Journal of Harbin Engineering University
ISSN: 1006-7043

Vol 45 No. 3
March 2024

Elastic Modulus Reduction Method-Based Plastic Limit Analysis for Reinforced Concrete Frame Structures

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Abstract

In this study, we explore an effective linear elastic iterative finite element technique known as the elastic modulus reduction method, tailored to estimate the limit load of reinforced concrete frame structures. The method determines the element bearing ratio by referencing the generalized yield criterion combined with the strain energy equilibrium principle. Recognizing the bending moment as the chief contributor to the reinforced concrete frame structure damage, we suggest a variant of the element bearing ratio that exclusively focuses on the bending moment yield. Through numerical illustrations, we highlight the method's precision and application for plastic limit analysis in the reinforced concrete frame structures.

Keywords: elastic modulus reduction method, plastic limit analysis, reinforced concrete structure, frame structure.

1. Introduction

The analysis of a building structure's plastic limit is crucial for both its design and the safety evaluation of engineering constructs [1]. Determining a structure's plastic limit load efficiently has been a focal point for many researchers. Although the structure's limit load value can be derived from allowable stress and displacement fields using upper and lower bound theorems, this approach is only applicable to relatively simple structures. Calculations become complex for structures with multiple static indeterminacies or intricate designs, making them less suitable for practical engineering applications.

With the development of computers, finite element methods have gradually been applied to the plastic limit analysis of structures. The Elastic-Plastic Incremental Method (EPIM) has become a popular tool in engineering [2]. However, a significant drawback of EPIM is the continuous need to adjust the stiffness and load matrices during staged loading, leading to extensive computational demands and increased costs. By the late 1980s, as linear elastic finite elements advanced, a new numerical method emerged for structural plastic limit analysis named the Elastic Modulus Adjustment Procedure (EMAP). EMAP, with its simplicity, efficiency, and accuracy, stands out when compared to traditional methods like mathematical programming and EPIM. It's now prevalently used in safety evaluations in the pressure vessel sector and civil and hydraulic engineering realms. Furthermore, global pressure vessel design standards such as the American Society of Mechanical Engineers (ASME) in the US and the European Union pressure vessel standard EN13445 in the European Union have recognized it as a viable method. As EMAP has evolved, it has spawned various algorithms, such as the reduced elastic modulus method, the generalized local stress-strain analysis method, the elastic compensation method, the modified elastic compensation method, the elastic modulus adjustment method based on the generalized yield criterion, the m_β method and reference body method, and the improved m_β method, among others [3]. A commonality among these is that they focus on stress as the pivotal parameter for adjusting the structural elastic modulus. These methods often overlook the material attributes and sectional resistance, lacking a standardized measure for the bearing capability of various element types. Therefore, these methods are only suitable for the plastic limit analysis of single-material structures and cannot be applied to structures with mixed types of elements.

Building on prior work, Lufeng Yang and his team introduced the concept of the "element load-bearing ratio" and established the Elastic Modulus Reduction Method (EMRM), further developing the elastic modulus adjustment method. EMRM has been successfully applied in structural reliability analysis in civil engineering, including frame structures, bridge structures, and safety assessment of hydraulic pressure pipelines [3-9]. The principles can also be applied to structures made of reinforced concrete.

2. Methodology

A. Elastic Modulus Reduction Method

The Elastic Modulus Reduction Method is derived from the element load-bearing ratio, which is defined by the cross-sectional yield strength. In the iterative process of structural limit analysis, this ratio acts as the guiding parameter for adjusting the element's elastic modulus. Based on this, formulas for the uniformity of the load-bearing ratio and the reference counterpart are established. By taking advantage of the consistency in deformation energy between consecutive iterations, a novel approach to modifying the elastic modulus is proposed. The foundational principle of this method is to use the reference load ratio as a benchmark. Only those elements whose elastic modulus exceeds the reference load ratio undergo reductions. As a result, there is a redistribution of the element load ratios and internal forces within the structure. This redistribution subsequently results in a series of statically allowable stress fields that approximate the optimal distribution, thereby determining the lower limit of the structure's ultimate load.

B. Element Load-bearing Ratio

For reinforced concrete frames, the structure can be discretized into beam elements. The spatial beam element has 6 degrees of freedom, while the planar beam element has 3 degrees of freedom. This article focuses on the spatial beam element as an example. Each node has 6 degrees of freedom, namely axial force N_x, shear forces Q_y , bending moments M_y and M_z, and torsional moment T_x . The element's load-bearing ratio can be defined based on the internal force of the element and the cross-sectional resistance:

$$\begin{cases} r_{N_x} = \dfrac{N_x}{N_{P_x}} \\ r_{Q_y} = \dfrac{Q_y}{Q_{P_y}}, r_{Q_z} = \dfrac{Q_z}{Q_{P_z}} \\ r_{T_x} = \dfrac{T_x}{T_{P_x}} \\ r_{M_y} = \dfrac{M_y}{M_{P_y}}, r_{M_z} = \dfrac{M_z}{M_{P_z}} \end{cases} \quad (1)$$

where,

N_{P_x} is tensile/compressive strength of the element.

Q_{P_y} and Q_{P_z} are shear strength of the element.

M_{P_y} and M_{P_z} are bending strength of the element.

T_{P_x} is torsion strength of the element.

If the type of structural failure has already been determined during the design phase, such as tensile-compressive, bending, or shear failure, it means that the above-mentioned internal forces-namely, tensile/compressive force, bending moment, or shear force will play a dominant role in the structural failure, the element load-bearing ratio corresponding to that particular internal force can be taken as the control variable during modulus adjustment. If the type of structural failure is not clear, the maximum value of the element load-bearing ratio in equation (1) can be chosen as the control variable. If the values of the various load ratios in equation (1) are relatively close, the upper limit of the generalized yield function of the beam element can be introduced to determine the element load-bearing ratio:

$$r_u = \sqrt{{r_{N_x}}^2 + {r_{Q_y}}^2 + {r_{Q_z}}^2 + {r_{T_x}}^2 + \frac{1}{\lambda_y}{r_{M_y}}^2 + \frac{1}{\lambda_z}{r_{M_z}}^2} \quad (2)$$

Where, for rectangular cross-section beam element:

$$\lambda_y = \lambda_z = 1 - {r_{N_x}}^2$$

C. Load-bearing Ratio Uniformity and Reference Load-bearing Ratio

Unlike previous elastic modulus adjustment methods that only considered the stress state of each individual unit, EMRM takes into account the impact of all units. Therefore, to improve computational efficiency, EMRM defines the load-bearing ratio uniformity d_k, with the following calculation expression:

$$d_k = \frac{\bar{r}_k + r_k^{min}}{\bar{r}_k + r_k^{max}} \quad (3)$$

$$\bar{r}_k = \frac{1}{N}\sum_{e=1}^{N} r_{k,e}$$

$$r_k^{max} = \max(r_{k,1}, r_{k,2}, \dots, r_{k,N})$$

$$r_k^{min} = \min(r_{k,1}, r_{k,2}, \dots, r_{k,N})$$

Where, the subscript k represents the number of iterations for adjusting the elastic modulus of the structure; N represents the total number of units after structural discretization; r_k^{max} and r_k^{min} respectively denote the maximum and minimum load-bearing ratios in the structure; $\bar{r}_k$ is the average load-bearing ratio of the structure.

Based on equation (2), the control parameter for elastic modulus adjustment, the reference load-bearing ratio, can be defined as:

$$r_k^0 = r_k^{max} - (r_k^{max} - r_k^{min}) \cdot d_k \tag{4}$$

Where, r_k^0 is the reference load-bearing ratio of the structure; the meanings of other symbols are the same as in equation (3).

D. Elastic modulus adjustment strategy

EMRM uses the element load-bearing ratio r_k^e and the reference load-bearing ratio r_k^0 as control parameters during element elastic modulus adjustment, ensuring the conservation of element strain energy as well as the Banach fixed-point principle, as follows:

$$E_{k+1}^e = \begin{cases} E_k^e \dfrac{2(r_k^0)^2}{(r_k^e)^2 + (r_k^0)^2}, r_k^e > r_k^0 \\ E_k^e, \quad r_k^e \le r_k^0 \end{cases} \tag{5}$$

Where, k represents the number of iterations for adjusting the elastic modulus of the structure; r_k^e is the element load-bearing ratio at the kth iteration; r_k^0 is the reference load-bearing ratio at the kth iteration; E_k^e and E_{k+1}^e are the element elastic moduli during the kth and (k+1)th adjustments, respectively.

E. Determination of Limit Load

During the iterative process of elastic modulus adjustment in EMRM, the maximum element load-bearing ratio r_k^{max} will be obtained based on each elastic iterative calculation. Based on this, the calculation expression for the limit load p_k^L in this iterative step can be further derived:

$$p_k^L = \frac{p_0}{r_k^{max}} \tag{6}$$

Where, p_0 is the initial load; r_k^{max} is the maximum element load-bearing ratio under the k^{th} iteration calculation; p_k^L is the limit load value of the structure when the calculation iteration proceeds to the kth time.

Repeat the above iterative calculations until the limit load meets the following convergence criterion:

$$\left| \frac{p_k^L - p_{k-1}^L}{p_{k-1}^L} \right| \le \varepsilon \tag{7}$$

Where, p_{k-1}^L and p_k^L are the limit load values of the structure during the kth and (k-1)th iteration calculations, respectively; ε is the preset convergence tolerance value for iterative calculation.

Suppose that after N iterative calculations, the limit load value solved by EMRM converges, then the calculation expression for the lower limit of the structure's limit load P^L is:

$$P^L = P_N^L \tag{8}$$

F. EMRM Model for Reinforced Concrete Materials

Reinforced concrete frame structures are widely used in various fields of the civil engineering industry due to their advantages of clear load transmission, flexible structural layout, good seismic resistance, and overall integrity. Aqueducts, bridges, and other projects that use stand-type support structures also often adopt the form of reinforced concrete frames. Most of the existing standards worldwide employ a load-bearing (strength) design concept for elastic and elastoplastic analysis of reinforced concrete structures, thereby calculating the internal forces of the structure and determining whether the structure meets the load-bearing requirements.

EMRM, when used for calculating the plastic limit load-bearing capacity, has the advantages of simple principles, high calculation efficiency, and good accuracy. However, it is only suitable for homogeneous isotropic structures. For heterogeneous materials like reinforced concrete, necessary assumptions must be made.

In the design process of reinforced concrete frame structures, it is usually required to adhere to the principles of 'strong in shear, weak in flexure' and 'strong columns, weak beams.' This means that the type of failure for this structure is predetermined as flexural failure during the design stage, where bending moments will play a dominant role in the failure process of the reinforced concrete structure. Based on this, the element load-bearing ratio, which considers only the yielding of bending moments, can be defined as a control variable in the adjustment of the elastic modulus r_u:

$$r_u = \frac{M_y}{M_{P_y}} \tag{9}$$

For reinforced concrete frames, the primary mode of failure is flexural, where bending moments play a dominant role in the failure. In the case of adequately reinforced beams undergoing flexural failure, it begins with the yielding of longitudinal tensile steel reinforcement and concludes with the crushing of the compressed concrete. By considering the equilibrium of forces, as shown in Equation (10):

$$\alpha_1 f_c bx = f_y A_s \quad (10)$$

We can determine that the pressure borne by concrete is equal in magnitude to the tensile force borne by steel reinforcement. However, since the resultant force of the steel reinforcement is located below the centroid of the concrete section's lower part, it can bear a greater moment. If we were to merely equate it as $\alpha_1 f_c bx$, it would be overly conservative. Therefore, we consider the capacity of this portion of the steel reinforcement to be equivalent to $f_c bh - \alpha_1 f_c bx$, making the entire section's capacity $f_c bh$, with the entire section being treated as a homogeneous material and its yield strength taken as the design value of concrete's compressive strength.

3. Case Study

Case 1: In reference [10], the test model used was a rectangular section beam, subjected to loading from two symmetric points, as shown in Fig. 1. The beam was reinforced with two Φ16 bars. Specific material parameters can be found in the Table 1.

Table 1 Geometry and Material Mechanical Properties

Span	1800 [mm]	
Sectional Dimensions	Width(B) [mm]	120
	Height(H) [mm]	250
Concrete	Poisson's Ratio (μ)	0.1667
	Design compressive strength (f_c) [MPa]	29.3
	Elastic modulus (E_c) [GPa]	25
Rebar	Design tensile strength (f_y) [MPa]	310
	Elastic modulus (E_c) [MPa]	200

Based on Fig. 1 and Table 1, the cross-sectional resistance of the beam is calculated to be $M_d = 24.0\ kN \cdot m$.

In reference [10], a layered finite element method is employed. This method involves dividing the beam along its length into n two-node beam elements and then subdividing the concrete into several layers along the height direction of the element, while the steel reinforcement is divided into equivalent layers based on its position and area. Specific assumptions and analysis methods are introduced as needed, and a finite element program is developed to calculate the ultimate load-carrying capacity of the reinforced concrete beam.

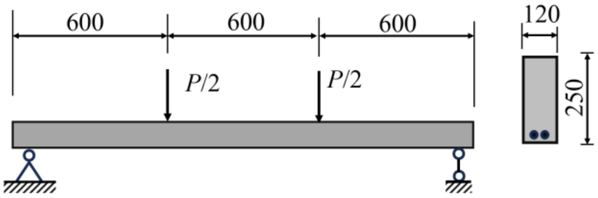


Fig. 1. Reinforced Concrete Beam Model.

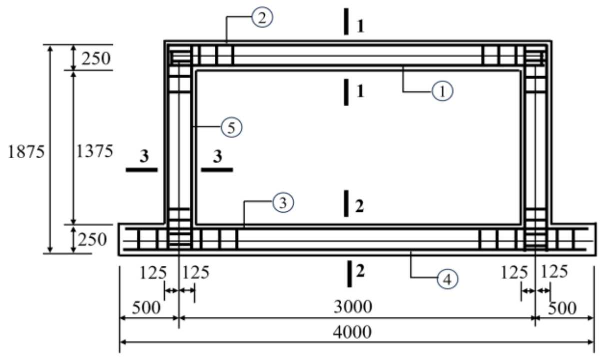
In this study, a continuum-based model is used for reinforced concrete, where the concrete elements are treated as homogeneous and continuous units with evenly distributed reinforcement. Therefore, an ANSYS finite element analysis model of this beam is established, taking into account geometric parameters, material properties, and structural constraints. The beam is simulated using 2-node beam elements (BEAM188). Arbitrary initial loads are applied, and the structural ultimate loads obtained through the EMRM proposed in this paper are presented in Table 2 as follows:

Table 2 Limit Load of The Reinforced Concrete Beam

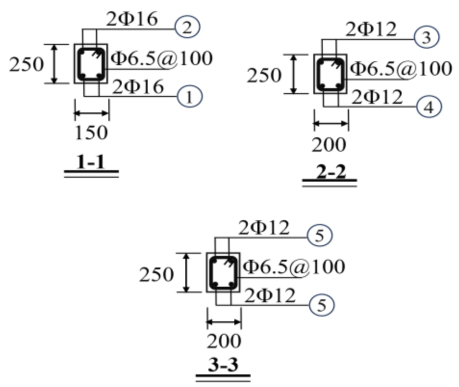
Method	EMRM	Ref. [10]	Experiment
Limit Load [kN]	106.2	100.0	110.0
Relative Error [%]	3.45	9.09	—

From the calculation results in Table 2, it can be observed that in the reference literature, the ultimate load of the reinforced concrete beam structure obtained using the layered finite element method has an error of 9.09% when compared to the model test results. In contrast, the ultimate load obtained using the EMRM proposed in this paper has a smaller error of 3.45% when compared to the model test results. This indicates that considering only the yielding of section bending moments in the EMRM results in reliable calculations and higher precision in determining the ultimate load of reinforced concrete beam structures.

Case 2: In reference [11], the experimental model consists of a reinforced concrete frame structure with symmetrically arranged 2Φ16 HRB335 grade steel bars in the beams and 2Φ12 HRB335 grade steel bars in the columns. A simplified diagram of the experimental model is shown in Fig. 2. Concrete and steel mechanical properties are provided in Table 3. Based on Fig. 2 and Table 3, the formulas for calculating the flexural capacity of rectangular cross-section reinforced concrete members can be used to determine the cross-sectional resistance of both columns and beams: the column $M_d = 15.4kN \cdot m$, the beam $M_d = 26.3kN \cdot m$.



(a)



(b)

Fig. 2. Reinforced Concrete Frame Model.

Table 3 Geometry and Material Mechanical Properties

Beam Section		
Dimensions	Width(B) [mm]	150
	Height(H) [mm]	250
Concrete	Poisson's Ratio (μ)	0.1667
	Design compressive strength (f_c) [MPa]	29.3
	Elastic modulus (E_c) [GPa]	32.3
Rebar	Design tensile strength (f_y) [MPa]	375.7
	Elastic modulus (E_c) [MPa]	195
Column Section		
Dimensions	Width(B) [mm]	200
	Height(H) [mm]	250
Concrete	Poisson's Ratio (μ)	0.1667
	Design compressive strength (f_c) [MPa]	29.3
	Elastic modulus (E_c) [GPa]	32.3
Rebar	Design tensile strength (f_y) [MPa]	383.3
	Elastic modulus (E_c) [MPa]	201

The experiments applied vertical loads to the frame beams using an actuator, which, after rigidly distributing the load, resulted in two equal vertical concentrated loads applied at the three-point locations on the experimental frame beam, as shown in Fig. 3. For the convenience of finite element modeling and to maintain the same constraints as the original experimental model, the pedestals used in the experiment were omitted in the modeling process, and the bottom of both columns was fully constrained.

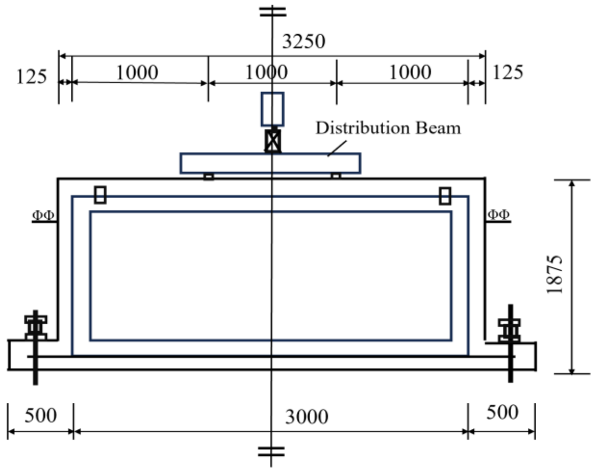


Fig. 3. Reinforced Concrete Frame Experiment Setting.

Similar to Case 1, an ANSYS finite element analysis model of this frame was established, taking into account geometric parameters, material properties, and structural constraints. Both columns and beams were simulated using 2-node beam elements (BEAM188). Arbitrary initial loads were applied, and the structural ultimate loads obtained through EMRM are presented in Table 4 as follows:

Table 4 Limit Load of The Reinforced Concrete Beam

Method	EMRM	Ref.[11]	Experiment
Limit Load [kN]	62.4	53.4	64.1
Relative Errors [%]	2.65	16.70	0

From the calculation results in Table 4, it can be observed that for reinforced concrete frame structures, the ultimate loads obtained using the "ultimate load method" from the reference literature show a relatively large error when compared to the model test results. In contrast, the ultimate loads obtained using the EMRM proposed in this paper exhibit an error of 2.65% when compared to the model test results. This indicates that, in the context of reinforced concrete frame structures, the ultimate load obtained by considering only the yielding of section bending moments as defined by the unit load ratio is reliable and possesses a high level of computational accuracy.

4. Conclusions

In accordance with the design principles of reinforced concrete frame structures, this paper assumes that the bending moment is the primary factor leading to structural failure. Following the design codes for reinforced concrete structures and the concept of unit load ratio, it presents an expression for calculating the unit load ratio for reinforced concrete frame structures, considering only the yielding of section bending moments. Additionally, in combination with the Elastic Modulus Reduction Method (EMRM) adjustment strategy, it establishes an EMRM applicable for the ultimate load analysis of reinforced concrete frame structures.

Through the analysis of two case studies, it is demonstrated that the ultimate loads obtained through the Elastic Modulus Reduction Method (EMRM) are very close to analytical results, with errors within 3%. Moreover, the ultimate loads obtained by considering only the yielding of section bending moments, as defined by the unit load ratio, are also in close agreement with analytical and model test results, with errors not exceeding 5%. This indicates that EMRM produces stable results, converges quickly, and has a high level of computational accuracy. When calculating the ultimate loads for reinforced concrete frame structures, it is feasible to consider only the yielding of section bending moments. The proposed calculation expression for the unit load ratio of reinforced concrete structures in this paper is reasonable and practical.

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