

# A Study of Bi-Ternary Gamma Semirings

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**Abstract:** We attempted to investigate the algebraic nature of a bi-ternary gamma and sub-bi-gamma ternary sub-semiring of a group in this research, and we addressed some of its features using a counterexample.

**Keywords:** Bi-ternary gamma semi-ring, Bi-ideal, Bi-ternary gamma sub-semi-ring.

## Introduction.

The evolution of the theory of semi rings is greatly influenced by ring theory and the idea of semigroups. Ternary semi-groups studied by (4). The concept of ternary semi-groups was studied and quasi and bi-ideals studied by (5). The study of rings, which are special semi rings reveals that multiplicative structures are quite independent of their additive structures through their additive structures are abelian groups. Expanding upon the concept of ternary semiring first presented by Lister and this idea extended by (6) and are studied by quasi and bi-ideals in ternary semi-ring by (7)(8). The concept of ternary semi rings was extended by (1) and then he studies three types of ideals in ternary semi rings (2, 3). In ternary semi-ring extended to essential ideals by (9).

Bi-groups are especially helpful because they offer answers for a problem that impacts all groups. Instead of producing an algebraic structure, the union of two sub-groups discovers a pleasing bi-algebraic structure in the form of bi-groups. Research with two groups was conducted from 1994-1996. Maggu was the one who first developed bi-group theory [18-19]. This idea was developed by VasanthaKandasamy and Meiyappan (1997) [11]. These writers corrected a number of conclusions that Maggu had previously shown. Among these results were the sub-bi-group characterization theorems.

Vasantha Kandasamy, however, recently looked at the concept of bi-algebraic structures [22]. In [12], Agboola and Akinola studied the bi-coset of a bi-

vector space. P. L. Maggu was the first to introduce the idea of a bi-group. Bi-algebraic structures were first described by (10–12), and Flourenche and VasanthaKandasamy(14), expanded upon it. The ternary operation is one of the operations in the algebraic framework that I used, and I then used the same idea in every way that I could.

## 1.Preliminaries:

**Definition1.1:** Let  $R \neq \emptyset$ . A structure  $(R, +, \cdot)$  with bi binary operations addition and multiplication is said to be a **Bi-ring** if  $R = R_1 \cup R_2$  if  $R_1$  and  $R_2$  are proper sub sets of  $R$  such that

- i.  $(R_1, +, \cdot)$  is a ring
- ii.  $(R_2, +, \cdot)$  is a ring

**Example1.2:** Let  $R = \{0, 2, 4, 5, 6, 8\}$  be a non-empty set under  $' +_{10}$  and  $' \times_{10}$ '.

If  $R_1 = \{0, 5\}$  and  $R_2 = \{0, 2, 4, 6, 8\}$  are rings under addition and multiplication modulo 10.

Hence  $R = R_1 \cup R_2$  is bi ring

$\times_{10}$		0	2	4	6	8
0	0	0	0	0	0	0
2	0	4	8	2	6	
4	0	8	6	4	2	
6	0	2	4	6	8	
8	0	6	2	8	4	

$+_{10}$		
	0	5
0	0	5
5	5	0

**Definition 1.3:** A set  $T \neq \emptyset$  together with a binary operation addition and ternary multiplication denoted by juxtaposition, is said to be ternary semi-ring if  $T$  is additive commutative semi group satisfying the following conditions

- i.  $[abc]de = a[bcd]e = ab[cde]$
- ii.  $(a + b)cd = acd + bcd$
- iii.  $a(b + c)d = abd + acd$
- iv.  $ab(c + d) = abc + abd$  for all  $a, b, c, d, e \in T$ .

**Note1.4:** Throughout out this paper, we write  $abc$  instead of  $[abc]$

**Example 1.5:** Let the set  $A = \{\emptyset, A, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$  in which we define the addition and ternary multiplication as follows:

- (i)  $P + Q = \min\{P, Q\} = P \cap Q$
- (ii)  $PQR = \max\{P, Q, R\} = P \cup Q \cup R \quad \forall P, Q, R \in A$  such that  $(A, +, [\cdot])$  is a ternary

Semiring.

## 2. BI-STRUCTURE ON GAMMA SEMIRINGS

**Definition 2.1:** Let  $T$  and  $\Gamma$  be two additive commutative semi groups. A Set  $T = T_1 \cup T_2$  is said to be a Bi-ternary gamma semi-ring, if both  $T_1$  and  $T_2$  are two ternary gamma semirings.  $(T_1, \Gamma, +, [\cdot])$  &  $(T_2, \Gamma, +, [\cdot])$  and if there exist a mapping from

$T_1 \times \Gamma_1 \times T_1 \times \Gamma_1 \times T_1 \rightarrow T_1$  Which maps  $(x_1, \alpha_1, y_1, \beta_1, z_1) \rightarrow [x_1 \alpha_1 y_1 \beta_1 z_1]$  satisfying the conditions

- i)  $[a_1 \alpha_1 b_1 \beta_1 c_1] \gamma_1 d_1 \delta_1 e_1 = [a_1 \alpha_1 [b_1 \beta_1 c_1 \gamma_1 d_1] \delta_1 e_1] = [a_1 \alpha_1 b_1 \beta_1 [c_1 \gamma_1 d_1 \delta_1 e_1]]$
- ii)  $[a_1 + b_1] \alpha_1 c_1 \beta_1 d_1 = [a_1 \alpha_1 c_1 \beta_1 d_1] + [b_1 \alpha_1 c_1 \beta_1 d_1]$

- iii)  $[a_1 \alpha_1 [b_1 + c_1] \beta_1 d_1] = [a_1 \alpha_1 b_1 \beta_1 d_1] + [a_1 \alpha_1 c_1 \beta_1 d_1]$
- iv)  $[a_1 \alpha_1 b_1 \beta_1 [c_1 + d_1]] = [a_1 \alpha_1 b_1 \beta_1 c_1] + [a_1 \alpha_1 b_1 \beta_1 d_1]$

Also the mapping  $T_2 \times \Gamma_2 \times T_2 \times \Gamma_2 \times T_2 \rightarrow T_2$  which maps  $(x_2, \alpha_2, y_2, \beta_2, z_2) \rightarrow [x_2 \alpha_2 y_2 \beta_2 z_2]$  satisfying the conditions

- i)  $[a_2 \alpha_2 b_2 \beta_2 c_2] \gamma_2 d_2 \delta_2 e_2 = [a_2 \alpha_2 [b_2 \beta_2 c_2 \gamma_2 d_2] \delta_2 e_2] = [a_2 \alpha_2 b_2 \beta_2 [c_2 \gamma_2 d_2 \delta_2 e_2]]$
  - ii)  $[a_2 + b_2] \alpha_2 c_2 \beta_2 d_2 = [a_2 \alpha_2 c_2 \beta_2 d_2] + [b_2 \alpha_2 c_2 \beta_2 d_2]$
  - iii)  $[a_2 \alpha_2 [b_2 + c_2] \beta_2 d_2] = [a_2 \alpha_2 b_2 \beta_2 d_2] + [a_2 \alpha_2 c_2 \beta_2 d_2]$
  - iv)  $[a_2 \alpha_2 b_2 \beta_2 [c_2 + d_2]] = [a_2 \alpha_2 b_2 \beta_2 c_2] + [a_2 \alpha_2 b_2 \beta_2 d_2]$
- $\forall a_i, b_i, c_i, d_i, e_i \in T$  &  $\forall \alpha_i, \beta_i, \gamma_i, \delta_i \in \Gamma$  where  $i = 1, 2$

**Definition2.2:** An element 0 of a bi ternary  $\Gamma$ -semiring  $T$  is said to be an absorbing zero of  $T$  provided  $0 + x = x = x + 0$  and  $0 \alpha a \beta b = a \alpha 0 \beta b = a \alpha b \beta 0 = 0 \quad \forall a, b, x \in T$  and  $\alpha, \beta \in \Gamma$

**Example2.3:** Let  $T = 2Z \cup 3Z$  be the bi ternary semi-ring and  $\Gamma = N \cup Z$  be the additive commutative semi group then  $T$  satisfies the conditions of bi ternary  $\Gamma$ -semiring.

**Example2.4:** Let  $T = Q \cup Z$  be the bi ternary semiring and  $\Gamma = W \cup N$  be the additive commutative semi group then  $T$  satisfies the conditions of bi ternary  $\Gamma$ -semi-ring.

**Example2.5:** Let  $T$  be the set of all  $2 \times 2$  upper and lower triangular matrices over the set of all non-positive integers  $Z_0^-$  and  $\Gamma$  be the set of all  $2 \times 2$  matrices over set off all negative integers forms a bi ternary  $\Gamma$ -semiring.

**Definition2.6:** Let  $T = T_1 \cup T_2$  be bi ternary  $\Gamma$ -semiring. A non-empty sub set  $S = S_1 \cup S_2$  is said to be a bi ternary  $\Gamma$ - subsemiring of  $T$ . If both  $S_1, S_2$  are additive sub semigroup of  $T_1, T_2$  respectively, also  $a_1 \alpha_1 b_1 \beta_1 c_1 \in S_1$  and  $a_2 \alpha_2 b_2 \beta_2 c_2 \in S_2 \quad \forall a_i, b_i \in S$  &  $\alpha_i, \beta_i \in \Gamma$

**Definition2.7:** A non-empty sub set  $S = S_1 \cup S_2$  of a bi ternary  $\Gamma$ -semiring  $T$  is a bi ternary  $\Gamma$ -

subsemiring if and only if  $S_1 + S_1 \subseteq S_1$  and  $S_1 \Gamma S_1 \Gamma S_1 \subseteq S_1$ .

Also  $S_2 + S_2 \subseteq S_2$  and  $S_2 \Gamma S_2 \Gamma S_2 \subseteq S_2$

Theorem 2.8: A non-empty intersection of two bi ternary  $\Gamma$ -sub semirings of a bi ternary  $\Gamma$ -semiring  $T$  is a bi ternary  $\Gamma$ -sub semiring of  $T$ .

Proof: Let  $S_1, S_2$  be two bi ternary  $\Gamma$ -sub semirings of  $T$ .

Let  $a, b, c \in S_1 \cap S_2$  and  $\alpha, \beta \in \Gamma$

$$a, b \in S_1 \cap S_2 \Rightarrow a, b \in S_1 \text{ and } a, b \in S_2$$

Since  $S_1$  is a bi ternary gamma sub semi ring of  $T$ ,  $a, b \in S_1$  then  $a + b \in S_1$

also  $S_2$  is a bi ternary gamma sub semi ring of  $T$ ,  $a, b \in S_2$  then  $a + b \in S_2$ , So  $a + b \in S_1 \cap S_2$

Here  $a, b, c \in S_1 \cap S_2$  then  $a, b, c \in S_1$  and  $a, b, c \in S_2$

$S_1$  is a bi ternary gamma sub semi ring of  $T$ ,  $a, b, c \in S_1$  and  $\alpha, \beta \in \Gamma$  then  $a\alpha b\beta c \in S_1$

also  $S_2$  is a bi ternary gamma sub semi ring of  $T$ ,  $a, b, c \in S_2$  and  $\alpha, \beta \in \Gamma$  then  $a\alpha b\beta c \in S_2$

So,  $a\alpha b\beta c \in S_1 \cap S_2 \forall a, b, c \in S_1 \cap S_2$  and  $\alpha, \beta \in \Gamma$

Hence  $S_1 \cap S_2$  is a bi ternary  $\Gamma$ -sub semi ring of  $T$ .

The intersection of two bi ternary  $\Gamma$ -sub semi ring of  $T$  is a bi ternary  $\Gamma$ -sub semi ring of  $T$ .

Theorem 2.9: The non-empty intersection of any family of bi ternary  $\Gamma$ -sub semi ring of a bi ternary  $\Gamma$ -semi ring  $T$  is a bi ternary  $\Gamma$ -sub semi ring of  $T$ .

Proof: Let  $\{S_\alpha\}_{\alpha \in \Delta}$  be a family of bi ternary  $\Gamma$ -subsemirings of  $T$ , And  $S = \bigcap_{\alpha \in \Delta} S_\alpha$ .

Let  $a, b, c \in S \Rightarrow a, b, c \in \bigcap_{\alpha \in \Delta} S_\alpha \Rightarrow a, b, c \in S_\alpha \forall \alpha \in \Delta$

$a, b, c \in S_\alpha$ ,  $S_\alpha$  is a bi ternary  $\Gamma$ -subsemiring  $T \Rightarrow a + b \in S_\alpha$  and  $a\alpha b\beta c \in S_\alpha \forall \alpha, \beta \in \Gamma$

Now  $\Rightarrow a + b \in S_\alpha$  and  $a\alpha b\beta c \in S_\alpha, \forall \alpha \in \Delta \Rightarrow a + b \in \bigcap_{\alpha \in \Delta} S_\alpha$  and  $a\alpha b\beta c \in \bigcap_{\alpha \in \Delta} S_\alpha$

Then  $a + b \in S$  and  $a\alpha b\beta c \in S$ .

Therefore  $S$  is a bi ternary  $\Gamma$ -sub-semi-ring of  $T$ .

Example 2.10: Let  $T = R \cup Z_0^-$  be bi ternary semi ring and  $\Gamma$  be the set of all natural numbers forms a bi ternary  $\Gamma$ -semiring. Let  $S = \{2k, k \in Z\} \cup \{3k, k \in Z\}$  be the bi ternary  $\Gamma$ -sub semi-ring.

Definition 2.11: A non-empty subset  $I = I_1 \cup I_2$  of a bi ternary  $\Gamma$ -semiring  $T$  is claimed to be an left ternary  $\Gamma$  ideal of  $T$  if

$$i) a, b \in I \Rightarrow a + b \in I$$

$$ii) a, b \in T, i \in I \text{ and } \alpha, \beta \in \Gamma \text{ then } i\alpha a\beta b \in I$$

Definition 2.12: A non-empty subset  $I = I_1 \cup I_2$  of a bi ternary  $\Gamma$ -semiring  $T$  is claimed to be an lateral ternary  $\Gamma$  ideal of  $T$  if

$$i) a, b \in I \Rightarrow a + b \in I$$

$$ii) a, b \in T, i \in I \text{ and } \alpha, \beta \in \Gamma \text{ then } a\alpha i\beta b \in I$$

Definition 2.13: A non-empty subset  $I = I_1 \cup I_2$  of a bi ternary  $\Gamma$ -semiring  $T$  is said to be an right ternary  $\Gamma$  ideal of  $T$  if

$$i) a, b \in I \Rightarrow a + b \in I$$

$$ii) a, b \in T, i \in I \text{ and } \alpha, \beta \in \Gamma \text{ then } i\alpha a\beta b \in I$$

Definition 2.14: A non-empty subset  $I = I_1 \cup I_2$  of a bi ternary  $\Gamma$ -semiring  $T$  is said to be two sided ternary  $\Gamma$  ideal of  $T$  if

$$i) a, b \in I \Rightarrow a + b \in I$$

$$ii) a, b \in T, i \in I \text{ and } \alpha, \beta \in \Gamma \text{ then } i\alpha a\beta b \in I \text{ and } a\alpha i\beta b \in I$$

Definition 2.15: A non-empty subset  $I = I_1 \cup I_2$  of a bi ternary  $\Gamma$ -semiring  $T$  is said to be ternary  $\Gamma$  ideal of  $T$  if

$$i) a, b \in I \Rightarrow a + b \in I$$

$$ii) a, b \in T, i \in I \text{ and } \alpha, \beta \in \Gamma \text{ then } i\alpha a\beta b \in I, a\alpha i\beta b \in I \text{ and } a\alpha i\beta b \in I$$

Definition 2.16: A bi ternary  $\Gamma$  ideal  $I$  of a bi ternary  $\Gamma$  semiring is said to be a principal ternary  $\Gamma$  ideal provided  $I$  is a ternary  $\Gamma$  ideal generated by  $\{a\}$  for some  $a \in T$ . It is denoted by  $J(a)$  or  $\langle a \rangle$

**Definition2.17:** A left ternary  $\Gamma$  ideal  $I$  of a bi ternary  $\Gamma$  semiring  $T$  is said to be the principal left ternary  $\Gamma$  ideal of  $T$

If  $I$  is generated by  $a$  if  $I$  is a left ternary  $\Gamma$  ideal generated by  $\{a\}$  for some  $a \in T$ . It is denoted by  $L(a)$  or  $\langle a \rangle_l$ .

**Note2.18:** A non-empty sub set  $I$  of a bi ternary  $\Gamma$  semiring  $T$  is said to be left ternary  $\Gamma$  ideal of

$T$  if and only if  $I$  is additive sub semigroup of  $T$ .  $T \Gamma T \Gamma T \subseteq I$

**Theorem2.19:** If  $T$  is a bi ternary  $\Gamma$  semiring and  $a \in T$  then  $\langle a \rangle_l = \{\sum_{i=1}^n r_i \alpha_i t_i \beta_i a + na : r_i, t_i \in T, \alpha_i, \beta_i \in \Gamma \text{ and } n \in \mathbb{Z}_0^+\}$  where  $\sum$  denotes a finite sum and  $\mathbb{Z}_0^+$  is the set of all positive integers with zero.

**Proof:** Let  $A = \{\sum_{i=1}^n r_i \alpha_i t_i \beta_i a + na : r_i, t_i \in T, \alpha_i, \beta_i \in \Gamma \text{ and } n \in \mathbb{Z}_0^+\}$

Let  $a, b \in A$  then  $a = \sum r_i \alpha_i t_i \beta_i a + na$  and  $b = \sum r_j \alpha_j t_j \beta_j a + na$  for all  $r_i, t_i, r_j, t_j \in T$  &  $\alpha_i, \beta_i, \alpha_j, \beta_j \in \Gamma$

Now  $a + b = \sum r_i \alpha_i t_i \beta_i a + na + \sum r_j \alpha_j t_j \beta_j a + na$  then  $a + b$  is a finite sum.

There fore  $a + b \in A$  and hence  $A$  is additive sub semigroup of  $T$ .

For  $t_1, t_2 \in T$  &  $a \in A$  then  $t_1 a t_2 \beta a = t_1 a t_2 \beta \sum r_i \alpha_i t_i \beta_i a + na$

$$= \sum r_i \alpha_i t_i \beta_i t_1 a t_2 \beta a + n t_1 a t_2 \beta a \in A$$

There fore  $t_1 a t_2 \beta a \in A$  and hence  $A$  is a left ternary  $\Gamma$  ideal of  $T$ .

Let  $L$  be a left ternary  $\Gamma$  ideal of  $T$  containing  $a$ .

Let  $r \in A \Rightarrow r = \sum r_i \alpha_i t_i \beta_i a + na$  for  $r_i, t_i \in T, \alpha_i, \beta_i \in \Gamma, n \in \mathbb{Z}_0^+$

If  $r = \sum r_i \alpha_i t_i \beta_i a + na \in L$

There fore  $A \subseteq L$ , and hence  $A$  is a smallest left ternary  $\Gamma$  ideal of  $T$  containing  $a$ .

There fore  $A = L(a) = \{\sum_{i=1}^n r_i \alpha_i t_i \beta_i a + na : r_i, t_i \in T, \alpha_i, \beta_i \in \Gamma \text{ and } n \in \mathbb{Z}_0^+\}$

**Note2.20:** A non-empty sub set  $I$  of a bi ternary  $\Gamma$  semiring  $T$  is said to be lateral ternary  $\Gamma$  ideal of  $T$  if and only if  $I$  is additive sub semigroup of  $T$  &  $T \Gamma I \Gamma T \subseteq I$

**Definition2.21:** A lateral ternary  $\Gamma$  ideal  $I$  of a bi ternary  $\Gamma$  semiring  $T$  is said to be the principal lateral ternary  $\Gamma$  ideal of  $T$  if  $I$  is generated by  $a$  if  $I$  is a lateral ternary  $\Gamma$  ideal generated by  $\{a\}$  for some  $a \in T$ . It is denoted by  $M(a)$  or  $\langle a \rangle_m$ .

**Theorem2.22:** If  $T$  is a bi ternary  $\Gamma$  semiring and  $a \in T$  then  $\langle a \rangle_m = \{\sum_{i=1}^n r_i \alpha_i a \beta_i t_i + \sum_{j=1}^n u_j \alpha_j v_j \beta_j a \gamma_j p_j \delta_j q_j + na : r_i, t_i, u_j, v_j, p_j, q_j \in T, \alpha_i, \beta_i, \alpha_j, \beta_j, \gamma_j, \delta_j \in \Gamma \text{ and } n \in \mathbb{Z}_0^+\}$  where  $\sum$  denotes a finite sum and  $\mathbb{Z}_0^+$  is the set of all positive integers with zero.

### 3. Quasi ternary $\Gamma$ ideal and bi ternary $\Gamma$ ideal in a bi ternary $\Gamma$ semiring

**Definition3.1:** An additive semigroup  $Q$  of a bi ternary  $\Gamma$  semi ring  $T$  is called quasi ternary  $\Gamma$  ideal of  $T$  if  $Q \Gamma T \Gamma T \cap (T \Gamma Q \Gamma T + T \Gamma T \Gamma Q \Gamma T \Gamma T) \cap T \Gamma T \Gamma Q \subseteq Q$ .

**Note 3.2:** Every quasi bi ternary  $\Gamma$  ideal of a bi ternary  $\Gamma$  semi ring  $T$  is a bi ternary  $\Gamma$  sub semi ring of  $T$ .

**Lemma3.3:** Every left, right, lateral bi ternary  $\Gamma$  ideal of a bi ternary  $\Gamma$  semiring  $T$  is a quasi bi ternary  $\Gamma$  ideal of  $T$

**Proof:** Assume that  $Q$  is a left bi ternary  $\Gamma$  ideal of  $T$ . Then  $T \Gamma T \Gamma Q \subseteq Q$ , but  $Q \Gamma T \Gamma T \cap (T \Gamma Q \Gamma T + T \Gamma T \Gamma Q \Gamma T \Gamma T) \cap T \Gamma T \Gamma Q \subseteq Q$ . Hence  $Q$  is a quasi bi ternary  $\Gamma$  Ideal of  $T$ . Similarly we can prove the remaining parts.

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