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Graphical Approach On Pattern Generation Using Edge Coloring

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Abstract: A graph's acyclic edge-coloring involves assigning colors to its edges in a way that ensures no bichromatic cycles exist. The acyclic chromatic index symbolized as $\chi'a(G)$, represents the minimum number of colors, required to effectively color the graph, denoted as 'm'. This study delves into the concept of edge coloring, particularly emphasizing the amalgamation of multiple graphs to create diverse patterns. Analyzing these patterns reveals their distinct characteristics. To achieve high symmetry, congruent polygons such as Isosceles right-angled triangles, regular pentagons, and regular hexagons are utilized. Notably, the findings establish a novel relationship: $\chi'a(G)$ equals the maximum degree of the graph, $\Delta(G)$, resulting in innovative pattern outcomes.

Keywords: Acyclic edge-coloring, Class one graph, Class two graph, Chromatic index, Bipartite graph, Catenation, Honeycomb networks

INTRODUCTION

Graph theory and networking are intimately connected fields, it plays a fundamental role in understanding, designing, and optimizing communication systems and networks. Graph theory provides a powerful framework for modeling and analyzing various types of networks. In graph theory, nodes and edges represent entities and connections, respectively, which align perfectly with the nodes (devices) and links (connections) in network topologies. Graph algorithms like Dijkstra's and Bellman-Ford are essential for routing and optimizing data transmission paths in networks. Moreover, graph theory aids in network design, capacity planning, and fault tolerance analysis by uncovering network topology properties. Social network analysis and wireless sensor network deployment also rely heavily on graph theory. Ultimately, the symbiotic relationship between graph theory and networking underpins the foundation of modern communication systems and their efficient operation. It provides a mathematical framework for modeling and representing various types of networks.

The simple, undirected, finite graphs are held in the study. Let $G = (V, E)$ a simple graph that has a vertex set $V = V(G)$ and an edge set $E = E(G)$. The terms order and size of G are represented by $n = |V|$ and $m = |E|$, independently. The volume of edges that cross a vertex, displayed by $d(v)$, is understood as its degree. Isolated vertex refers to a vertex u, that has $d(u) = 0$. Let $\delta(G)$ and $\Delta(G)$ characterize the degree of at the smallest and loftiest degree, respectively. The diameter of G is represented by the diam () symbol and represents the most significant distance between any two vertices in a set of vertices (or edges) that aren't conterminous to one another and is appertained to as a vertex (or edge) independent set.

Literature Survey:

The concept of Acyclic Edge Coloring was studied by Fiamcik [2] and he bounced the acyclic edge coloring conjecture in 1978. He answered the guess for subcubic graphs. His papers weren't available in English till lately and hence were unknown. Alon, McDiarmid, and Reed [6] presented it singly and using probabilistic styles demonstrated that $\chi'a((G) \leq 64\Delta$. They also adverted that the constant 64 could be bettered with the more alert operation of the Lovasz Local Lemma. Later Molloy and Reed exhibited that a $\chi'a((G) \leq 16\Delta$. This is the best-known set presently for arbitrary graphs. Muthu, Narayanan, and Subramanian [3] demonstrated that $\chi'a((G) \leq 4.52\Delta$ for graphs G of girth at least 220 (Girth is the length of the shortest cycle in a graph). All the above results use probabilistic styles. The best-known formative bound is by Subramanian who showed that $\chi'a(G) \leq 5\Delta(\log \Delta + 2)$. Though the best-known upper bound for the general case is far from the conjectured Δ + 2, the conjecture is true for some special classes of graphs. Alon, Sudakov, and, Zaks [8] proved that there exists a constant k such that $\chi'a(G) \leq \Delta + 2$ for any graph G whose girth is at least kΔ log Δ. They also proved that $\chi'a(G) \leq \Delta+2$ for almost all Δ-regular graphs. This result was improved by Neset ˇ ˇril and Wormald [9] who showed that for a random Δ-regular graph $\chi'a(G) \leq \Delta + 1$. Muthu, Narayanan, and Subramanian proved the conjecture for grid-like graphs [10] and outerplanar graphs [11]. They gave a better bound of Δ + 1 for those classes of graphs. From Burnstein's [12] result it follows that the conjecture is true for subcubic graphs. Skulrattankulchai [13] gave a polynomial time algorithm to color a subcubic graph using Δ + 2 = 5 colors. Deciding a'(G) is a hard challenge both from a theoretical and an algorithmic point of view. Even for the simple and highly structured class of complete graphs, the value of a'(G) is still not determined exactly. The hardship in determining $\chi'a(G)$ for complete graphs could be observed by its equivalence to the Perfect 1-factorization Conjecture. It has also been shown by Alon and Zaks [14] that judging whether $\chi'a(G) \leq 3$ is NP-complete for an arbitrary graph G. A generalization of the acyclic edge chromatic number has also been studied.

K. Bhuvaneswari [1] undertook a thorough investigation into the generation of patterns created through tile pasting. Various P systems were utilized to form two-dimensional visual languages. The realm of picture processing and scene analysis, as explored by S. Kuberal [15], delves into octagonal arrays and patterns. T. Kalyani [17] delved into the realm of literature with the introduction of the k-Tabled Tetrahedral Tile Pasting System (k-TTTPS) and Tetrahedral Tile Pasting P System (TetTPPS), both designed to generate tetrahedral picture patterns.

Our research establishes an intriguing link between this concept and the realm of graph theory's acyclic edge coloring, thereby facilitating the creation of an extensive variety of distinct graph patterns."

1.1. Preliminaries

In this section, specific fundamental definitions are reviewed and key ideas are illustrated.

A graph G = (V, E) consists of a set V of vertices (also called nodes) and a set E of edges. A graph G = (V, E) consists of a set V of vertices (also called nodes) and a set E of edges.

Definition 1:
A **simple graph** is a graph with no loop edges or multiple edges. Edges in a simple graph may be specified by a set {vi, vj} of the two vertices the edge makes adjacent. A graph with more than one edge between a pair of vertices is called a multigraph while a graph with loop edges is called a pseudograph.

Definition 2:
The **degree** of a vertex is the number of edges incident to the vertex and is denoted deg(v), Let $\Delta(G)$ denote the maximum degree of a graph G.

Definition 3: In graph theory, **proper edge coloring** of a graph is an assignment of "colors" to the edges of the graph so that no two incident edges have the same color.

Definition 4: The minimum required number of colors for the edges of a given graph is called the **chromatic index** of the graph, denoted by $\chi'(G)$.

Definition 5: Union of two graphs: Given two graphs G1 and G2 their union will be a graph such that

$$V(G1 \cup G2) = V(G1) \cup V(G2)$$

And $$E(G1 \cup G2) = E(G1) \cup E(G2)$$

The union of G1 and G2 is denoted by **G1 UG2**

Proposition: For any graph G, $\chi'(G) \geq \Delta(G)$.

Definition 6: A proper edge coloring of G = (V, E) is a map c: E → C (where C is the set of available colors) with c(e) ≠c(f) for any adjacent edges e,f. The minimum number of colors needed to properly color the edges of G is called the chromatic index of G and is denoted by $\chi'(G)$.

Definition 7: A proper edge coloring c is called **acyclic** if there are no bichromatic cycles in the graph. In other words, an edge coloring is acyclic if the union of any two color classes induces a set of paths (i.e., linear forest) in G. The acyclic edge chromatic number (also called acyclic chromatic index), denoted by a′ (G), is the minimum number of colors required to acyclically edge color G.

Theorem 1:1 Vizing's Theorem:
Let $\Delta(G)$. be the maximum degree of a graph G. Then the number of colors χ' needed to edge color is

$$\Delta(G) \leq \chi'(G) \leq \Delta(G) + 1$$

Theorem 1:2 (K˝onig, 1916):
If G is a bipartite graph with a maximum degree Δ then $\chi'(G) = \Delta$

Theorem 1:3
A regular graph on an odd number of vertices is class two.

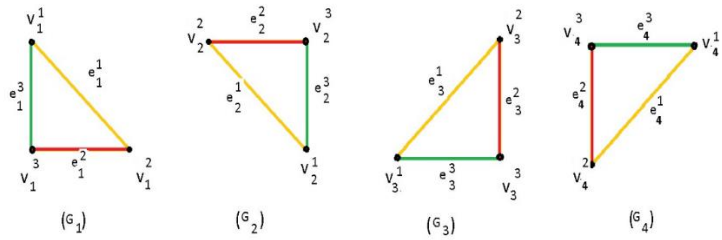
Definition 8: Class one graph and class two graph: If $\chi'(G) = \Delta(G)$ then G is said to be a class one graph, and if $\chi'(G) = \Delta(G) + 1$ then G is said to be a class two graph.

A complete graph, denoted as Kp, belongs to Class 1 when p is an even number, and it falls into Class 2 when p is an odd number.
According to Vizing's Theorem, the chromatic index of any graph G can be precisely determined as either equal to its maximum degree Δ(G) or Δ (G) + 1. A straightforward approach for calculating the exact chromatic index of a graph with 2s + 2 vertices and a maximum degree of 2s.

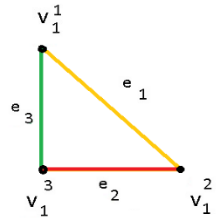
2. Triangular graphs

a. Definition

The labeled isosceles right-angled triangular graph is defined as G1= (V, E) where V= v_1^1, v_1^2, v_1^3 are vertices, E = e_1^1, e_1^2, $e_1^3$are edges whose horizontal (Vertical) and side edges are of length 1 unit and $\sqrt{2}$ unit respectively.



The isosceles right-angled triangular graph has the path P = v1,v2,v3 by joining vi and vi+1. It is a regular graph. We denote this graph by G1.



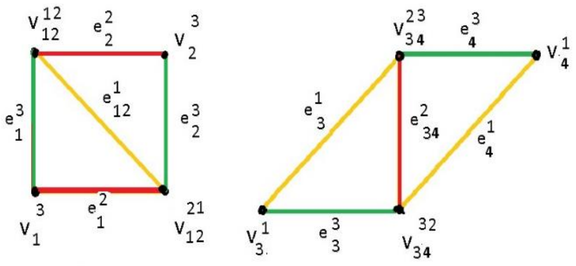
Maximum degree Δ [G1] = 2
Number of vertices in G1 = 3
Number of edges in G1 = 3
Chromatic number $\chi'(G1)$=3
Clearly, $\chi'(G1) > \Delta(G1)$. if $\chi'(G1) = \Delta(G1) + 1$ then G1 is said to be a class two graph.

b. Catenation of triangular graphs:

Definition 9:
The process of catenating two triangular graphs involves taking two separate triangular graphs and connecting them by adding additional edges in a manner that they share at least one common edge, resulting in a larger interconnected graph. This operation can be used to create a more intricate network structure for various graph theory applications.

On one of the three sides of the first graph G1 with the same edge color, the second graph G2 could be positioned. The resulting graph is square. A parallelogram will result from placing the third graph G3 and the fourth graph G4 side by side and combining their edges with the same color.

Example:



(G1UG2)   (G3UG4)

Result (1)

Result (2)

Example1:

Consider two graphs with three vertices, G1 and G2. The complete union of two graphs, GI and G2, is what we refer to as G1 UG2 being a connected graph.

V(G1) = { v_1^1, v_1^2, v_1^3}

E(G1) = { e_1^1, e_1^2, e_1^3}

V(G2) = { v_2^1 , v_2^2, v_2^3}

E(G2) = { e_2^1, e_2^2, e_2^3}

V(G1 U G2) = { v_1^3, v_{12}^{21}, v_2^3, v_{12}^{12}}

E (G1 U G2) = {$e_1^2, e_2^3, e_2^2, e_1^3, e_{12}^1$}

When G1 and G2 are combined, results in a square pattern. Note that the merged edges are all the same length. Edges $e_1^1$and e_2^1 with the same yellow colour are pasted together to form edge e_{12}^1.

Maximum degree Δ [G1UG2] = 3

Number of vertices in G1UG2 = 4

Number of edges in G1UG2 = 5

Chromatic number $\chi(G1UG2)$ =3

Clearly, $\chi'(G1UG2) = \Delta(G1UG2)$.

Example 2:

Let G3 and G4 be two graphs with 3 vertices. We say that G3 UG4 is a joined graph is the complete union of two graphs G3 and G4

V(G3) = { v_3^1, v_3^2, v_3^3}

E(G3) = { e_3^1, e_3^2, e_3^3}

V(G4) = { v_4^1 , v_4^2, v_4^3}

E(G4) ={ e_4^1, e_4^2, e_4^3}

V(G1 U G2) = { v_3^1, v_{34}^{32}, v_4^1, v_{34}^{23}}

E (G1 U G2) = {$e_3^3, e_4^1, e_4^3, e_3^1, e_{34}^2$}

When G3 and G4 are combined, results in a parallelogram pattern. Note that the merged edges are all the same length. Edges e_3^2 and e_4^2 with the same red colour are pasted together to form edge e_{34}^2.

Maximum degree Δ [G3UG4] = 3

Number of vertices in G3UG4 = 4

Number of edges in G3UG4 = 5

Chromatic number $\chi(G3UG4)$ = 3

Clearly, $\chi'(G3UG4) = \Delta(G3UG4)$.

Observation 1.

In graphs G1, G2, G3, and G4 maximum degree was Δ = 2, and the Chromatic number χ =3. Clearly, $\chi'(G) > \Delta(G)$, it's a **bipartite graph.** While combining the graphs maximum degree was Δ = 3 and the Chromatic number χ =3. Clearly, $\chi'(G1UG2) = \Delta(G1UG2)$ then the joined graph is said to be a **class one.**

Catenation of 4 vertices graphs :

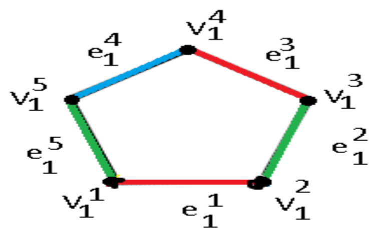
The highest degree and the minimum number of colours in a four-vertex appropriate edge-colored graph are both 2 = $\chi'(G) = \Delta(G)$. A proper edge coloration is impossible with only two colours if we join another graph with four vertices, which has a maximum degree of 3 = $\Delta(G)$, According to **Vizing's Theorem** $\Delta(G)$ should be less than $\chi'(G)$.

. Therefore, Pattern generating is not feasible.

3. Pentagon Graphs

Definition 11:The labeled pentagon graph is a simple undirected graph with no loops and no multiple edges between the same pair of vertices. The graph is formed by five vertices v_1^1, $v_1^2, v_1^3, v_1^4, v_1^5$. Each vertex is then connected to the two adjacent vertices by edges e_1^1, e_1^2, e_1^3, e_1^4, e_1^5 resulting in a cycle of five vertices.

The pentagon graph is denoted as "P"= (V, E) to indicate that it is a cycle with five vertices. Red, Blue, and Green colors have been allocated to graph P1's edge coloring so that adjacent edges are colored differently.

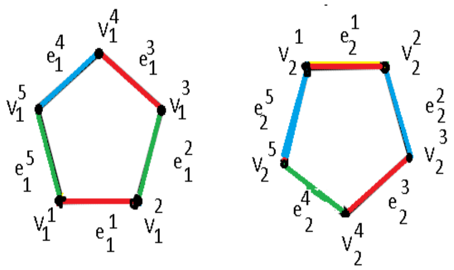


(P_1)

Maximum degree Δ [P1] = 2
Number of vertices in P1 = 5
Number of edges in P1 = 5
Chromatic number $\chi'(P1)$.=3
Clearly, $\chi'(P1) > \Delta(P1)$.

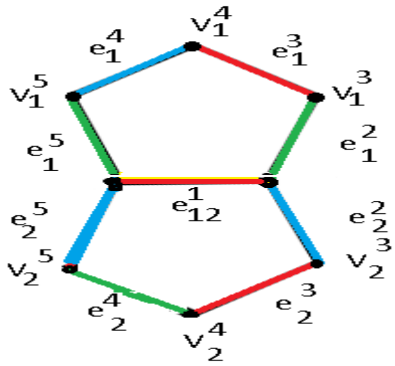
1.6. Catenation of pentagon graphs :

In this process, where two or more pentagon graphs are connected in a specific way to create a unique pattern. This process involves linking the graphs together by identifying one or more vertices from each graph and adding edges between them which has the same edge colour. Let's consider the catenation of two pentagon graphs (P1) and (P2) to form a larger graph:



(P_1) (P_2)

A new pattern is created when the P1 graph is positioned on the P2 graph by connecting the edges of the two graphs with the same edge colour on both.



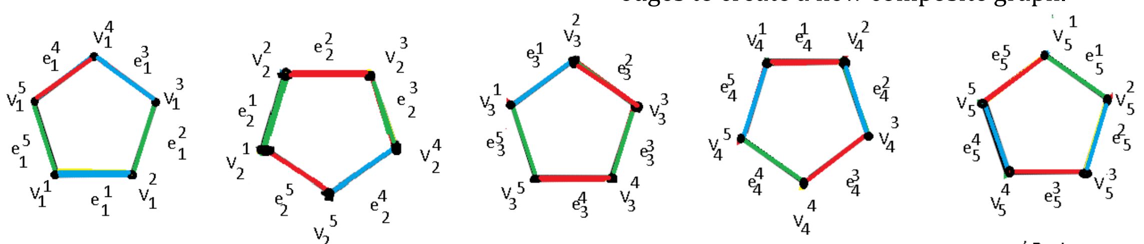
(P_1 U P_2)

Example 3:

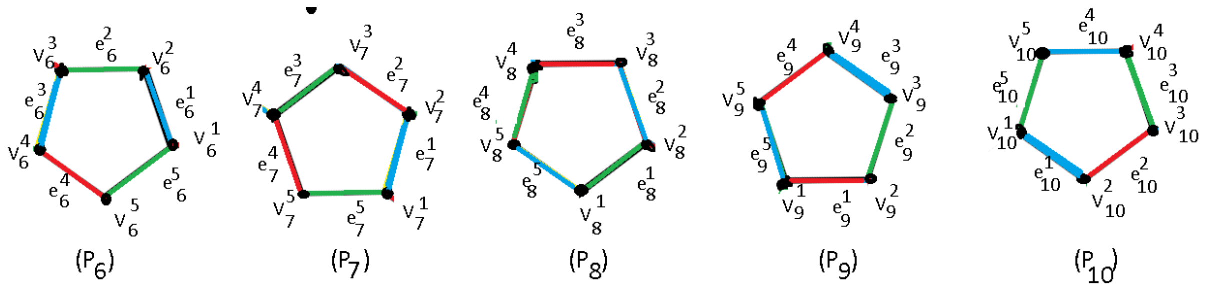
Let P1 and P2 be two graphs with 5 vertices. We say that P1UP2 is a joined graph is the complete union of two graphs P1 and P2
V(P1) = { v_1^1, $v_1^2, v_1^3, v_1^4, v_1^5$}
E(P1) = { $e_1^1, e_1^2, e_1^3, e_1^4, e_1^5$}
V(P2) = { v_2^1 , v_2^2, v_2^3, v_2^4, v_2^5}
E(P2) = { $e_2^1, e_2^2, e_{2,}^3 e_2^4, e_2^5$}
V(P1 U P2) = { v_1^3, v_1^4, v_1^5, v_{12}^1, v_2^5, v_2^4, v_2^3, v_{12}^{22}}
E (P1 U P2) = {$e_1^2, e_1^3, e_1^4, e_1^5, e_2^5, e_2^4$, e_2^3, e_2^2, e_{12}^1}

When P1 and P2 are combined, it results in a new pattern. Note that the merged edges are all the same length. Edges e_1^1 and e_2^1 with the same red colour are pasted together to form edge e_{12}^1.
Maximum degree Δ [P1 U P2] = 3
Number of vertices in P1 U P2 = 8
Number of edges in P1 U P2 = 9
Chromatic number χ(P1 U P2).=3
Clearly, χ′(P1 U P2) = Δ(P1 U P2).

Example 4: Consider a set of pentagon graphs with proper edge coloring red, green, and blue, the operation of "joining multiple pentagon graphs edges with same edge color" entails fusing the individual graphs by joining their edges to create a new composite graph.



(P_1) (P_2) (P_3) (P_4) (P_5)



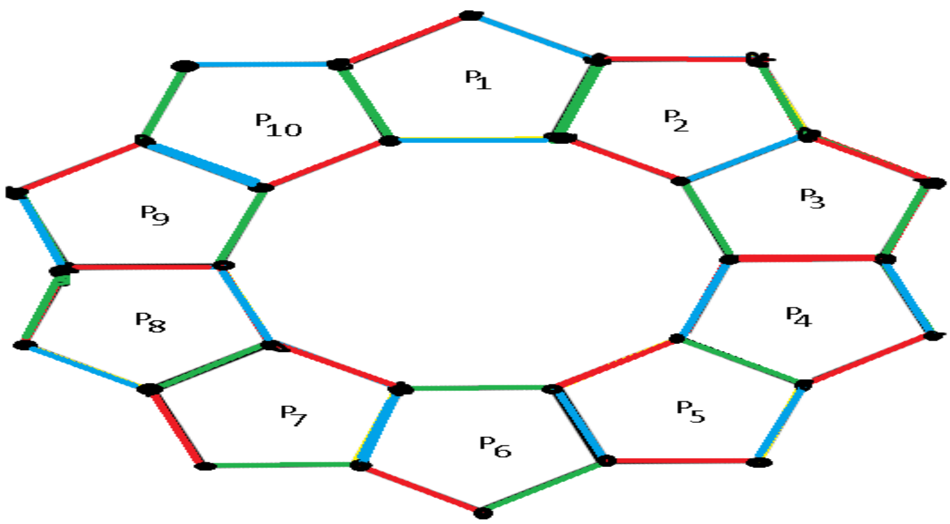
By connecting the edges of the same color, we link together the other 10 pentagon graphs. As a result, an exquisite flower design is created.

Let P1, P2, P3, P4, P5, P6, P7, P8, P9 and P10 be two graphs with 5 vertices.

V(P_1) = { $v_1^1, v_1^2, v_1^3, v_1^4, v_1^5$} E($P_1$) = { $e_1^1, e_1^2, e_1^3, e_1^4, e_1^5$}
V(P_2) = { v_2^1 , $v_2^2, v_2^3, v_2^4, v_2^5$} E($P_2$) = { $e_2^1, e_2^2, e_{2,}^3 e_2^4, e_2^5$}
V(P_3) = { $v_3^1, v_3^2, v_3^3, v_3^4, v_3^5$} E($P_3$) = { $e_3^1, e_3^2, e_3^3, e_3^4, e_3^5$}
V(P_4) = { v_4^1 , $v_4^2, v_4^3, v_4^4, v_4^5$} E($P_4$) = { $e_4^1, e_4^2, e_{4,}^3 e_4^4, e_4^5$}
V(P_5) = { $v_5^1, v_5^2, v_5^3, v_5^4, v_5^5$} E($P_5$) = { $e_5^1, e_5^2, e_5^3, e_5^4, e_5^5$}
V(P_6) = { v_6^1 , $v_6^2, v_6^3, v_6^4, v_6^5$} E($P_6$) = { $e_6^1, e_6^2, e_{6,}^3 e_6^4, e_6^5$}
V(P_7) = { $v_7^1, v_7^2, v_7^3, v_7^4, v_7^5$} E(P_7) = { $e_7^1, e_7^2, e_7^3, e_7^4, e_7^5$}
V(P_8) = { v_8^1 , $v_8^2, v_8^3, v_8^4, v_8^5$} E($P_8$) = { $e_8^1, e_8^2, e_{8,}^3 e_8^4, e_8^5$}
V(P_9) = { $v_9^1, v_9^2, v_9^3, v_9^4, v_9^5$} E(P_9) = { $e_9^1, e_9^2, e_9^3, e_9^4, e_9^5$}
V(P_{10}) = { v_{10}^1 , $v_{10}^2, v_{10}^3, v_{10}^4, v_{10}^5$} E($P_{10}$) = { $e_{10}^1, e_{10}^2, e_{10,}^3 e_{10}^4, e_{10}^5$}

The joining rule for graphs P1,P2,P3,P4,P5, P6,P7, P8, P9 and P10 are given below.

1) Graph P1 U P2, green-colored edges e_1^2 and e_2^1 from the P1 and P2 joins, respectively.
2) Graph P2 U P3, blue-colored edges e_2^4 and e_3^1 from the P2 and P3 joins, respectively.
3) Graph P3 U P4, red-colored edges e_3^4 andee_4^1 from the P3 and P4 joins, respectively.
4) Graph P4 U P5, green-colored edges e_4^4 and e_5^1 from the P4 and P5 joins, respectively.
5) Graph P5 U P6, blue-colored edges e_5^4 and e_6^1 from the P5 and P6 joins, respectively
6) Graph P6 U P7, blue-colored edges e_6^3 and e_7^1 from the P6 and P7 joins, respectively.
7) Graph P7 U P8, green-colored edges e_7^3 and e_8^1 from the P7 and P8 joins, respectively.
8) Graph P8 U P9, red-colored edges e_8^3 and $e_9^1$from the P8 and P9 joins, respectively.
9) Graph P9 U P10, blue-colored edges e_9^3 and e_{10}^1 from the P9 and P10 joins, respectively.
10) Graph P10 U P1, green-colored edges e_{10}^3 and e_1^5 from the P10 and P1 joins, respectively



By connecting the edges of the same color, we link together the other 10 pentagon graphs. As a result, an exquisite flower design is created.

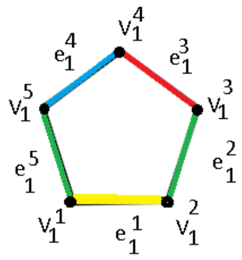
Let graph G1 = P1 U P2 U P3 U P4 U P5 U P6 U P7 U P8 U P9 U P10
Maximum degree Δ [G1] = 3
Number of vertices in G1 = 30
Number of edges in G1 = 40
Chromatic number χ(P1 U P2).=3
Clearly, χ′(P1 U P2) = Δ(P1 U P2).

Observation 2:

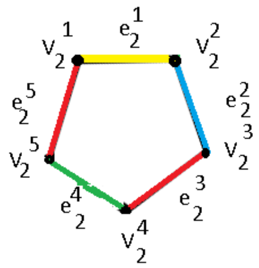
In this graph P1 U P2 and P1 U P2 U P3 U P4 U P5 U P6 U P7 U P8 U P9 U P10, 3 colours are sufficient to colour the edges, then the graph is said to be 3-colourable, Since χ′= 3, it is said to be 3-edge-chromatic. It satisfies **Vizing's Theorem ,** Δ(G) ≤ χ′(G) ≤ Δ(G) +1. While combining the graphs maximum degree was Δ = 3 and the Chromatic number χ =3. Since χ′ = Δ, then joined graph is said to be a **class one.**

When χ′ = Δ very few pattern is possible.

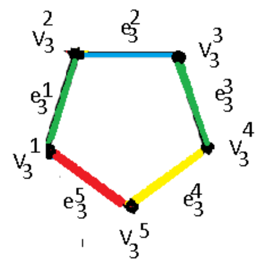
Example 5:
The operation of "joining numerous pentagon graphs edges with same edge color" entails fusing the individual graphs by joining their edges to create a new composite graph. Assume a set of pentagon graphs, for which 4 colors are sufficient to color the edges, the graph is said to be 4-colourable. The edges are colored red, green, yellow, and blue.



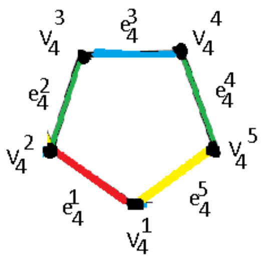
(P_1)



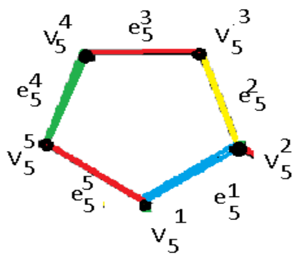
(P_2)



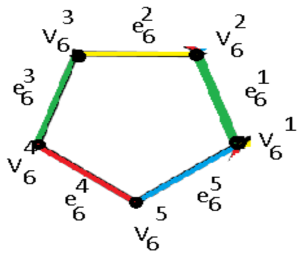
(P_3)



(P_4)



(P_5)

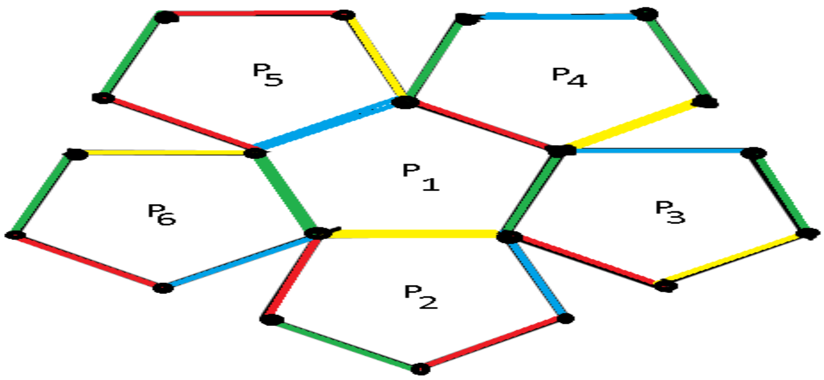


(P_6)

We are adhering the other five pentagon graphs to the five edges of P1 by linking the edges of the same color. Consequently, a lovely flower design is produced.

The joining rule for graphs P1,P2,P3,P4,P5 and P6 are given below.

1) Graph P1 U P2, yellow-colored edges e_1^1 and e_2^1 from the P1 and P2 joins, respectively.
2) Graph P1 U P3, green-colored edges e_1^2 and e_3^1 from the P1 and P3 joins, respectively.
3) Graph P1 U P4, red-colored edges e_1^3 and e_4^1 from the P1 and P4 joins, respectively.
4) Graph P1 U P5, blue-colored edges e_1^4 and e_5^1 from the P1 and P5 joins, respectively.
5) Graph P1 U P6, green-colored edges e_1^5 and e_6^1 from the P1 and P6 joins, respectively.



G2

Let G2 = P1 U P2 U P3 U P4 U P5 U P6 results in a new patten.

Maximum degree Δ [G2] = 4

Number of vertices in G2 = 20

Number of edges in G2 = 25

Chromatic number χ(G2).= 4

Clearly, χ′(G2) = Δ(G2).

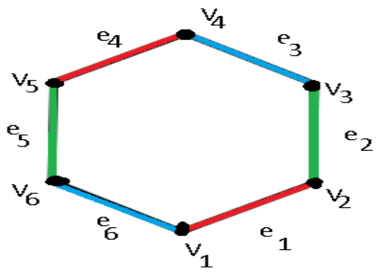
Since the degree of this new graph is 4, we are focused on using the four colors red, blue, green, and yellow to effectively color the edges, ensuring that every pair of neighboring edges has a distinct color.

Observation 3:

While combining the graphs maximum degree was Δ = 4 and the Chromatic number χ =4. Clearly, χ′ = Δ then the joined graph is said to be a **class one**.

Hexagonal Graphs

Definition 12: A hexagon is a six-sided polygon, while a graph is a mathematical structure composed of vertices (nodes) and edges (connections) between these vertices. Given this, a hexagon graph could be understood as a graph in which the vertices and/or edges form a hexagonal pattern. V_1, V_2, V_3, V_4, V_5 and V_6 are vertices the path formed by joining vi and vi+1. It is a regular graph. We denote this graph by H. e_1, e_2, e_3, e_4 , e_5 and $e_6$are edges.



(H)

Proper coloring

1) Colour the vertical opposite edges e_2 and e_5 by green
2) Colour the alternate obliqued edges in the top left e_4 and bottom right e_1 by red
3) Colour alternate obliques edge in the top right e_3 and bottom left e_6 by blue.

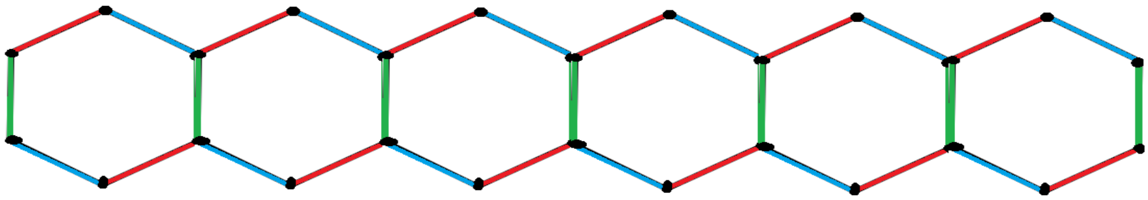
1.6. Catenation of hexagonal graphs :

Honeycomb networks are built recursively from hexagonal tessellation. The honeycomb network HC(1) is a hexagon. The honeycomb network is obtained by adding several hexagons to the boundary edges.

Example 6:

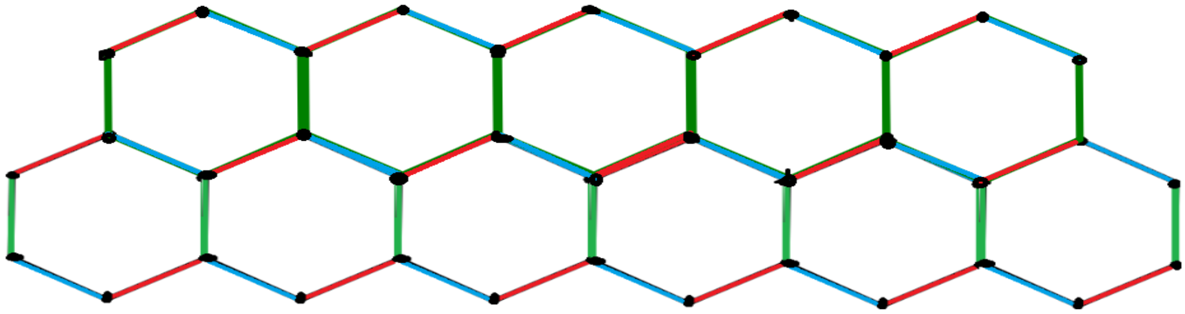
The joining rule for several hexagon graphs H to make a honeycomb network is given below.

1) Join the vertical edges e_2 and e_5 which has the same green colour



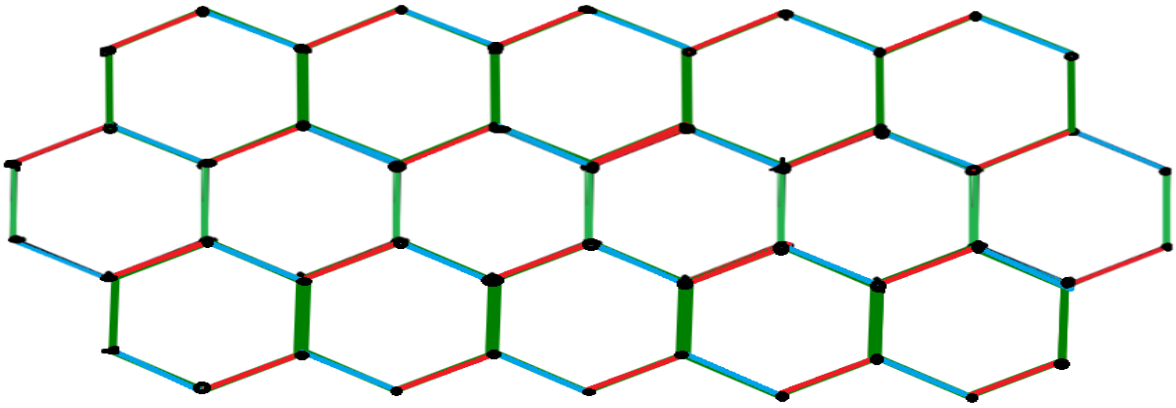
2) Join the alternative obliqued edges top right e_3 and bottom left e_6 which has the same blue colour.

Join the alternative oblique edges top left e_4 and bottom right e_1 which has the same red colour



1) Join the alternative oblique edges bottom right e_1 and top right e_4 which has the same red colour.

Join the alternative oblique bottom left e_6 and top right e_3 which has the same blue colour



HC

Every cycle includes at least two oblique acute, two oblique obtuse and two vertical edges. Thus the edges of any cycle are colored with at least three colors.
Maximum degree Δ [HC] = 3
Number of vertices in HC = 47
Number of edges in HC = 63
Chromatic number $\chi(HC)$.=3
Clearly, $\chi'(HC) = \Delta(HC)$.

Observation 4: Since $\chi'((HC) \geq \Delta(HC) = 3$, we have a $\chi'((HC) = 3$.

Conclusion:
When a graph features a vertex with a degree of 'k', it necessitates a minimum of 'k' distinct colors for the proper coloring of edges linked to that vertex. Nonetheless, the chromatic index is not solely dictated by the degree. Certain graphs exhibit uniform vertex degrees, yet their chromatic index surpasses expectations due to

additional structural attributes. A regular graph on an odd number of vertices is **class two**. A regular graph on an even number of vertices is class one. Remarkably, in all the innovative patterns we formulated, the degree aligns with the chromatic index, denoted as $\chi' = \Delta$, thereby categorizing the merged graph as a **"class one"** structure. Analyzing graph patterns helps in understanding network connectivity, identifying influential nodes, and optimizing network performance.

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