

## Cototal Litact Domination in Graphs

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### Abstract

**Introduction:** In the realm of graph theory, the study of domination has been a fundamental pursuit, exploring the notion of how certain vertices in a graph control or influences others. One such variant is cototal domination, which extends the concept of domination to consider sets of vertices that together control the entire graph. In this paper, we delve into the intriguing interplay of cototal domination within litact graphs.

**Objectives:** This work's primary goal is to ascertain the new domination parameter known as cototal domination on a litact graph.

### Motivation

The exploration of cototal domination within litact graphs arises from the intersection of two intriguing areas of graph theory. Understanding how cototal domination manifests within litact graphs not only contributes to the theoretical foundation of graph domination but also unveils practical implications for network design, fault tolerance, and optimization.

**Results:** Several results on cototal domination on a litact graph are obtained in the present study, both in terms of different graph  $G$  parameters (vertices, edges, diameter, maximum degree, and many more) and domination parameters (connected domination, total domination, edge domination, and many more). A few fundamental definitions, findings, and the ideas of several dominations parameters have been used in this.

The current study aims to determine some relations between cototal on a litact graph and graph  $G$ 's different parameters, and domination parameters. Furthermore, results comparable to Nordhaus-Gaddum's were also established.

### Summary of Findings

We have established the formal definitions and properties of cototal domination and litact graphs, laying the groundwork for our analysis. Our computational exploration has revealed the complexity of cototal domination in litact graphs and highlighted the challenges associated with determining cototal domination sets efficiently.

### Conclusion

In conclusion, our exploration of cototal domination in litact graphs has shed light on the intricate dynamics of graph control within this unique class of graphs. Through our investigation, we have achieved several key insights and contributions.

**Keywords:** Graphs, Litact graph, Litact domination number, cototal litact domination number

### 1. Introduction

In advanced era, graph has grown up as the most effective leading tool of mathematics in various subjects. In the interim it has also turned out as a full-fledged substantial field of Mathematics. In graphs, the domination theory has extensive applications in different fields. Now days, it can be considered as the most basic concepts in the theory of graphs and its practical significance in web graphs, social networks, biological patterns

etc., shows the increased curiosity towards the topic. In general the domination appears in problems like locating the facilities in which one aims to reduce the distance a person requires to reach the nearest facility when the facility is fixed. A similar kind of problem arises where the maximum distance to a particular facility remains constant and a person tries to see that everyone is facilitated with the minimum number of facilities. Also the concept of domination arises in the

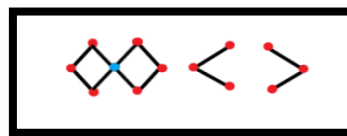
problems of obtaining a set of representatives, in electrical networks or in land survey etc., Many graph theorists (Konig, Ore, Bauer, Harary, Lasker, Berge, Cockayne, Hedetniemi, Alavi, Allan, Chartrand, Kulli, Muddebihal, Sampathkumar, Walikar, Armugam, Acharya, Neeralgi, Nagaraja Rao, Vangipuram) put forth many interesting concepts of domination theory and related topics. Some of them worked in introducing a new domination parameter and obtaining the boundaries of the defined variable in terms of the graph parameters. And some of them worked on the graph algorithms to study the complexity results of domination parameters. The combination of domination with other graph theoretical properties resulted in several domination parameters and many of them are defined by inflicting an added constraint on the dominating set.

The concept of Cototal domination number in graphs was introduced by Kulli, Janakiram and Iyer in [5]. A study on "Connected cototal domination number of a graph" was given by B. Basavana Goud and S.M.Hosamani in [3]. Some results on "Equitable Cototal domination in graphs" is obtained by B. Basavanagoud and Vijay V Telli in [11]. A few bounds on "Global Cototal Domination in Graphs" and "1, Semi global Cototal Domination on Graphs" were obtained by T.Nicholas, T. Sheeba Helen in [8 & 9]. A survey of discussion was given on "Cototal edge domination number" and the "Cartesian product of independent cototal edge domination number" by Anupama S.B., Y.B. Maralabhavib and Venkanagouda M. Goudarin [2]. A new graph valued function "Degree Equitable Connected cototal dominating graph" on a graph was defined and some of the properties were studied by Shigehalli V.S and Vijayakumar Patil in [10]. "Inverse and Disjoint Connected Cototal Domination number of the Jump Graph of a Graph" was studied by Annie Jasmine S.E., K. Ameenal Bibi in [1].

**2. Preliminaries**

Only connected, simple, finite, non-trivial, and undirected graphs are examined in this analysis. The definitions of the undefined notations used in the article are from F.Harary and V.R.Kulli.

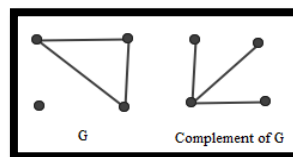
**2.1: Cut vertex:** A vertex which divides the entire graph into two or more components through its removal is called cut vertex.



**Figure 2.1**

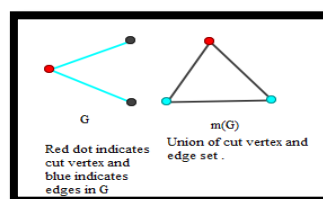
The blue coloured vertex in the figure is a cut vertex as it leads to more components after its removal from original graph.

**2.2: Complement:** A graph  $\bar{G}$  so formed with  $V(G) = V(\bar{G})$ , and for any  $u, v \in V(G)$ , if  $uv \in E(G)$  ,  $uv \notin E(\bar{G})$  then the graph  $\bar{G}$  is complement to the given graph.



**Figure 2.2**

**2.3: Litact Graph:** A graph denoted by  $m(G)$  with  $V(m(G)) = E(G) \cup C(G)$  where  $C(G)$  is the graph cut vertex set whereas the edges of the graph are formed with the incidence and adjacency of the edges and cut vertices of the graph  $G$  is called the litact graph of the given graph.



**Figure 2.3**

**2.4: Litact domination number:** A set  $D \subseteq V(m(G))$  of vertices dominating in  $m(G)$  is litact dominating, if every vertex  $u$  in  $V(m(G)) - D$  is connected to some vertex  $v$  in  $D$ . The number  $\gamma_m(G) = \min|D|$  is the litact domination number. Example: In figure 4,  $\gamma_m(G) = 1$

**2.5: Cototal litact domination number:** A set  $D$  of vertices dominating in  $m(G)$  is Cototal dominating if there exists no vertex  $v$  in the induced subgraph of  $V(m(G)) - D$  with  $d(v) = 0$ . The Cototal litact domination number of  $G$ ,  $\gamma_{ctm}(G)$  is given by  $\gamma_{ctm}(G) = \min|D|$ .

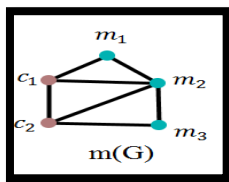


Figure 2.4 cototal Litact Domination Number

In the Figure 4 Cototal litact dominating set,  $D = \{m_2\}$  and so  $\gamma_{ctm}(G) = 1$

### 3. Existing Results

**Theorem 3.1:** For each graph  $G$ ,  $\gamma(G) \leq \beta_0[G]$ .

**Theorem 3.2:** For each graph  $G$ ,  $p - q \leq \gamma(G)$  only if each component of  $G$  is a star.

**Theorem 3.3:** For any graph  $G$ ,  $diam(G) - 1 \leq \gamma_c(G) \leq p - \Delta(G)$

**Theorem 3.4:** For any connected graph  $G$  with at least two vertices,  $\gamma_t(G) \leq \gamma_c(G)$

### 4. Proved Results

#### 4.1 Particular values

The exact values of the defined variant, cototal litact domination number for various graph families is given below.

(i) For each Cycle graph  $C_p$ , with a minimum of three vertices,  $\gamma_{ctm}(C_p) = p - 2$

(ii) For each Wheel graph  $W_p$ , with at least four vertices,  $\gamma_{ctm}(W_p) = \lfloor \frac{p}{3} \rfloor$

(iii) For each Path  $P_p$ , with at least four vertices,  $\gamma_{ctm}(P_p) = \lfloor \frac{p-1}{2} \rfloor$

(iv) For any complete graph  $K_p$ , with at least three vertices,  $\gamma_{rem}(K_p) = \lfloor \frac{p}{4} \rfloor$

(v) For any complete bipartite graph  $K_{m,n}$ , with  $m, n \geq 2$ , vertices,  $\gamma_{rem}(K_{m,n}) = \min\{m, n\}$

(vi) For each Star graph  $K_{1,p}$ , with a minimum of three vertices,  $\gamma_{rem}(K_{1,p}) = 1$

The result below relates  $\gamma_{ctm}(G)$  with  $p$

**Theorem 4.2:** In each graph,  $\gamma_{ctm}(G) \geq \frac{p-2}{6}$ .

**Proof :** Let  $D$  be a set with vertices of the litact graph  $m(G)$  which is formed by joining the edges and cut vertices of the graph  $G$  when they are incident or adjacent with one another. If there exists no vertex  $u$  in the graph  $(V(m(G)) - D)$  with  $deg(u) = 0$  then the set  $D$  will be a cototal dominating set in  $m(G)$  and  $|D| = \gamma_{ctm}(G)$ . Then we have

$$6 |D| \geq V(m(G)) - 2$$

(1)

and also it is clear that

$$|V(m(G))| \geq p$$

(2)

and hence from (1) and (2) we get

$$6 |D| \geq p - 2 \text{ which implies } \gamma_{ctm}(G) \geq \frac{p-2}{6}$$

The next theorem gives a lower bound and upper bound in terms of  $q$ ,  $\Delta'(G)$  and  $\delta'(G)$

**Theorem 4.3:** In any graph  $G$ ,  $\frac{q}{\Delta'(G)+2} \leq \gamma_{ctm}(G) \leq q - \delta'(G)$  except for path and complete bipartite graph with  $p \leq 4$  vertices.

**Proof:** Let  $u \in E(G)$  with minimum degree in  $G$  and so  $deg(u) = \delta'(G)$ . By the definition of litact graph  $m(G)$ , there exists a vertex  $v$  in  $m(G)$  corresponding to the edge  $u$  in  $G$  and so  $u = v \in V(m(G))$ . Let  $D$  be the cototal litact dominating set in  $G$  such that  $|D| = \gamma_{ctm}(G)$ .

Case (i):  $\delta'(G) \leq 2$

We have clearly  $|V(m(G))| \geq 2$  &  $q - \gamma_{ctm}(G) \geq 2$ .

So,  $\gamma_{ctm}(G) \leq q - 2 \leq q - \delta'(G)$ . Hence the result.

Case (ii) :  $\delta'(G) > 2$

For any edge  $r \in N(u)$ ,  $r = s \in N(v)$ .

Let  $D \subseteq V(m(G)) - N(v) \cup \{s\}$ . Then  $D$  will be a cototal litact dominating set in  $G$ . So,  $|D| = \gamma_{ctm}(G)$ .

$$\text{Hence } |D| \leq |V(m(G))| - |N(v) \cup \{s\}|$$

$$\Rightarrow \gamma_{ctm}(G) \leq q - (\delta'(G) + 1) + 1 = q - \delta'(G)$$

$$\Rightarrow \gamma_{ctm}(G) \leq q - \delta'(G)$$

(1)

By the definition of dominating set, every vertex of  $V(m(G)) - D$  is adjacent to at least one vertex in  $D$ . And so every vertex in  $V(m(G)) - D$  contributes at least one to the sum of degrees of vertices of  $D$  and therefore  $|V(m(G)) - D| \leq \sum_{v \in D} deg(v)$ . Also let  $e \in E(G)$  with  $deg(e) = \Delta'(G)$ .

We have,

$$q - \gamma_{ctm}(G) = |V(m(G)) - D| \leq$$

$$\sum_{v \in D} deg(v)$$

$$\text{and } \sum_{v \in D} deg(v) \leq$$

$$2 \gamma_{ctm}(G) \cdot \Delta'(G)$$

$$\text{Therefore } \frac{q}{\Delta'(G)+2} \leq \gamma_{ctm}(G) \quad (2)$$

From (1) and (2) we get the required result.

The following theorem relates  $\gamma_{ctm}(G)$  with  $p$  and  $\Delta(G)$

**Theorem 4.4:** In any graph  $G$ ,  $\frac{p}{\Delta(G)+4} \leq \gamma_{ctm}(G)$ .

**Proof :** Let  $D \subseteq V(m(G))$  be a cototal dominating set in the litact graph  $m(G)$ . Then  $|D| = \gamma_{ctm}(G)$ . Also let  $v$  be the vertex in  $G$  with degree,  $\deg(v) = \Delta(G)$ .

Clearly we have,  $p \leq |V(m(G))|$  and  $|V(m(G))| \leq \gamma_{ctm}(G)(\Delta(G) + 4)$  And therefore it implies  $p \leq \gamma_{ctm}(G)(\Delta(G) + 4)$  and hence  $\frac{p}{\Delta(G)+4} \leq \gamma_{ctm}(G)$ .

The next theorem is a relation for  $\gamma_{ctm}(G)$  interms of  $\gamma_t(G)$  and  $\beta_0(G)$

**Theorem 4.5:** In any graph  $G$ ,  $\gamma_{ctm}(G) \leq \gamma_t(G) + \beta_0(G)$ .

**Proof :** Let  $M$  be a set of vertices in  $G$  in which for any two vertices  $x, y$  with  $dist(x, y) \geq 2$  we have  $N(x) \cap N(y) = \emptyset$ ,  $a \in V(G) - M$ . Then  $|M| = \beta_0(G)$ . Let  $R \subseteq V(G) - M$  be the subset of vertices in  $G$  which forms a graph with no isolated vertices. If  $R$  is a total dominating set in  $G$  then  $|R| = \gamma_t(G)$ . Otherwise,  $R \cup \{w\}$  forms  $\gamma_t$ -set for any vertex  $w \in N(R)$ . Now by the definition of litact graph  $m(G)$ , the edges which are incident with the vertices of  $R$  corresponds to the vertices  $R' = \{v_1, v_2, \dots, v_i\} \subseteq V(m(G))$ . Further let  $D \subseteq R'$  be the set of vertices such that the induced subgraph of  $V(m(G)) - D$  has no isolated vertices. Then  $D$  forms cototal dominating set in  $m(G)$ . Thus clearly we have  $|D| \leq |R \cup \{w\}| + |M|$  and hence  $\gamma_{ctm}(G) \leq \gamma_t(G) + \beta_0(G)$ .

The next theorem gives a relation for  $\gamma_{ctm}(G)$  with  $\delta(G)$

**Theorem 4.6:** In any graph  $G$ ,  $\gamma_{ctm}(G) + 2 \geq \delta(G)$ .

**Proof :** Let  $D \subseteq V(m(G))$  be the set of vertices such that the subgraph

$\langle V(m(G)) - D \rangle$  has no isolated vertices. Then  $|D| = \gamma_{ctm}(G)$ . Let  $v$  be a vertex in  $G$  with  $deg(v) = \delta(G)$ .

Then we have clearly

$$|D| \leq |V(m(G))| \quad (1)$$

Also we have

$$\begin{aligned} \delta(G) &< p \leq |V(m(G))| \\ \Rightarrow \delta(G) &< |V(m(G))| \end{aligned} \quad (2)$$

Subtracting (1) from (2) we get

$$\begin{aligned} \delta(G) \leq |D| &\Rightarrow \delta(G) \leq \gamma_{ctm}(G) < \gamma_{ctm}(G) + 2 \\ \text{which gives } &\gamma_{ctm}(G) + 2 \geq \delta(G). \end{aligned}$$

The next theorem relates  $\gamma_{ctm}(G)$  with  $p$  &  $q$ .

**Theorem 4.7:** In any graph  $G$ ,  $\gamma_{ctm}(G) \leq 4p - q$ .

**Proof :** Let  $D$  be a minimum dominating set in  $m(G)$  and  $D' \subseteq V(m(G)) - D$  be the corresponding cototal dominating set of  $m(G)$  such that  $|D| = \gamma_m(G)$  and  $|D'| = \gamma_{ctm}(G)$ . If  $D$  is a dominating set in  $m(G)$  then there exists at least  $\gamma_{ctm}(G)$  edges between  $D$  and  $D'$ . If  $D'$  is a cototal dominating set of  $m(G)$  then every vertex in  $V(m(G)) - D - D'$  has at least one edge to  $D$  and one edge to  $D'$ . Then the number of edges from  $V(m(G)) - D - D'$  is at least  $2|V(m(G)) - D - D'|$ .

Hence  $q < 2[|V(m(G))| - |D| - |D'|] + \gamma_{ctm}(G)$

$$\Rightarrow q + 2\gamma_m(G) + \gamma_{ctm}(G) < 2|V(m(G))|$$

Since  $|V(m(G))| \leq 2p$  we have

$$q + 2\gamma_m(G) + \gamma_{ctm}(G) < 4p \quad (1)$$

Since  $\gamma_m(G) \leq \gamma_{ctm}(G)$  and from (1) we have

$$\gamma_{ctm}(G) \leq \frac{4p-q}{3} < 4p - q$$

And hence  $\gamma_{ctm}(G) \leq 4p - q$ .

The next theorem relates  $\gamma_{ctm}(G)$  with  $diam(G)$  and  $\beta_0(G)$

**Theorem 4.8:** In any graph  $G$ ,  $\gamma_{ctm}(G) \leq diam(G) + \beta_0(G)$ .

**Proof:** From Theorem 3.3 we have  $diam(G) - 1 \leq \gamma_c(G)$  (1)

From Theorem 3.4 we have

$$\begin{aligned} \gamma_t(G) &\leq \gamma_c(G) \\ (2) \end{aligned}$$

From Theorem 4.5 we have

$$\gamma_{ctm}(G) \leq \gamma_t(G) + \beta_0(G) \quad (3)$$

Subtracting (2) from (1) we get

$$\begin{aligned} diam(G) - 1 &\leq \gamma_t(G) \\ (4) \end{aligned}$$

Subtracting (4) from (3) we get

$$\gamma_{ctm}(G) \leq diam(G) + \beta_0(G) - 1 < diam(G) + \beta_0(G)$$

Hence the result.

The following theorem is a relation for  $\gamma_{ctm}(G)$  with  $\gamma_c(G)$

**Theorem 4.9:** For each graph  $G$ ,  $\gamma_{ctm}(G) \leq 2\gamma_c(G)$ .

**Proof:** From Theorem 3.1 we have

$$\gamma(G) \leq \beta_0(G) \quad (1)$$

Since  $\gamma(G) \leq \gamma_c(G)$

(2)

Subtracting (2) from (1) we get

$$\gamma_c(G) \leq \beta_0(G) \quad (3)$$

From Theorem 4.5 we have

$$\gamma_{ctm}(G) \leq \gamma_t(G) + \beta_0(G) \quad (4)$$

Subtracting (3) from (4) we get

$$\gamma_{ctm}(G) \leq \gamma_t(G) + \gamma_c(G) \quad (5)$$

From Theorem 3.4 we have

$$\gamma_t(G) \leq \gamma_c(G) \quad (6)$$

From (5) and (6) we get the result.

The following theorem is a relation for  $\gamma_{ctm}(G)$  with  $diam(G)$

**Theorem 4.10:** For each graph  $G$ ,  $\left\lfloor \frac{\gamma_{ctm}(G)}{2} \right\rfloor \leq diam(G)$ .

**Proof:** From Theorem 3.3 we have

$$2(diam(G) - 1) \leq 2\gamma_c(G) \quad (1)$$

From Theorem 4.9 we get

$$\gamma_{ctm}(G) \leq 2\gamma_c(G) \quad (2)$$

Subtracting (1) from (2) we get

$$\gamma_{ctm}(G) \leq 2(diam(G) - 1) < 2 diam(G)$$

$$\left\lfloor \frac{\gamma_{ctm}(G)}{2} \right\rfloor \leq \frac{\gamma_{ctm}(G)}{2} < diam(G)$$

Hence the result.

The succeeding result relates  $\gamma_{ctm}(G)$  with  $\gamma(G)$  and  $\gamma_t(G)$

**Corollary 4.1:** For each graph  $G$ ,  $\gamma_{ctm}(G) \leq \gamma(G) + \gamma_t(G)$ .

**Proof:** From Theorem 3.1 we have

$$\gamma(G) \leq \beta_0(G) \quad (1)$$

From Theorem 4.5 we have

$$\gamma_{ctm}(G) \leq \gamma_t(G) + \beta_0(G) \quad (2)$$

Subtracting (1) from (2) we get the result.

The following theorem relates  $\gamma_{ctm}(G)$ ,  $p$  and  $\gamma(G)$

**Corollary 4.2:** For each graph  $G$ ,  $p - \gamma_{ctm}(G) \leq \gamma(G) + \delta'(G)$ .

**Proof:** From Theorem 4.3, we have

$$\gamma_{ctm}(G) \leq q - \delta'(G) \quad (1)$$

From Theorem 3.2,

$$p - q \leq \gamma(G) \Rightarrow p \leq q + \gamma(G) \quad (2)$$

Subtracting (1) from (2) we get the result.

The next theorem relates  $\gamma_{ctm}(G)$  and  $q$

**Theorem 4.11:** For each graph  $G$ ,  $\gamma_{ctm}(G) < q$ .

**Proof:** From Theorem 4.2 we have,  $\gamma_{ctm}(G) \geq \frac{p-2}{6}$

$$\text{Clearly we have, } \frac{p-2}{6} < p \quad (2)$$

Subtracting (2) from (1) we get  $\gamma_{ctm}(G) < p$  and

$$\text{so } 4\gamma_{ctm}(G) < 4p \quad (3)$$

From Theorem 4.7,  $\gamma_{ctm}(G) \leq 4p - q$

Subtracting (4) from (3) we get,

$$3\gamma_{ctm}(G) \leq q \Rightarrow \gamma_{ctm}(G) \leq \frac{q}{3} < q$$

Hence the result.

The following corollary relates  $\gamma_{ctm}(G)$  and  $\alpha_1(G)$ ,  $\beta_1(G)$

**Corollary 4.3:** For each graph  $G$ ,  $\gamma_{ctm}(G) < \alpha_1(G) + \beta_1(G) = \alpha_0(G) + \beta_0(G)$ .

**Proof:** From Theorem 4.2 we have,

$$\gamma_{ctm}(G) \geq \frac{p-2}{6} \quad (1)$$

$$\text{Clearly we have, } \frac{p-2}{6} < p \quad (2)$$

Subtracting (2) from (1) we get  $\gamma_{ctm}(G) < p$

By known theorem we have,  $\alpha_1(G) + \beta_1(G) = p$ ,  $\alpha_0(G) + \beta_0(G) = p$  and from (3) we get the result.

The succeeding theorem relates  $\gamma_{ctm}(G)$  and  $\alpha_0(G)$

**Theorem 4.12:** For each graph  $G$ ,  $\gamma_{ctm}(G) < 2\alpha_0(G)$ .

**Proof:** From Theorem 4.9

$$\left\lfloor \frac{\gamma_{ctm}(G)}{2} \right\rfloor \leq diam(G) \quad (1)$$

Since  $\left\lfloor \frac{\gamma_{ctm}(G)}{2} \right\rfloor < \frac{\gamma_{ctm}(G)}{2}$  and from (1) we get

$$\frac{\gamma_{ctm}(G)}{2} \leq diam(G) \quad (2)$$

(2)

From Theorem 4.7 we have

$$\gamma_{ctm}(G) \leq diam(G) + \beta_0(G) \quad (3)$$

(3)

From (2) and (3), we get,

$$\frac{\gamma_{ctm}(G)}{2} \leq \beta_0(G) \quad (4)$$

From Corollary 4.3 we get,  $\gamma_{ctm}(G) < \alpha_0(G) + \beta_0(G)$  (5)

Subtracting (4) from (5) we get the result.

The theorem below establishes an upper bound for  $\gamma_{ctm}(G)$

**Theorem 4.13:** In a graph  $G$ ,  $\gamma_{ctm}(G) \leq \beta_1(G) + 1$ , where  $\beta_1$  is the edge independence number of  $G$ .

**Proof:** Let  $B = \{e_1, e_2, \dots, e_i\}$  be the set of maximum number of edges which are not adjacent to each other. Then  $|B| = \beta_1(G)$ . Consider a vertex set  $D$  of  $m(G)$  corresponding to the edges in  $B$ . Then clearly  $D$  is a  $\gamma_{ctm}$ -set in  $m(G)$  and so  $|D| = \gamma_{ctm}(G)$ .

From Theorem 4.2 we have,

$$\gamma_{ctm}(G) \geq \frac{p-2}{6} \quad (1)$$

Clearly we have,  $\frac{p-2}{6} < p \quad (2)$

Subtracting (2) from (1) we get  $\gamma_{ctm}(G) < p$

That is,  $\gamma_{ctm}(G) = |D| < p = |V(G)| \Rightarrow |D| < |V(G)| < |V(G)| + |B| + 1 \quad (3)$

Also  $|B| < |V(G)| + |B| \quad (4)$

Subtracting (4) from (3) we get  $|D| - |B| \leq 1$

Hence the result  $\gamma_{ctm}(G) \leq \beta_1(G) + 1$ .

The following is another upper bound for  $\gamma_{ctm}$

**Theorem 4.14:** In a graph  $G$ ,  $\gamma_{ctm}(G) \leq \alpha_1(G)$ , where  $\alpha_1$  is the edge cover of  $G$ .

**Proof:** Let  $R = \{e_1, e_2, \dots, e_j\}$  be the set of edges which are which covers all the vertices of  $G$ . Then  $|R| = \alpha_1(G)$ . Consider a vertex set  $D$  of  $m(G)$  where the induced sub graph  $\langle V(m(G)) - D \rangle$  has no isolates. Then clearly  $D$  is a  $\gamma_{ctm}$ -set in  $m(G)$  and so  $|D| = \gamma_{ctm}(G)$ .

From Theorem 4.2 we have

$$\gamma_{ctm}(G) \geq \frac{p-2}{6} \quad (1)$$

Clearly we have,  $\frac{p-2}{6} < p \quad (2)$

Subtracting (2) from (1) we get  $\gamma_{ctm}(G) < p$

We have clearly,  $\alpha_1(G) < \alpha_1(G) + \beta_1(G) = p$

That is  $\alpha_1(G) < p \quad (4)$

Subtracting (4) from (3), we get the result.

## 5. Nordhaus- Gaddum type results

**Theorem 5.1::**

In any graph  $G$ , (i)  $\gamma_{ctm}(G) + \gamma_{ctm}(\bar{G}) < (p - 2)^2$

(ii)  $\gamma_{ctm}(G) \cdot \gamma_{ctm}(\bar{G}) = p + q$

## 6. Discussion

This article introduces a strange domination parameter on the litact graph for stated graphs. The estimations of standard graphs and several general graphs were acquired. In addition, a range of results were found in the form of boundaries affixing the new variables to multiple graph variants. Because domination theory occupies

many fields of Science and Engineering and its application has been studied by many researchers who have made the domination area of the research field, the current work is worth study. While our study has made important strides in understanding cototal domination in litact graphs, several avenues for future research remain open. Further investigation into the computational complexity of cototal domination algorithms, as well as the development of heuristic approaches for large-scale graphs, could enhance the practical applicability of our findings. Additionally, exploring connections between cototal domination and other graph parameters, such as edge domination or total domination, may uncover deeper insights into the nature of graph control in litact graphs.

In closing, our examination of cototal domination in litact graphs has enriched our understanding of graph domination theory and its applications. We hope that our findings inspire continued exploration in this fascinating area of graph theory, paving the way for new discoveries and advancements in the field.

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