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Fuzzy Game Problem with Octagonal Fuzzy Numbers in Operations Research Environment

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Abstract
In this paper we deal with fuzzy numbers in which the payoff represented as octagonal fuzzy numbers. We use Beta Distribution to convert the octagonal fuzzy numbers to crisp numbers. By applying algebraic method, the value of the game is obtained.

Keywords: Fuzzy sets, octagonal fuzzy numbers, Fuzzy game problem, Algebraic method.

1 Introduction

Game theory is developed for decision making under conflicting situation where there are one or more opponents. Game theory provides optimal solution to such games assuming that each of the players wants to maximise his profit or minimize his loss.

In this paper we have proposed an approach for solving the fuzzy game problem by converting in to its equivalent form.

2 Preliminaries

2.1 Fuzzy numbers

A fuzzy number $\breve{A}$ is a fuzzy set on the real line R must satisfy the following conditions

(i) There exist atleast one $x \in X$ with $\mu_A(x) = 1$
(ii) $\mu_A(x)$ is piecewise continuous

2.2 Octagonal fuzzy numbers

A fuzzy number $\tilde{A}$ is an octagonal fuzzy number denoted by (a, b, c, d, e, f, g, h) where $a \le b \le c \le d \le e \le f \le g \le h$ are real numbers and its membership function $\mu_{\tilde{A}}(x)$ is given below

$$\mu_{\breve{A}}(x) = \begin{cases} 0 & for\ x < a \\ k\left(\frac{x-a}{b-a}\right) & for\ a \le x \le b \\ k & for\ b \le x \le c \\ k + (1-k)\left(\frac{x-c}{d-c}\right) & for\ c \le x \le d \\ 1 & for\ d \le x \le e \\ k + (1-k)\left(\frac{f-x}{f-e}\right) & for\ ex \le f \\ k & for\ f \le x \le g \\ k\left(\frac{h-x}{h-g}\right) & for\ g \le x \le h \\ 0 & for\ x \ge h \end{cases} \quad 0$$

< k < l

2.3 Graphical representation of octagonal fuzzy numbers

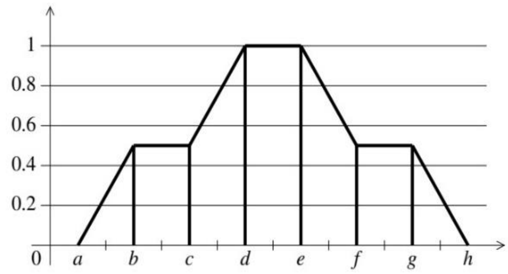

Figure 1

3 Defuzzification for Octagonal Fuzzy Numbers

3.1 Definition

Let $\tilde{A}$ be a octagonal fuzzy number. The value of $\beta(\tilde{A})$ called the measure of $\tilde{A}$ is calculated as follows:

$$\beta(\tilde{A}) = \frac{3a+6b+4c+5d+5e+4f+6g+3h}{36} \qquad (1)$$

3.2 Pure strategy

Pure strategy is a decision making rule in which one particular course of action is selected

3.3 Saddle point

If the maxi-min value equals the min-maxi then the game is said to be saddle point and the corresponding strategies are called optimum strategies. The amount of payoff at an equilibrium point is known as the value of the game.

3.4 Solution of 2 × 2 games without

saddle point

Consider the 2×2 game $\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$

To solve the game, we proceed as follows

(i) To verify the saddle point

(ii) If there is no saddle point solve by finding equalizing strategies

Let A's play with probability x and T with probability $1-x$. If player B plays H all the time, then A's gain expected as $E(A,H) = x.a_{11} + (1-x)a_{12}$

Let B's play with probability T all the time A's expected gain will be

$E(A,T) = x.a_{21} + (1-x)a_{22}$

$E(A,H) = E(A,T) = E(A)$ be the strategy for A

The same procedure as follows for player B. The strategy foe player B

$E(B,H) = E(B,T) = E(B)$

3.5 Description of the paper

In this paper game problem considered as octagonal fuzzy numbers which can be converted into crisp fuzzy numbers. To determining game value by using 3.4.

4 Numerical Example

4.1 Fuzzy game with payoffs as octagonal fuzzy numbers

$$\begin{array}{cc} & Player\ B \\ Player\ A & \begin{vmatrix} 5,6,7,9,10,11,12 & 0,1,2,3,4,5,6,7 \\ 2,3,4,5,6,7,8,9 & 4,5,6,7,8,9,10,11 \end{vmatrix} \end{array}$$

The following table gives the crisp values of fuzzy octagonal numbers

Strategy	Octagonal fuzzy numbers	$3a_1 + 6a_2 + 4a_3 + 5a_4 + 5a_5 + 4a_6 + 6a_7 + 3a_8/36$
a_{11}	(5, 6, 7, 8, 9, 10, 11,12)	8.5
a_{12}	(0, 1, 2, 3, 4, 5, 6, 7)	3.6
a_{21}	(2, 3, 4, 5, 6, 7, 8, 9)	5.3
a_{22}	(4, 5, 6, 7, 8, 9, 10, 11)	7.75

The given fuzzy number in the payoff matrix is reduced in to crisp number using beta distribution and solved by maxi-min criterion as follows:

$$\begin{pmatrix} 8.5 & 3.6 \\ 5.3 & 7.75 \end{pmatrix}$$

To find a saddle point

Minimum of 1st row = 3.6

Minimum of 2nd row = 5.3

Maxi(mini) = 5.3

Maximum of 1st column = 8.5

Maximum of 2nd column = 7.75

Mini(max) = 7.75

Saddle point does not exist

By applying section 3.4 to find the value of the game

Let B play 8.5 with probability x and 3.6 with probability as $1-x$

$E(A, 8.5) = 8.5x + 3.6(1-x)$

$E(A, 5.3) = 5.3x + 7.75(1-x)$

$E(A) = E(A, 8.5) = E(A, 5.3)$

$\Rightarrow 8.5x + 3.6 - 3.6x = 5.3x + 7.75 - 7.75x$

$\Rightarrow 4.9x + 3.6 = 7.75 - 2.45x$

$\Rightarrow 4.9x + 2.45x = 7.75 - 3.6$

$\Rightarrow 7.35x = 4.15 - x = \frac{4.15}{7.35}$

$1 - x = 1 - \frac{4.15}{7.35}$

$= \frac{3.2}{7.35}$

$B's$ strategy $= \left(\frac{4.15}{7.35}, \frac{3.2}{7.35}\right)$

Let A play 8.5 with probability y and 3.6 with probability $(1-y)$

$E(B, 8.5) = 8.5y + (1-y)5.3$

$E(B, 3.6) = 3.6y + (1-y)7.75$

$\Rightarrow 8.5y + 5.3 - 5.3y = 3.6y + 7.75 - 7.75y$

$\Rightarrow 3.2y + 5.3 = -4.15y + 7.75$

$\Rightarrow 3.2y + 4.15y = 7.75 - 5.3$

$\Rightarrow 7.35y = 2.45$

$\Rightarrow y = \frac{2.45}{7.35}$

$1 - y = \frac{4.9}{7.35}$

$A's$ strategy $= \left(\frac{2.45}{7.35}, \frac{4.9}{7.35}\right)$

$$\textbf{Value of the game} = 8.5 * \frac{2.45}{7.35} + 5.3 * \frac{4.9}{7.35}$$
$$= 8.5 * 0.33 + 5.3 * 0.66$$
= 2.80 + 3.49
$$\boldsymbol{V = 5.5}$$

5. Conclusion

In this method we have obtained the payoff octagonal fuzzy numbers. The given problem defuzzied by using algebraic method to find the optimum solution by using section 3.4. The value of the game is 5.5. This is the alternate method to find the value of the game and also easy for execution.

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