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A Mathematical Model for Prevention the Population Growth of Aphids in Plants

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Abstract:

The current article presents a mathematical model that illustrates how plant diseases pose a serious threat to biodiversity. Among insects, aphids and mazus are significant contributors to plant disease. Insects have a natural tendency to feed on plants and pose a threat to their growth rate, but plants also respond to these attacks by initiating their defense mechanisms. The population growth of Hemipetra like aphids and mazus discharges a type of lethal material that smeared over that surface of the leaf, preventing the individuals from further sucking and movements and so causing death due to starvation. Aphids are among the most conspicuous and important pests in green houses. Aphids are known to transmit over 100 different kinds of plant viruses, including beet mosaic, cabbage black ringspot, carnation latent, cauliflower mosaic, cherry ringspot, cucumber mosaic, onion yellow dwarf, pea wilt, potato Y, tobacco etch, tobacco mosaic, tomato spotted wilt, and turnip yellow mosaic. Insects are the main problem for our environment because they prevent the growth of plants and destroy crops and vegetables. For agriculture, the high growth rate of the insect population is very fatal. There are different types of insects, like aphids, moths, caterpillars, cutworms, etc., that continuously destroy the growth of crops and damage grains and vegetables in our daily lives. In this paper, an attempt has been made to prevent the population growth of insects (Aphids) that destroy plants through a mathematical model.

Key words: Population growth, insect,aphids,moths, mortality, density, toxic substances, Aphids • Virus • Transmission • Biology • Lifecycle • Virus-vector relationships

Introduction:

All over the world, there are more than 5,000 aphid species known. In the Indian subcontinent, 825 aphid species of 223 general subfamilies were known until 2014, and out of these, 818 species of 216 genera were found in the Himalayan region [Chakrabarti: 1]. Aphids are no longer than about 4.5 mm and have a round abdomen with different colours. They are among the most foetal pests to affect cultivated plants in temperate regions. Aphids with wings are very dangerous for crops since they destroy plants much faster than regular aphids.

Aphids can be winged or wingless. Usually, the first generation to hatch after winter is wingless. However, after several generations, there can be a lack of space on the host plant. This triggers the birth of a generation of winged aphids, which can migrate to other hosts. All the aphids born from the winter eggs are female. Several more generations of female aphids are born during spring and summer. Females can live for 25 days, during which they can each produce up to 80 new aphids. Spring and summer reproduction occur asexually, without males. Many mathematical models characterise the qualification behaviour of the interactions of Aphids verity.[Dani: 2]

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There are different cultivation methods that we can use to prevent or minimise an infestation of aphids [Blackman:3]. These include:

(1.) Reducing weeds which is responsible for collecting the aphids eggs and adults.

(2.) Using insect nets to cover crops.

(3.) Prohibiting excessive use of nitrogen fertilizer for crops growth.

(4.) Removing crop residues from their surrounding

(5.) Establishing plant species that can serve as a reservoir for predators (banker plants)

All types of aphidsget their foods by piercing the leaves with their stylus and sucking the juice from leaf of plants.[Bell:4] . Also, they have a couple of tubes in the back called cornicles through which the aphids excrete a kind of honeydew called cornicle wax. For nutrition, aphids generally feed on the plant's phloem sap, which is rich in sugars, minerals, and other elements. The phloem is responsible for distributing this kind of sap throughout the differents parts of plant. For water, aphids take liquid from xylem, where raw sap runs directly from the roots. due to which aphids remains hydrated even in dryand hot environment.

Generally aphids eggs survives in winter season, during this environmental conditions with temperature and moisture. In spring the eggs on the plant hatch, which is initial population of Aphids. All the aphids born from the winter eggs are females. Many female aphids are born during the spring and summer. A female can live for 25to 30 days, during this period it produce up to more than 90 new aphids. Spring and summer reproduction occurs asexually – without males. Therefore, the resulting aphids are basically clones of the mother. In this way Females fertilized and lay winter eggs on the plant .The most harmful consequence for the crop is the transmission of viruses. Aphids can transmit more than dozens of viruses from a diseased plant to the healthy plants in few seconds.

Fig1 aphids feeding on a plant.

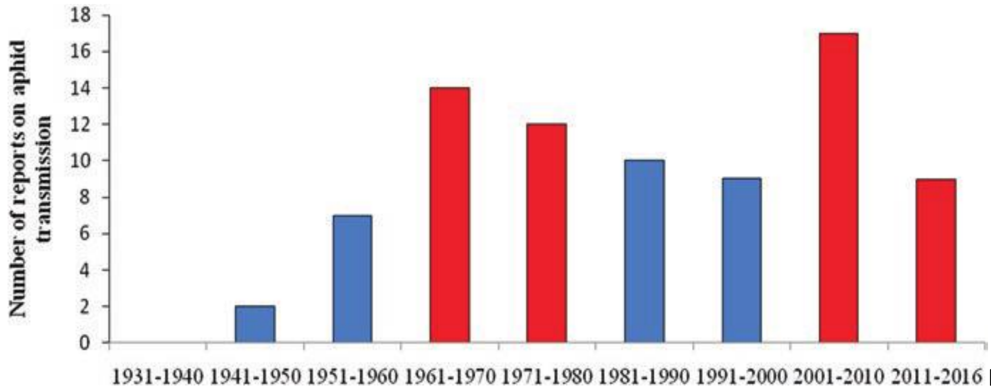

Fig:2 Trend of publications in India on aphid transmission of plant viruses. Maximum numbers of reports are made during 1961-1980 and 2001-2016 which are marked by red colour bar[A Ghosh: 5]

Plant diseases are diseases in plants caused by pathogens (infectious organisms) and environmental conditions (physiological factors). Organisms that cause infectious disease include fungi, oomycetes, bacteria, viruses, viroids, virus-like organisms, phytoplasmas, protozoa, nematodes an d parasitic plants.

Table 1:List of a phids transmitting plant viruses in India:

S. No	**Name of the virus**	**Aphid species transmitting the virus**	**Type of transmission**
1	Cardamom mosaic virus	Pentalonianigronervosa	Non- persistent
		Aphis craccivora	
		Aphis gossypii	
		Aphis nerii	
		Aphis rumicis	
		Brachycaudushelichrysi	
		Greenidiaartocarpi	
		Macrosiphumpisi	
		Macrosiphumrosaeformis	
		Macrosiphumsonchi	
		Toxopteracyperi)	
		Schizaphisgraminum	
		Pentalonianigronervosa	
		typica	
		Pentalonianigronervosa	
		caladii	
2	Large cardamom	Rhopalosiphummaidis	Non- persistent
		Brachycaudushelichrysi	

	chirke virus	Rhopalosiphumpadi	
		Sitobionavenae	
		Myzuspersicae	
		Pentalonianigronervosa	
3	Sugarcane mosaic virus	Rhopalosiphummaidis	Non- persistent
		Melanaphissacchari	
		Aphis gossypii	
		Longiunguissacchari	
4	Barley mosaicvirus	Rhopalosiphummaidis	Non-persistent
5	Potato virus Y	Myzuspersicae	Non- persistent
		Aphis gossypii	
6	Watermelon mosaic virus	Myzuspersicae	Non- persistent
7	Turnip mosaic virus	Myzuspersicae	Non- persistent
		Brevicorynebrassicae	
		Lipaphiserysimi	
8	Dasheen mosaic virus	Myzuspersicae	Non- persistent
		Aphis gossypii	
9	Onion yellow dwarf virus	Aphis gossypii	Non- persistent
		Myzuspersicae	
		Aphis craccivora	
10	Pepper vein banding virus	Aphis gossypii	Non- persistent
		Myzuspersicae	
11	Pepper veinal mottle virus	Aphis gossypii	Non- persistent
		Myzuspersicae	
12	Bean common mosaic virus was observed	Aphis craccivora	Non- persistent
		Aphis fabaesolanella	
		Aphis gossypii	
		Myzuspersicae	
		Lipaphiserysimi	
13	Sunflower mosaic virus	Aphis gossypii	Non- persistent
		Aphis craccivora	
		Myzuspersicae	
14	Garlic mosaic virus	Aphis craccivora	Non- persistent
		Myzuspersicae	
15	Cucumber mosaic virus	Myzuspersicae	Non- persistent
		Myzusornatus	
		Macrosiphumeuphorbiae	
		Aphis gossypii	
		Myzusascalonicus	

		Acyrthosiphonpisum Aphis craccivora, Aphis citricola (= Aphis spiraecola) Macrosiphumeuphorbiae	
16	Papaya ringspot virus	Myzuspersicae Aphis gossypii Aphis craccivora Myzusnicotianae	Non- persistent
17	Cowpea mosaic virus	Aphis craccivora Aphis gossypii Aphis evonymi	Non- persistent
18	Banana bract mosaic virus	Rhopalosipummaidis Aphis gossypii Pentalonianigronervosa Aphis craccivora	Non- persistent
19	Urdbean leaf crinckle virus	Aphis craccivora Aphis gossypii	Non- persistent
20	Citrus tirsteza virus	Toxopteracitricida Aphis gossypii Myzuspersicae Aphis craccivora Dactynotusjaceae (= Uroleuconjaceae) Aphis craccivora Toxopteraaurantii Macrosiphumcompositae (= Uroleuconcompositae)	Semi- persistent
21	Banana bunchytop virus	Pentalonianigronervosa	Persistent- circulative
22	Cardamom bushy dwarf virus	Pentalonianigronervosa Micromyzuskalimpongensis	Persistent- circulative
23	Barley yellow dwarf virus	R. maidis R. padi Sitobionavenae	Persistent- circulative
24	Chickpea stunt disease associated virus	Aphis craccivora	Persistent- circulative

(**Black Spot:** Black spot is one of the most common diseases found on roses, but it can also occur on other ornamental and garden plants.)

(Powdery Mildew: Powdery mildew is a fungal disease that affects many of our landscape plants, flowers, vegetables and fruits)

(Blight: Blight is a fungal disease that spreads through spores that are windborne.)

(**Canker:** Canker is often identified by an open wound that has been infected by fungal or bacterial pathogens)

In field of agriculture there are many diseases in plants and crops generated by insects(aphids)which are of pathogens. The aphids are responsible for two ways of damage to growing crops. First is direct damaged to the plant by feeding insect, which eats leaves in stems, fruit, or roots. There are more than hundred verity of pest species, in the form of larvae and adults. There was many attempts have been made to develop the mathematical models by [Barlo: 6], [Barlow and Dixon: 7], In an experiment it has been observed in the end of a week and after that that total number of individuals in the populations of insects rapidly increased in starting reaching its maximum and due to mortality there population is decreases

The population of insect has a birth rate and death rate. The first and simplest law of population by [Malthus: 8]Aphids population lies between birth rate and death rate. The first and the simplest law of population growth of a species was given by Malthusas

$$\frac{dA}{dt} = aA$$

Which leads the exponential growth equation is $A(t) = A_0 e^{at}$

This model is valid only in limited resources

Where $A(t)$ denotes the population at time t and $a > 0$ is the specific growth rate. andA_0 is the population at some arbitrary time. $t = 0$. According to [Perl:9] developed another model for limited resources.

$\frac{dA(t)}{dt} = \big(a - qA(t)\big)A(t)$, t>0

The population $A(t)$ asymptotically approaches then a limiting value $\frac{a}{q}$ for $q > 0\ as\ t \to \infty$. There are many models for growth by [Kapur:10] assuming different law of Moths can play as biodiversity indicator of the environmental impacts of human activity [Sudhanshu 11]. The Aphids produced toxic substances by bacteria become a limiting factor to their further growth by increasing rapidly the mortality rate of organism because Aphids spread saliva on the leaf preventing further sucking and movement on the individuals so causing death by eating toxic substance. Aphid populations show periodic fluctuations, and many causes are attributed to their dynamic.To elucidate this we have developed the following Mathematical model for Aphid population growth control [Alfonso:12]

Formulation of the model and its solution by two species competition:

When the initial competition in a species for a limited resources are takan into account, the two species competition growth modal is given by [Volterra:13] as

$\frac{dA_i}{dt} = \left(a_i - \sum_{j=1}^{n} q_{ij} \int_0^t A_j\right) \int_0^t A_i \;\; ds \ldots (1)\ where\ i - 1,2,3 \ldots, n\ where\ q_{ij} > 0$ is the effect of the density of j^{th}species on the population growth rate of i^{th}species.where $A_i(t)$ denotes the Aphid population at time t of i^{th} species. To obtain an explicit solution of non linear differential equation (1) we suppose that there is environmental condition such that theeffect of the density of the other aphid species on a species rate of increase is the same as the effect of the density of a species on its own rate of increase then from above equation (1) we have

$$A_1' = \left(a_1 - q_{11}\int_0^t A ds\right)\int_0^t A_1 \, ds$$

$$A_2' = \left(a_2 - q_{22}\int_0^t A ds\right)\int_0^t A_2 \, ds$$

$A_3' = \left(a_3 - q_{33}\int_0^t A ds\right)\int_0^t A_3$ ds

Similarly

Where $A_n' = \left(a_n - q_{nn}\int_0^t A ds\right)\int_0^t A_n \, ds$

Now we shall assume two Aphids populations verity u and v then the competition between them as presented here

$A_u' = \left(a_u - q_{uu}\int_0^t A ds\right)\int_0^t A_u ds$...(2)

&$A_u' = \left(a_v - q_{vv}\int_0^t A ds\right)\int_0^t A_v \, ds$...(3)

Divide above equation by $\frac{1}{q_{uu}\int_0^t A_u \, ds}$ and $\frac{1}{q_{vv}\int_0^t A_v \, ds}$, we have

$$\frac{1}{q_{uu}\int_0^t A_u \, ds} A_u' - \frac{1}{q_{vv}\int_0^t A_v \, ds} A_u' = \frac{a_u}{q_{uu}} - \frac{a_v}{q_{vv}}$$

or $\left\{ log \frac{A_u^{\frac{1}{q_{uu}}}}{A_v^{\frac{1}{q_{vv}}}} \right\} = \frac{a_u}{q_{uu}} - \frac{a_v}{q_{vv}}$, where $D \equiv \frac{d}{dt}$

$= p_{uv}$ (say)

Thus $\frac{A_u^{\frac{1}{q_{uu}}}}{A_v^{\frac{1}{q_{vv}}}} = P \exp(p_{uv} t) = R(Say)$ where P is an arbitrary Constant of Integration. In Starting at time $t = 0$ the initial value of A_u &A_v be $(A_u)_0^{\frac{1}{q_{uu}}}$&$(A_v)_0^{\frac{1}{q_{vv}}}$ where $P = \frac{(A_u)_0^{\frac{1}{q_{uu}}}}{(A_v)_0^{\frac{1}{q_{vv}}}}$

Result and Discussion:

Result No. 1: If $p_{uv} = 0$ this implies that $R = \text{constant}$this concern that both the ratio of two verity of Aphids population being constant with time.

Result No. 2:If$p_{uv} > 0$ as time increases infinitely $R = \infty$ i.e; $A_v \to 0$ this concern that Aphids population of $v$verity become no longer active.

Result No. 3: $p_{uv} < 0$ as time increases infinitely then $R = 0$ i.e;$A_u \to 0$ this concern that Aphids population of u verity become no longer active.

So the factor p_{uv} helps us to predict the behavior of the Aphids yerity as time increases as infinitely. From above results if any one of the factor p_{uv} where $v = 1,2,3 \dots, n$ is negative then v verity of Aphids population become no longer active that is terminated. So uth row from the matrix in a space will

Remove as well as from column. Thus the competition among Aphids populations continuously till one row left.

In the above Mathematical model, we observed that the population of Aphids is not fixed it changed, in different intervals. we observed that it initially increases with constant ratio, the Aphids population decreases and tends to zero which is the population tended to extinction. So, there is to find control its population growth is possible which prevented a high growth rate of Aphids population growth by which our crops and plants can be protected from harm.

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