mathbb{S}_2(\widehat{A}_{bp}) \subseteq \mathbb{S}_1(\widehat{A}_{bp})$$

Proof:

$$\text{Let } x \in \mathbb{S}_2(\widehat{A}_{bp})$$

Then $\widehat{A}_{bp}^+(x)(\alpha) > 0, \widehat{A}_{bp}^-(x)(\alpha) < 0$, for every $\alpha \in I$

Therefore $x \in \mathbb{S}_1(\widehat{A}_{bp})$

implies $\mathbb{S}_2(\widehat{A}_{bp}) \subseteq \mathbb{S}_1(\widehat{A}_{bp})$.

Definition 3.5

For a second-order bipolar fuzzy set \widehat{A}_{bp} on X , the complement of \widehat{A}_{bp} is defined in two different ways,

(i) $(\widehat{A}_{bp})_c = ((\widehat{A}_{bp}^+)_c, (\widehat{A}_{bp}^-)_c)$, where
 $(\widehat{A}_{bp}^+)_c(x)(\alpha) = \widehat{A}_{bp}^+(x)(1 - \alpha)$, for every
 $x \in X$ and for every $\alpha \in I$.

$(\widehat{A}_{bp}^-)_c(x)(\alpha) = \widehat{A}_{bp}^-(x)(1 - \alpha)$, for every
 $x \in X$ and for every $\alpha \in I$.

(ii) $(\widehat{A}_{bp})^c = ((\widehat{A}_{bp}^+)^c, (\widehat{A}_{bp}^-)^c)$,
 where

$(\widehat{A}_{bp}^+)^c(x)(\alpha) = 1 - \widehat{A}_{bp}^+(x)(\alpha)$, for every
 $x \in X$ and for every $\alpha \in I$.

$(\widehat{A}_{bp}^-)^c(x)(\alpha) = -1 - \widehat{A}_{bp}^-(x)(\alpha)$, for every
 $x \in X$ and for every $\alpha \in I$.

Remark 3.6

(i) $((\widehat{A}_{bp})_c)_c = \widehat{A}_{bp}$

(ii) $((\widehat{A}_{bp})^c)^c = \widehat{A}_{bp}$

Definition 3.7

Let X be a nonempty set. A collection $\widehat{\tau}_{\mathfrak{B}}$ of second-order bipolar fuzzy set on X defines a **second-order bipolar fuzzy topology (Chang)** on X if the following conditions are satisfied:

- (i) $\widehat{0}_{bp}, \widehat{1}_{bp} \in \widehat{\tau}_{\mathfrak{B}}$
- (ii) $(\widehat{A}_{bp})_\lambda \in \widehat{\tau}_{\mathfrak{B}}$, for each $\lambda \in \Lambda$ implies $\bigcup_{\lambda \in \Lambda} (\widehat{A}_{bp})_\lambda \in \widehat{\tau}_{\mathfrak{B}}$.
- (iii) $(\widehat{A}_{bp})_i \in \widehat{\tau}_{\mathfrak{B}}$, for each $i = 1$ to m implies that $\bigcap_{i=1}^m (\widehat{A}_{bp})_i \in \widehat{\tau}_{\mathfrak{B}}$.

The pair $(X, \widehat{\tau}_{\mathfrak{B}})$ is called a **second-order Chang bipolar fuzzy topological space**.

A **second-order Lowen bipolar fuzzy topology** on X is defined by replacing axiom (i), in the above definition by (i)'.

(i)' All constant second-order bipolar fuzzy sets $\in \widehat{\tau}_{\mathfrak{B}}$.

The pair $(X, \widehat{\tau}_{\mathfrak{B}})$ is called **second-order Lowen bipolar fuzzy topological space**.

The elements of $\widehat{\tau}_{\mathfrak{B}}$ are called **second-order bipolar fuzzy open sets**.

Example 3.8

Let X be a non empty set. Let

1. $\widehat{\tau}_{\mathfrak{B}} = \{\widehat{0}_{bp}\} \cup \{\widehat{A}_{bp} \in \text{SBPF}(X) / \widehat{A}_{bp}^+(x)(\alpha) = 1, \widehat{A}_{bp}^-(x)(\alpha) = -1,$
 for every $x \in X$ and for every $\alpha \in I\}$

Then $\widehat{\tau}_{\mathfrak{B}}$ is a second-order bipolar fuzzy topology on X .

2. $\widehat{\tau}_{\mathfrak{B}} = \{\widehat{1}_{bp}\} \cup \{\widehat{A}_{bp} \in \text{SBPF}(X) / \widehat{A}_{bp}^+(x)(\alpha) = 0, \widehat{A}_{bp}^-(x)(\alpha) = 0,$
 for every $x \in X$ and for every $\alpha \in I\}$

Then $\widehat{\tau}_{\mathfrak{B}}$ is a second-order bipolar fuzzy topology on X .

Definition 3.9

Let $(X, \widehat{\tau}_{\mathfrak{B}})$ be a second-order bipolar fuzzy topological space. A family $\widehat{\mathfrak{B}} \in \widehat{\tau}_{\mathfrak{B}}$ is called a **base** for $\widehat{\tau}_{\mathfrak{B}}$ iff for each $\widehat{A}_{bp} \in \widehat{\tau}_{\mathfrak{B}}$, there exists a family $\{(\widehat{A}_{bp})_\lambda / \lambda \in \Lambda\}$ in $\widehat{\tau}_{\mathfrak{B}}$ such that $\widehat{A}_{bp} = \bigcup_{\lambda \in \Lambda} (\widehat{A}_{bp})_\lambda$

Definition 3.10

Let $(X, \widehat{\tau}_{\mathfrak{B}})$ be a second-order bipolar fuzzy topological space. A family $\widehat{\mathcal{S}}$ of second-order bipolar fuzzy open sets in $(X, \widehat{\tau}_{\mathfrak{B}})$ is called a **sub-base** for $\widehat{\tau}_{\mathfrak{B}}$ iff the family of finite intersection of members of $\widehat{\mathcal{S}}$ forms a base for $\widehat{\tau}_{\mathfrak{B}}$.

Definition 3.11

Let X and Y be a nonempty set, let $\widehat{A}_{bp} \in \text{SBPF}(X)$ and $\widehat{B}_{bp} \in \text{SBPF}(Y)$ and let

$\theta: X \rightarrow Y$ be a mapping. Then

- (i) The **image** of \widehat{A}_{bp} under θ , denoted by $\theta(\widehat{A}_{bp}) = (\theta(\widehat{A}_{bp}^+), \theta(\widehat{A}_{bp}^-))$, is a second-order bipolar fuzzy set on Y defined as follows:
 for every $y \in Y$

$$\begin{aligned} & (\theta(\widehat{A}_{bp}^+))(y) \\ &= \begin{cases} \bigvee_{x \in \theta^{-1}(y)} \widehat{A}_{bp}^+(x), & \text{if } \theta^{-1}(y) \neq \emptyset \\ \mathbf{0} & \text{otherwise} \end{cases} \end{aligned}$$

and

$$\begin{aligned} & (\theta(\widehat{A}_{bp}^-))(y) \\ &= \begin{cases} \bigwedge_{x \in \theta^{-1}(y)} \widehat{A}_{bp}^-(x), & \text{if } \theta^{-1}(y) \neq \emptyset \\ \mathbf{0} & \text{otherwise} \end{cases} \end{aligned}$$

- (ii) The **pre-image** of \widehat{B}_{bp} under θ ,

denoted by

$$\theta^{-1}(\widehat{B}_{bp}) = (\theta^{-1}(\widehat{B}_{bp}^+), \theta^{-1}(\widehat{B}_{bp}^-)),$$

is a second-order bipolar fuzzy set on X defined as follows: for every $x \in X$.

$$(\theta^{-1}(\widehat{B}_{bp}^+))(x) = \widehat{B}_{bp}^+(\theta(x))$$

and

$$(\theta^{-1}(\widehat{B}_{bp}^-))(x) = \widehat{B}_{bp}^-(\theta(x))$$

Definition 3.12

For any two second-order bipolar fuzzy sets $\hat{A}_{bp}, \hat{B}_{bp} \in SBPF(X)$

- (i) $\hat{A}_{bp} \cap_1 \hat{B}_{bp} = \hat{O}_{bp}$ means given $x \in X$, either $\hat{A}_{bp}^+(x) = 0, \hat{A}_{bp}^-(x) = 0$ or $\hat{B}_{bp}^+(x) = 0, \hat{B}_{bp}^-(x) = 0$
- (ii) $\hat{A}_{bp} \cap_2 \hat{B}_{bp} = \hat{O}_{bp}$ means given $x \in X$ and $\alpha \in I$, either $\hat{A}_{bp}^+(x)(\alpha) = 0, \hat{A}_{bp}^-(x)(\alpha) = 0$ or $\hat{B}_{bp}^+(x)(\alpha) = 0, \hat{B}_{bp}^-(x)(\alpha) = 0$.

Remark 3.13

If $\hat{A}_{bp} \cap_1 \hat{B}_{bp} = \hat{O}_{bp}$ then $\hat{A}_{bp} \cap_2 \hat{B}_{bp} = \hat{O}_{bp}$.

Proof:

Let $\hat{A}_{bp} = (\hat{A}_{bp}^+, \hat{A}_{bp}^-)$ and $\hat{B}_{bp} = (\hat{B}_{bp}^+, \hat{B}_{bp}^-)$

Assume $\hat{A}_{bp} \cap_1 \hat{B}_{bp} = \hat{O}_{bp}$, then by the definition given $x \in X$,

either $\hat{A}_{bp}^+(x) = 0, \hat{A}_{bp}^-(x) = 0$ or $\hat{B}_{bp}^+(x) = 0, \hat{B}_{bp}^-(x) = 0$.

Then either $\hat{A}_{bp}^+(x)(\alpha) = 0, \hat{A}_{bp}^-(x)(\alpha) = 0$ or $\hat{B}_{bp}^+(x)(\alpha) = 0, \hat{B}_{bp}^-(x)(\alpha) = 0$, for every $\alpha \in I$. Therefore $\hat{A}_{bp} \cap_2 \hat{B}_{bp} = \hat{O}_{bp}$.

Following example shows that $\hat{A}_{bp} \cap_2 \hat{B}_{bp} = \hat{O}_{bp}$ but $\hat{A}_{bp} \cap_1 \hat{B}_{bp} \neq \hat{O}_{bp}$

Let $X = \{x_1, x_2, x_3\}$. Define

$$\hat{A}_{bp}^+(x_1)(\alpha) = 0.1, \hat{A}_{bp}^-(x_1)(\alpha) = -0.2$$

$$\hat{A}_{bp}^+(x_2)(\alpha) = 0.6, \hat{A}_{bp}^-(x_2)(\alpha) = -0.4$$

$$\hat{A}_{bp}^+(x_3)(\alpha) = 0.7, \hat{A}_{bp}^-(x_3)(\alpha) = -0.3$$

$$\hat{B}_{bp}^+(x_1)(\alpha) = 0.2, \text{ for } \alpha = 1, \\ = 0, \text{ for } \alpha \neq 1$$

$$\hat{B}_{bp}^-(x_1)(\alpha) = -0.5, \text{ for } \alpha = 1, \\ = 0, \text{ for } \alpha \neq 1$$

$$\hat{B}_{bp}^+(x_2)(\alpha) = 0.2, \hat{B}_{bp}^-(x_2)(\alpha) = -0.8$$

$$\hat{B}_{bp}^+(x_3)(\alpha) = 0.9, \hat{B}_{bp}^-(x_3)(\alpha) = -0.4$$

Here for $x_1 \in X$ and $\alpha \neq 1$,

$$\hat{A}_{bp}^+(x_1)(\alpha) = 0.1, \hat{A}_{bp}^-(x_1)(\alpha) = -0.2 \text{ and}$$

$$\hat{B}_{bp}^+(x_1)(\alpha) = 0, \hat{B}_{bp}^-(x_1)(\alpha) = 0$$

Therefore $\hat{A}_{bp} \cap_2 \hat{B}_{bp} = \hat{O}_{bp}$

For all $x \in X$, $\hat{A}_{bp}^+(x) \neq 0, \hat{A}_{bp}^-(x) \neq 0$ and $\hat{B}_{bp}^+(x) \neq 0, \hat{B}_{bp}^-(x) \neq 0$

Therefore $\hat{A}_{bp} \cap_1 \hat{B}_{bp} \neq \hat{O}_{bp}$

Proposition 3.14

For any two second order bipolar fuzzy sets $\hat{A}_{bp}, \hat{B}_{bp} \in SBPF(X)$

- (i) $\hat{A}_{bp} \cap_1 \hat{B}_{bp} = \hat{O}_{bp}$ iff $S_1(\hat{A}_{bp}) \cap S_1(\hat{B}_{bp}) = \emptyset$
- (ii) $\hat{A}_{bp} \cap_2 \hat{B}_{bp} = \hat{O}_{bp}$ implies $S_1(\hat{A}_{bp}) \cap S_2(\hat{B}_{bp}) = \emptyset$

which in view $S_2(\hat{A}_{bp}) \subseteq S_1(\hat{A}_{bp})$ implies $S_2(\hat{A}_{bp}) \cap S_2(\hat{B}_{bp}) = \emptyset$.

Proof:

- (i) Assume $\hat{A}_{bp} \cap_1 \hat{B}_{bp} = \hat{O}_{bp}$

Take $x \in S_1(\hat{A}_{bp})$.

implies

$\hat{A}_{bp}^+(x)(\alpha) > 0, \hat{A}_{bp}^-(x)(\alpha) < 0$, for some $\alpha \in I$

implies

$\hat{A}_{bp}^+(x)(\alpha) \neq 0, \hat{A}_{bp}^-(x)(\alpha) \neq 0$, for some $\alpha \in I$

implies

$\hat{A}_{bp}^+(x) \neq 0, \hat{A}_{bp}^-(x) \neq 0$

implies

$\hat{B}_{bp}^+(x) = 0, \hat{B}_{bp}^-(x) = 0$ (since $\hat{A}_{bp} \cap_1 \hat{B}_{bp} = \hat{O}_{bp}$)

implies

$\hat{B}_{bp}^+(x)(\alpha) = 0, \hat{B}_{bp}^-(x)(\alpha) = 0$, for every $\alpha \in I$

implies $x \notin S_1(\hat{B}_{bp})$

therefore $S_1(\hat{A}_{bp}) \cap S_1(\hat{B}_{bp}) = \emptyset$

Conversely,

assume

$$S_1(\hat{A}_{bp}) \cap S_1(\hat{B}_{bp}) = \emptyset \text{----- (1)}$$

Take any $x \in X$, Suppose $\hat{A}_{bp}^+(x) \neq 0, \hat{A}_{bp}^-(x) \neq 0$. Then

$\hat{A}_{bp}^+(x)(\alpha) \neq 0, \hat{A}_{bp}^-(x)(\alpha) \neq 0$, for some $\alpha \in I$

Therefore

$\hat{A}_{bp}^+(x)(\alpha) > 0, \hat{A}_{bp}^-(x)(\alpha) < 0$, for some $\alpha \in I$

implies $x \in S_1(\hat{A}_{bp})$

implies $x \notin S_1(\hat{B}_{bp})$

implies

$\hat{B}_{bp}^+(x)(\alpha) = 0, \hat{B}_{bp}^-(x)(\alpha) = 0$, for every $\alpha \in I$

implies $\hat{B}_{bp}^+(x) = 0, \hat{B}_{bp}^-(x) = 0$

implies $\hat{A}_{bp} \cap_1 \hat{B}_{bp} = \hat{O}_{bp}$

- (ii) Assume $\hat{A}_{bp} \cap_2 \hat{B}_{bp} = \hat{O}_{bp}$

Take $x \in S_2(\hat{B}_{bp})$. Then

$\hat{B}_{bp}^+(x)(\alpha) > 0, \hat{B}_{bp}^-(x)(\alpha) < 0$, for every $\alpha \in I$

implies

$\hat{B}_{bp}^+(x)(\alpha) \neq 0, \hat{B}_{bp}^-(x)(\alpha) \neq 0$, for every $\alpha \in I$

implies

$$\widehat{A}_{bp}^+(x)(\alpha) = 0, \widehat{A}_{bp}^-(x)(\alpha) = 0, \text{ for every } \alpha \in I$$

(since $\widehat{A}_{bp} \cap_1 \widehat{B}_{bp} = \widehat{0}_{bp}$)

implies $x \notin S_1(\widehat{A}_{bp})$

$$\text{implies } S_1(\widehat{A}_{bp}) \cap S_2(\widehat{B}_{bp}) = \emptyset.$$

Proposition 3.15

For a family $\{(\widehat{A}_{bp})_\lambda \in \text{SBPF}(X)/\lambda \in \Lambda\}$,
 $S_1(\bigcup_{\lambda \in \Lambda} (\widehat{A}_{bp})_\lambda) = \bigcup_{\lambda \in \Lambda} S_1((\widehat{A}_{bp})_\lambda)$.

Proof:

$$\text{Let } x \in S_1(\bigcup_{\lambda \in \Lambda} (\widehat{A}_{bp})_\lambda)$$

\Leftrightarrow there exists an $\alpha \in I$ such that

$$(\bigvee_{\lambda \in \Lambda} (\widehat{A}_{bp}^+)_\lambda)(x)(\alpha) > 0,$$

$$(\bigwedge_{\lambda \in \Lambda} (\widehat{A}_{bp}^-)_\lambda)(x)(\alpha) < 0$$

\Leftrightarrow there exists an $\alpha \in I$ such that

$$\bigvee_{\lambda \in \Lambda} ((\widehat{A}_{bp}^+)_\lambda(x)(\alpha)) > 0,$$

$$\bigwedge_{\lambda \in \Lambda} ((\widehat{A}_{bp}^-)_\lambda(x)(\alpha)) < 0$$

\Leftrightarrow there exists an $\alpha \in I$ and $\lambda \in \Lambda$ such that

$$(\widehat{A}_{bp}^+)_\lambda(x)(\alpha) > 0, (\widehat{A}_{bp}^-)_\lambda(x)(\alpha) < 0$$

$\Leftrightarrow x \in S_1(\widehat{A}_{bp})_\lambda$ for some $\lambda \in \Lambda$

$$\Leftrightarrow x \in \bigcup_{\lambda \in \Lambda} S_1((\widehat{A}_{bp})_\lambda)$$

Therefore

$$S_1(\bigcup_{\lambda \in \Lambda} (\widehat{A}_{bp})_\lambda) = \bigcup_{\lambda \in \Lambda} S_1((\widehat{A}_{bp})_\lambda).$$

Proposition 3.16

For a collection $(\widehat{A}_{bp})_1, (\widehat{A}_{bp})_2, \dots, (\widehat{A}_{bp})_m \in \text{SBPF}(X)$,
 $S_1(\bigcap_{i=1}^m (\widehat{A}_{bp})_i) \subseteq \bigcap_{i=1}^m S_1(\widehat{A}_{bp})_i$

Proof :

$$\text{Let } x \in S_1(\bigcap_{i=1}^m (\widehat{A}_{bp})_i)$$

\Rightarrow there exists an $\alpha \in I$ such that

$$(\bigwedge_{i=1}^m (\widehat{A}_{bp}^+)_i)(x)(\alpha) > 0,$$

$$(\bigvee_{i=1}^m (\widehat{A}_{bp}^-)_i)(x)(\alpha) < 0$$

\Rightarrow there exists an $\alpha \in I$ such that

$$\bigwedge_{i=1}^m ((\widehat{A}_{bp}^+)_i(x)(\alpha)) > 0,$$

$$\bigvee_{i=1}^m ((\widehat{A}_{bp}^-)_i(x)(\alpha)) < 0$$

\Rightarrow there exists an $\alpha \in I$ such that

$$((\widehat{A}_{bp}^+)_i)(x)(\alpha) > 0, ((\widehat{A}_{bp}^-)_i)(x)(\alpha) < 0,$$

for every $i = 1, 2, \dots, m$

$$\Rightarrow x \in S_1(\widehat{A}_{bp})_i, \text{ for every } i = 1, 2, \dots, m$$

$$\Rightarrow x \in \bigcap_{i=1}^m S_1(\widehat{A}_{bp})_i$$

Therefore

$$S_1(\bigcap_{i=1}^m (\widehat{A}_{bp})_i) \subseteq \bigcap_{i=1}^m S_1(\widehat{A}_{bp})_i$$

Following example shows that $S_1(\bigcap_{i=1}^m (\widehat{A}_{bp})_i) \neq \bigcap_{i=1}^m S_1(\widehat{A}_{bp})_i$

Let $X = \{x_1, x_2\}$.

Define $(\widehat{A}_{bp})_1, (\widehat{A}_{bp})_2 \in \text{SBPF}(X)$,

And for every $x \in X$

$$(\widehat{A}_{bp}^+)_1(x_1)(\alpha) = 0.1, \text{ for } \alpha \neq 1$$

$$= 0, \text{ for } \alpha = 1$$

$$(\widehat{A}_{bp}^-)_1(x_1)(\alpha) = -0.3, \text{ for } \alpha \neq 1$$

$$= 0, \text{ for } \alpha = 1$$

$$(\widehat{A}_{bp}^+)_1(x_2)(\alpha) = 0.7, \text{ for } \alpha \neq 1$$

$$= 0, \text{ for } \alpha = 1$$

$$(\widehat{A}_{bp}^-)_1(x_2)(\alpha) = -0.8, \text{ for } \alpha \neq 1$$

$$= 0, \text{ for } \alpha = 1$$

$$(\widehat{A}_{bp}^+)_2(x_1)(\alpha) = 0, \text{ for } \alpha \neq 1$$

$$= 0.4, \text{ for } \alpha = 1$$

$$(\widehat{A}_{bp}^-)_2(x_1)(\alpha) = 0, \text{ for } \alpha \neq 1$$

$$= -0.6, \text{ for } \alpha = 1$$

$$(\widehat{A}_{bp}^+)_2(x_2)(\alpha) = 0, \text{ for } \alpha \neq 1$$

$$= 0.9, \text{ for } \alpha = 1$$

$$(\widehat{A}_{bp}^-)_2(x_2)(\alpha) = 0, \text{ for } \alpha \neq 1$$

$$= -0.2, \text{ for } \alpha = 1$$

Here $x_1 \in S_1(\widehat{A}_{bp})_1, S_1(\widehat{A}_{bp})_2$

$$\Rightarrow x_1 \in S_1(\widehat{A}_{bp})_1 \cap S_1(\widehat{A}_{bp})_2$$

$$\text{But } ((\widehat{A}_{bp}^+)_1 \wedge (\widehat{A}_{bp}^+)_2)(x_1)(\alpha) = 0$$

and

$$((\widehat{A}_{bp}^-)_1 \vee (\widehat{A}_{bp}^-)_2)(x_1)(\alpha) = 0, \text{ for every } \alpha \in I$$

implies

$$\left((\widehat{A}_{bp}^+) \right)_1 \wedge \left((\widehat{A}_{bp}^+) \right)_2 (x_1)(\alpha) \neq 0$$

and

$$\left((\widehat{A}_{bp}^-) \right)_1 \vee \left((\widehat{A}_{bp}^-) \right)_2 (x_1)(\alpha) < 0, \text{ for every } \alpha \in I$$

$$\text{implies } x_1 \notin S_1 \left((\widehat{A}_{bp})_1 \cap (\widehat{A}_{bp})_2 \right)$$

Therefore

$$S_1(\widehat{A}_{bp})_1 \cap S_1(\widehat{A}_{bp})_2 \neq S_1 \left((\widehat{A}_{bp})_1 \cap (\widehat{A}_{bp})_2 \right)$$

Therefore

$$S_1 \left(\bigcap_{i=1}^m (\widehat{A}_{bp})_i \right) \neq \bigcap_{i=1}^m S_1(\widehat{A}_{bp})_i.$$

Proposition 3.17

(i) For a family $\{(\widehat{A}_{bp})_\lambda \in \text{SBPF}(X) / \lambda \in \Lambda\}$,

$$S_2 \left(\bigcup_{\lambda \in \Lambda} (\widehat{A}_{bp})_\lambda \right) \supseteq \bigcup_{\lambda \in \Lambda} S_2 \left((\widehat{A}_{bp})_\lambda \right)$$

(ii) For a collection $(\widehat{A}_{bp})_1, (\widehat{A}_{bp})_2, \dots, (\widehat{A}_{bp})_m \in \text{SBPF}(X)$,

$$S_2 \left(\bigcap_{i=1}^m (\widehat{A}_{bp})_i \right) = \bigcap_{i=1}^m S_2(\widehat{A}_{bp})_i$$

Proof:

(i) Let $x \in \bigcup_{\lambda \in \Lambda} S_2 \left((\widehat{A}_{bp})_\lambda \right)$

implies $x \in S_2(\widehat{A}_{bp})_\lambda$, for all $\lambda \in \Lambda$

implies

$$\left((\widehat{A}_{bp}^+) \right)_\lambda (x)(\alpha) > 0,$$

$$\left((\widehat{A}_{bp}^-) \right)_\lambda (x)(\alpha) < 0,$$

for every $\alpha \in I$

implies

$$\bigvee_{\lambda \in \Lambda} \left((\widehat{A}_{bp}^+) \right)_\lambda (x)(\alpha) > 0,$$

$$\bigwedge_{\lambda \in \Lambda} \left((\widehat{A}_{bp}^-) \right)_\lambda (x)(\alpha) < 0,$$

for every $\alpha \in I$

implies

$$\left(\bigvee_{\lambda \in \Lambda} (\widehat{A}_{bp}^+) \right)_\lambda (x)(\alpha) > 0,$$

$$\left(\bigwedge_{\lambda \in \Lambda} (\widehat{A}_{bp}^-) \right)_\lambda (x)(\alpha) < 0,$$

for every $\alpha \in I$

$$\text{implies } x \in S_2 \left(\bigcup_{\lambda \in \Lambda} (\widehat{A}_{bp})_\lambda \right)$$

$$\text{Therefore } \bigcup_{\lambda \in \Lambda} S_2 \left((\widehat{A}_{bp})_\lambda \right) \subseteq S_2 \left(\bigcup_{\lambda \in \Lambda} (\widehat{A}_{bp})_\lambda \right)$$

Therefore

$$S_2 \left(\bigcup_{\lambda \in \Lambda} (\widehat{A}_{bp})_\lambda \right) \supseteq \bigcup_{\lambda \in \Lambda} S_2 \left((\widehat{A}_{bp})_\lambda \right).$$

(ii) Let $x \in S_2 \left(\bigcap_{i=1}^m (\widehat{A}_{bp})_i \right)$

$$\Leftrightarrow \left(\bigwedge_{i=1}^m (\widehat{A}_{bp}^+) \right)_i (x)(\alpha) > 0,$$

$$\left(\bigvee_{i=1}^m (\widehat{A}_{bp}^-) \right)_i (x)(\alpha) < 0,$$

for every $\alpha \in I$

$$\Leftrightarrow \bigwedge_{i=1}^m \left((\widehat{A}_{bp}^+) \right)_i (x)(\alpha) > 0,$$

$$\bigvee_{i=1}^m \left((\widehat{A}_{bp}^-) \right)_i (x)(\alpha) < 0,$$

for every $\alpha \in I$

$$\Leftrightarrow \left((\widehat{A}_{bp}^+) \right)_i (x)(\alpha) > 0,$$

$$\left((\widehat{A}_{bp}^-) \right)_i (x)(\alpha) < 0,$$

for every $\alpha \in I$ and

for every $i = 1, 2, \dots, m$

$$\Leftrightarrow x \in S_2(\widehat{A}_{bp})_i,$$

for every $i = 1, 2, \dots, m$

$$\Leftrightarrow x \in \bigcap_{i=1}^m S_2(\widehat{A}_{bp})_i.$$

Therefore

$$S_2 \left(\bigcap_{i=1}^m (\widehat{A}_{bp})_i \right) = \bigcap_{i=1}^m S_2(\widehat{A}_{bp})_i$$

4. Decision Making in Medical Diagnosis

Medical diagnosis involves complex decision-making processes with uncertainty, imprecision, vagueness, etc... To explore this type of problems bipolar-valued fuzzy set was introduced. It defines the relationships and interactions between different elements within the diagnostic framework. The topology provides a structured way to analyze the connections among test results and patient history, allowing healthcare professionals to make decisions. The proposed algorithm helps to find the efficiency in two medicines which are given for the arthritis patient. This model facilitates an accurate representation of uncertain information leading to improved decision making in the medical field.

Algorithm :4.1

Step 1: Consider the attributes

$$X = \{x_1, x_2, x_3, \dots\}.$$

Step 2 : Write the readings in [0,1].

Step 3 : Build the second order bipolar fuzzy topology by converting the readings in the range $[-1,0) \times (0,1]$

➤ convert the reading within the reference level as positive membership function in the range [0,1]

- convert the reading below or above the reference level as negative membership function in the range [-1,0]

Step 4: Test 1 - Efficiency of medicine I

Test 2 - Efficiency of medicine II

Step 5 : Compare the analysis of Test 1 & Test 2

Result:

- ✓ If Test 1 < Test 2, then medicine I is more efficient.
- ✓ If Test 1 > Test 2, then medicine II is more efficient.
- ✓ If Test 1 = Test 2, then there is no improvement.

Problem: 4.2

Let us consider the decision-making situation in medical diagnosis of Low arthritits patient which desires to make a decision whether the improvement takes place or not after diagnosis by using the proposed algorithm (SBPFTS).

Blood test	Test1	Reference level	Mean Value
Haemoglobin	9.0	12-15.5	13.75
Haemotocrit	30.9	35.0-45.0	40
Total white blood cell count	9070	4500-11000	7750
Total Red blood cell count	4.58	4-5.5	4.75
ESR	49	5-20	15

Step 1: Attributes {x1, x2, x3, x4, x5} stands for the blood test of Haemoglobin, Haemotocrit, Total white blood cell count, Total red blood cell count, ESR value.

Step 2 : Convert it as follows:

Attributes	Test result in [0,1]	Efficiency if Medicines	
		Med I	Med II

x1	0	9.4	14.28
x2	0	35.1	42.3
x3	0.8	9070	5810
x4	0.4	4.58	4.9
x5	0	18.55	25.27

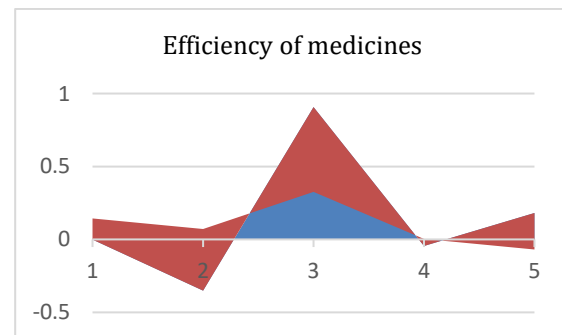
Step 3 : Let us write the above table in terms of second order bipolar fuzzy points

$[-1,0) \times (0,1]$

Attributes	Test result in [0,1]	Efficiency if Medicines	
		Med I	Med II
x1	0	0	0.1428
x2	0	-0.351	0.423
x3	0.8	0.9070	-0.5810
x4	0.4	-0.0458	0.049
x5	0	0.18	-0.25

Step 4: Test 1 - Efficiency of medicine I

Test 2 - Efficiency of medicine II



The above figure represents the second order bipolar fuzzy graph of the given case study

Step 5: After analysis of test 1 and test 2, maximum attributes x1, x2, x4 and x5 in test 2 are greater which is chosen as optimal decision.

Result: Test 1 < Test 2. therefore medicine II is more efficient than medicine I.

4. Conclusion

In this paper, the concept of second-order bipolar fuzzy set and second order bipolar fuzzy topological spaces are defined and the theorems are proved. The integration of second order bipolar fuzzy sets and their topology enhances the precision and reliability of medical diagnosis for arthritis, offering a sophisticated approach to handle uncertainty. Here the case study of a arthritis patient is analysed after diagnosis of arthritis and comparative analysis of test1 and test 2 are done to find the efficiency of the medicines.

References

- [1] Chang, C. L. (1968), Fuzzy Topological Spaces, *Journal of Mathematical Analysis and Applications*, **24**, 182-190.
- [2] Kalaichelvi, A. (2007), Second Order Fuzzy Topological Spaces – I, *ActaCientiaIndica*, 33(3), 819-826.
- [3] Kalaichelvi, A. (2011), Second Order Fuzzy Topological Spaces – III, *Journal of Mathematical Sciences and Applications*, 1(2), 625 – 633
- [4] Kim, J., Samanta, S. K., Lee, P. K., Hur, K. (2019), Bipolar Fuzzy Topological Spaces, *Annals of Fuzzy Mathematics and Informatics*, 17, 205-229.
- [5] Lowen, R. (1976), Fuzzy Topological Spaces and Compactness, *Journal of Mathematical Analysis and Applications*, **56**, 621-633.
- [6] Wong, C.K. (1974b), Fuzzy Points and Local Properties of Fuzzy Topology, *Journal of Mathematical Analysis and Applications*, **46 (2)**, 316 - 328.
- [7] Zadeh, L.A. (1965), Fuzzy Sets, *Information and Control*, **8(3)**, 338-353.
- [8] Zadeh, L.A. (1975), The Concept of a Linguistic Variable and its Application to Approximate Reasoning-1, *Information Sciences*, **8**, 199-249.
- [9] Zhang, W.R. (1994), Bipolar Fuzzy Sets and Relations: a Computational Framework for Cognitive Modelling and Multiagent Decision Analysis, *Proc. of IEEE*, 305-309.