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Prime Fractional Labeling of Fuzzy Graph

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Abstract: We present the idea of prime fractional labeling of fuzzy graphs in this study. It is possible to do prime fractional magic labeling from prime fractional fuzzy labeling.

Keywords: Fuzzy Labeling, Prime Fractional Labeling, Magic Labeling, Prime Fractional Magic Labeling.

1. Introduction

Zadeh Introduced the fuzzy set as a class of object with a continuum of grades of membership. The fuzzy approach pertains to grade of membership between [0, 1] described in terms of membership function of a fuzzy number, as opposed to conventional crisp sets where a set is characterized by either membership or non-membership. The concept of a fuzzy relation on a set was initially introduced by Zadeh in 1965. Kaufmann established the concept of a fuzzy graph in 1973, while Rosenfield created the structure of a fuzzy graph in 1975. Fuzzy graphs find numerous uses in real-time modeling systems when the system's intrinsic knowledge varies with varying degrees of accuracy.

In this paper we discuss about the prime fractional labeling of fuzzy graph and some of its properties.

2. PRELIMINARIES

Definition.2. 1

Let C^* = (V, E) be a simple graph. Then C * = (φ,ψ) is called a fuzzy graph on C^*, if φ: V → [0, 1] and ψ: E → [0, 1] and for all *x,y* Є *E*.

ψ (x, y) ≤ = min [φ (x), φ (y)]

A fuzzy graph C= (φ, ψ)) on C * is called a fuzzy labeling graph if φ and are ψ one to one maps for all *x, y* Є*E.*

Definition. 2.2

Prime labeling graph Let C = (V|C), (E|C) be a graph with P vertices. A bijection f: V(C) → {1, 2,…p} is called a prime labeling if for each edge e = uv, gcd (f|u), (f|v) = 1. A graph which admits prime labeling is called prime graph.

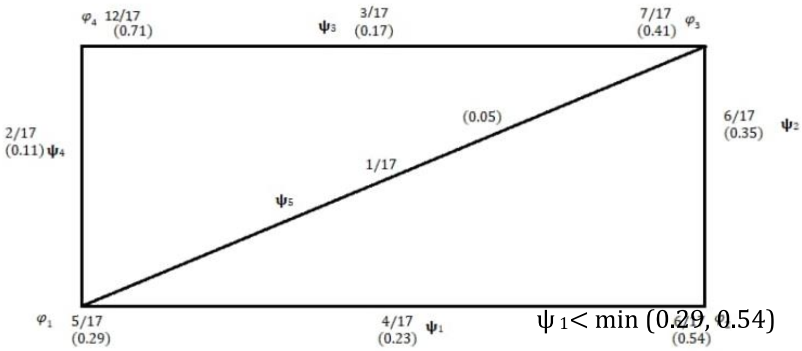
3. PRIME FRACTIONAL LABELLING OF FUZZY GRAPHS

Definition. 3.1

Consider a prime fractional number $\frac{P_i}{17}$ that satisfying the conditions of prime labeling and fuzzy labeling conditions are called prime fractional fuzzy labeling graph.

Example:3.1

Consider a square graph S4 for all $1 \leq I \leq 17$, we define $\psi(ei) = \frac{i}{17}$ and $\varphi(V_1)$ = 5/17, $\varphi(V_2)$ = 16/(17), $\varphi(V_3)$ = 7/(17), $\varphi(V_4)$ = 12/17



$\psi_1 <$ min (0.29, 0.54)

0.23 < min (0.29, 0.54)

Gcd (0.29, 0.54) = 0.01

ψ_2 < min (0.54, 0.41)

0.35 < min (0.54, 0.41)

Gcd (0.54, 0.41) = 0.01

ψ_3 < min (0.41, 0.71)

0.17 < min (0.41, 0.71)

Gcd (0.41, 0.71) = 0.01

ψ_4 < min (0.71, 0.29)

0.11 < min (0.71, 0.29)

Gcd (0.71, 0.29) = 0.01

ψ_5 < min (0.29, 0.41)

0.05 < min (0.29, 0.41)

Gcd (0.41, 0.29) = 0.01

Therefore the square graph S_4 satisfying the prime fractional labeling of fuzzy graph with Gcd 0.01.

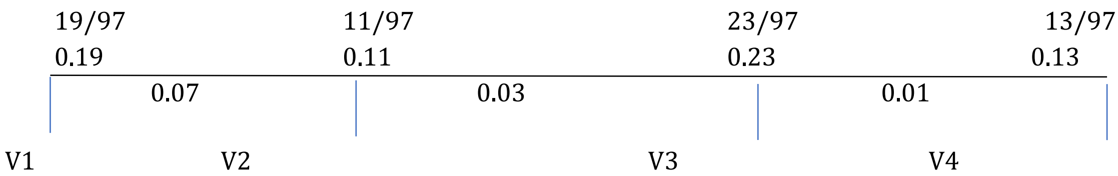
Definition. 3.2

A prime fractional fuzzy labeling graph C = (φ, ψ) on C is called prime fractional fuzzy magic labeling graph if there exists m∈ (0, 3), which is called magic value such that for all x, y ∈E.

φ (x) + φ (y) + ψ (x, y) = m

Example:

Consider $\frac{1}{97}$, $\frac{2}{97}$,$\frac{3}{97}$, $\frac{4}{97}$ be the prime fractional.



C* = (φ, ψ)

A simple graph satisfying the prime fractional labeling and magic labeling conditions.

φ_1 (x) + ψ_1 (x) + φ_2 (x) = 0.37

0.19 + 0.07 + 0.11 = 0.37

φ_2 (x) +φ_3 (x) + ψ_2 (x) = 0.37

φ_3 (x) + ψ_3 (x) + φ_4 (x) = 0.37

Proposition:3.1

If (S^w, t^w) is prime fractional fuzzy labeling subgraph of (φ^w ψ^w) then

t^{w^∞}(u, v)≤ ψ^{w^∞}(u, v) for all u, v∈v.

Proof:

Let G^w = (φ^w ψ^w) be any prime fractional fuzzy labeling of graph and H^w =(S^w, t^w) be its sub graph. Let (u, v) be any path in C^w then its strength be ψ^{w^∞}(u, v). Since it is a sub graph S^w(u)≤ φ^w (u) and t^w (u, v)≤ ψ^w (u, v) which implies t^{w^∞}(u, v)≤ ψ^{w^∞}(u, v) for all u, v ∈v.

Definition:3.3

A cycle graph C* is said to be prime fractional fuzzy labeling cycle graph if is has prime factional fuzzy labeling.

Preposition:3.2

If C* is a cycle then the prime fractional fuzzy labeling cycle C*has exactly only weakest arc.

Proof:

Let C^w be a fuzzy labeling cycle and ψ^w(x, y) $= \wedge_{i-1}^{n} \psi^w$($x_i$, y_i). Since C^w has a fuzzy labeling, it will have only one arc with ψ^w(x_i, y). If we remove ψ^w(x_i, y) from C^w it will not reduce the strength of connectedness therefore ψ^w(x_i,

y) is a weakest arc. Hence there exist only one weakest arc in prime fractional fuzzy labeling cycle.

Proposition:3.3

Let C^w be a fuzzy labeling cycle such that $C^*$is a cycle, then it has (n-1) bridges.

Proof:

Let C^*be a cycle with fuzzy labeling by preposition, we get only one weakest arc, it is not a fuzzy bridge. Therefore, the removal of any arc except the weakest arc will reduces the strength of connectedness. Hence, every prime fractional fuzzy labeling cycle have (n-1) bridges.

4. Conclusion

This study has examined the definitions of magic labeling conditions and prime fractional fuzzy labeling graphs. We expand upon this research by examining more unique classes of graphs.

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