Journal of Harbin Engineering University
ISSN: 1006-7043

Vol 45 No. 7
July 2024

Study of Transient and Steady State Solution of M/G/1 Queue with Two Phase Repairs

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Abstract: We study a single server M/G/1 queueing system with Poisson input subject to random breakdowns. The server provides service to all arriving customers on a first come first served basis and service times follow general distribution. The system may breakdown at random and it must be sent to repair process immediately. Further the repair process involves two phases of repairs with different general repair time distributions. The supplementary variable technique is applied to find explicitly probability generating function of the number in the system and the corresponding steady state results are computed explicitly.
Keywords: General Times, Poisson Arrivals, Probability Generating Function, Random Breakdowns, Steady State, Supplementary Variable Technique.

1. Introduction

Queues and queueing networks occupy a prominent role in the performance analysis of a wide range of systems in computer-communications, logistics and flexible manufacturing systems. Further, queueing network models have been successfully applied in the optimization of computer systems, telecommunication systems and broadcasting. A classic example is the celebrated Erlang loss model, first studied by [1] in the beginning of the 20th century in the context of telephone networks. Another successful branch of queueing theory is the study of networks of queues motivated by computer - communication systems evolved in the 60's and 70's, which have seen breakthroughs in several important fields. (Refer [2] and [3]).

There is a natural interest in studying queueing systems subject to server interruptions. In real life situations, a queueing system might suddenly break down and hence the server will not be able to continue providing service unless the system is repaired. The situations where such kind of interruptions are common may be encountered in communication systems, in telephone traffic, in Industry and in the operation of electronic computers etc. Single server queueing systems subject to interruptions have been studied by numerous researchers due to their wide applications. Many authors have studied queues with server breakdowns which include [4], [5], [6].

[7] has considered discrete-time queueing systems operating under first-come-first-served queueing discipline with deterministic service times of one slot and subject to independent server interruptions. For such systems, they have derived a relationship between the probability generating functions of the system content during an arbitrary slot and of the system delay of an arbitrary customer. M/G/∞ queue with altering renewal breakdowns where the service station is subject to occasional interruptions is studied by [8].

A single server retrial queue where the server is subject to breakdowns and repairs is analyzed by [9]. Two models are studied in this paper. In model I, the failed server cannot be occupied and the customer whose service is interrupted must either leave the system or rejoin the retrial group. In model II, the customer whose service is interrupted by a failure stays at the server and restarts the service when repair is completed. A detailed analysis for reliability of retrial queues is given by [10]. Using supplementary variable technique, the explicit expressions for some main reliability indexes such as availability and frequency are computed.

A single server M/G/1 queueing models subject to server breakdowns have paid attention of many researchers due to its numerous applications in our day-to-day life (Refer [11],[12] and [13]). The most realistic aspect in modelling of an unreliable server is phase type of repairs. A queue with active breakdown and delay time subject to two phases of repair under multiple vacation policy is studied by

[14]. In the proposed model, the probability generating function of the queue size at an arbitrary and departure epoch in steady state are obtained using the supplementary variable technique.

An M/G/1 retrial queue with two types of breakdowns is studied by [15]. In this model, When the server is idle, it is subject to breakdowns according to a Poisson process with rate δ and it cannot be repaired immediately. While when the server is busy, it may break down according to a Poisson process with rate θ and can be immediately repaired. The necessary and sufficient condition for the system to be stable and the probability of the orbit size at the departure epochs are computed.

A single server M/G/1 queueing system with Poisson input subject to compulsory server vacation and three phases of repair is studied by [16]. The supplementary variable technique is applied to find explicitly probability generating function of the number in the system and the corresponding steady state results. Not much work has been dealt with queueing systems subject to phase types of repairs.

In this regard, we consider a single server queueing model subject to two phases of repairs. The system may breakdown at random with breakdown rate α >0 . As soon as the system is broken down it is immediately sent for repair where the repair is provided in two phases. After the first stage of repair, every failed unit must undergo the second phase of repair. After completion of the second phase of repair, the server resumes its work immediately. Also, the customer whose service is interrupted comes back to the head of the queue.

The rest of the paper is organized as follows. The mathematical description of our model is in section 2 and the equations governing the model are given in section 3. The time dependent solution has been obtained in section 4 using supplementary variable technique and the corresponding steady state results have been derived explicitly in section 5.

2. Assumptions Underlying the Model

The following assumptions describe the mathematical model

- Customers arrive at the system one by one in according to a Poisson stream with arrival rate $\lambda > 0$.
- The service to customers is provided one by one on a first come, first served basis and their service times follow a general (arbitrary) distribution with distribution function $B(v)$ and the density function $b(v)$.
- Let $\mu(x)dx$ be the conditional probability of completion of a service during the interval $(x + dx]$ given that elapsed time is x, so that

$$\mu(\text{x}) = \frac{b(x)}{1-B(x)} \quad (1)$$

and therefore

$$b(v) = \mu(v)e^{-\int_0^v \mu(x)dx} \quad (2)$$

- The customers are served according to the first come, first served rule.
- The server is subject to random breakdowns such that αdt is the first order probability that the service channel will fail during the short interval of time $(t, t + dt]$.
- We assume that whenever the service channel breaks down, it instantly undergoes a repair process and the customer whose service gets interrupted comes back to the head of the queue.
- We assume that the repair process comprises of two phases of repairs, the first phase followed by the second phase.
- The duration of repairs follows a general (arbitrary) distribution with distribution functions $H(r_i)$ and density functions $h(r_i)$, $i = 1,2$.

$$\beta_i(\text{x}) = \frac{r_i(x)}{1-R_i(x)}, i = 1,2 . \quad (3)$$

and therefore

$$r_i(t) = \beta_i(t)e^{-\int_0^t \beta(x)dx}, i = 1,2 . \quad (4)$$

- Various Stochastic Processes involved in the system are independent of each other.

3. Definitions, Notations and Time Dependant Equations

$P_n(x,t)$: Probability that at time t, there are $n \geq 0$ customers in the queue excluding one customer in service and the elapsed served time of this customer is x. Consequently $P_n(t) = \int_0^\infty P_n(x,t)dx$ denotes the probability that at time t, there are n customers in the queue excluding the one customer in service irrespective of the value of x .

$R_n^{(1)}(x,t)$: Probability that at time t, the server is inactive due to system breakdown and the server is under first phase of repair with elapsed repair time x and there are n customers waiting in the queue for repair. Consequently $R_n^{(1)}(t)=\int_0^\infty R_n^{(1)}(x,t)dx$ denotes the probability that at time t, there are customers in the queue and the server is under first phase of repair irrespective of the value of x.

$R_n^{(2)}(x,t)$: Probability that at time t, the server is inactive due to system breakdown and the server is under second phase of repair with elapsed repair time x and there are n customers waiting in the queue for repair. Consequently $R_n^{(2)}(t)=\int_0^\infty R_n^{(2)}(x,t)dx$ denotes the probability that at time t, there are n customers in the queue and the server is under second phase of repair irrespective of the value of x.

$Q(t)$: Probability that at time t, there is no customer in the system and the server is idle but available in the system.

The model is then governed by the following time dependent forward system of equations

$$\frac{\partial}{\partial x}P_n(x,t)+\frac{\partial}{\partial t}P_n(x,t)+(\lambda+\mu(x)+\alpha)P_n(x,t)=\lambda P_{n-1}(x,t), n=1,2,\ldots \quad (5)$$

$$\frac{\partial}{\partial x}P_0(x,t)+\frac{\partial}{\partial t}P_0(x,t)+(\lambda+\mu(x)+\alpha)P_0(x,t)=0, \quad (6)$$

$$\frac{\partial}{\partial x}R_n^{(1)}(x,t)+\frac{\partial}{\partial t}R_n^{(1)}(x,t)+\big(\lambda+\beta_1(x)\big)R_n^{(1)}(x,t)=\lambda R_{n-1}^{(1)}(x,t), n=1,2,\ldots, \quad (7)$$

$$\frac{\partial}{\partial x}R_0^{(1)}(x,t)+\frac{\partial}{\partial t}R_0^{(1)}(x,t)+\big(\lambda+\beta_1(x)\big)R_0^{(1)}(x,t)=0, \quad (8)$$

$$\frac{\partial}{\partial x}R_n^{(2)}(x,t)+\frac{\partial}{\partial t}R_n^{(2)}(x,t)+\big(\lambda+\beta_2(x)\big)R_n^{(2)}(x,t)=\lambda R_{n-1}^{(2)}(x,t), n=1,2,\ldots, \quad (9)$$

$$\frac{\partial}{\partial x}R_0^{(2)}(x,t)+\frac{\partial}{\partial t}R_0^{(2)}(x,t)+\big(\lambda+\beta_2(x)\big)R_0^{(2)}(x,t)=0, \quad (10)$$

$$\frac{d}{dt}Q(t)=-\lambda Q(t)+\int_0^\infty R_0^{(2)}(x,t)\beta_2(x)dx. \quad (11)$$

Equations (5) to (11) are to be solved subject to the following boundary conditions:

$$P_0(0,t)=\lambda Q(t)+\int_0^\infty R_1^{(2)}(x,t)\beta_2(x)dx \quad (12)$$

$$P_n(0,t)=\int_0^\infty R_{n+1}^{(2)}(x,t)\beta_2(x)dx, n=1,2,\ldots, \quad (13)$$

$$R_0^{(1)}(0,t)=0, \quad (14)$$

$$R_n^{(1)}(0,t)=\alpha\int_0^\infty P_{n-1}(x,t)dx, n=1,2,\ldots, \quad (15)$$

$$R_n^{(2)}(0,t)=\int_0^\infty R_n^{(1)}(x,t)\beta_1(x)dx, n=0,1,\ldots, \quad (16)$$

Next, we assume that initially the server is available but idle because of no customers so that the initial condition is

$$Q(0)=1. \quad (17)$$

4. Generating Functions of the Queue Length: The Time Dependent Solution

In this section we define the transient solution for the above set of differential-difference equations. We define the probability generating functions as follows.

$$\left.\begin{aligned}R^{(1)}(x,z,t)=\sum_{n=0}^\infty z^n R^{(1)}(x,t), R^{(1)}(z,t)=\sum_{n=0}^\infty z^n R^{(1)}(t),\\ R^{(2)}(x,z,t)=\sum_{n=0}^\infty z^n R^{(2)}(x,t), R^{(2)}(z,t)=\sum_{n=0}^\infty z^n R^{(2)}(t),\\ P(x,z,t)=\sum_{n=0}^\infty z^n P(x,t), P(z,t)=\sum_{n=0}^\infty z^n P(t).\end{aligned}\right\} \quad (18)$$

which are convergent inside the circle given by $|z|\le 1$ and define the Laplace transform of a function $f(t)$ as

$$\overline{f(s)}=\int_0^\infty e^{-st}f(t)dt, \Re(s)>0 \quad (19)$$

We take Laplace transforms of equations (5) to (16) and using (17) we obtain

$$\frac{\partial}{\partial x}\overline{P_n}(x,s)+(s+\lambda+\mu(x)+\alpha)\overline{P_n}(x,s)=\lambda\overline{P}_{n-1}(x,s), n=1,2,\ldots, \quad (20)$$

$$\frac{\partial}{\partial x}\overline{P_0}(x,s)+(s+\lambda+\mu(x)+\alpha)\overline{P_0}(x,s)=0, \quad (21)$$

$$\frac{\partial}{\partial x}\overline{R_n}^{(1)}(x,s)+\big(s+\lambda+\beta_1(x)\big)\overline{R_n}^{(1)}(x,s)=\lambda\,\overline{R}_{n-1}^{(1)}(x,s), n=1,2,\ldots, \quad (22)$$

$$\frac{\partial}{\partial x}\overline{R_0}^{(1)}(x,s)+\big(s+\lambda+\beta_1(x)\big)\overline{R_0}^{(1)}(x,s)=0, \quad (23)$$

$$\frac{\partial}{\partial x}\overline{R_n}^{(2)}(x,s)+\big(s+\lambda+\beta_2(x)\big)\overline{R_n}^{(2)}(x,s)=\lambda\overline{R}_{n-1}^{(2)}(x,s), n=1,2,\ldots, \quad (24)$$

$$\frac{\partial}{\partial x}\overline{R_0}^{(2)}(x,s)+\big(s+\lambda+\beta_2(x)\big)\overline{R_0}^{(2)}(x,s)=0, \quad (25)$$

$$(s+\lambda)Q(s)=1+\int_0^\infty\overline{R}_O^{(2)}(x,s)\beta_2(x)dx, \quad (26)$$

$\overline{P}_0(0,s) = \lambda\overline{Q}(s) + \int_0^\infty \overline{R}_1^{(2)}(x,s)\beta_2(x)dx,$
(27) $\overline{P}_n(0,s) = \int_0^\infty \overline{R}_{n+1}^{(2)}(x,s)\beta_2(x)dx\,, n = 1,2,\ldots,$ (28)
$\overline{R_0}^{(1)}(0,s) = 0\,,$ (29)
$\overline{R_n}^{(1)}(0,s) = \alpha\int_0^\infty P_{n-1}(x,s)dx, \quad n = 1,2,\ldots,$
(30)
$\overline{R_n}^{(2)}(0,s) = \int_0^\infty R_n^{(1)}(x,s)\beta_1(x)dx, \quad n = 0,1,\ldots,$
(31)

We multiply equations (20) – (25) and (27) – (31) by suitable powers of z, using equations (11), (18) and then simplifying we obtain

$\frac{\partial}{\partial x}\overline{P}(x,z,s) + (s + \lambda - \lambda z + \mu(x) + \alpha)\overline{P}(x,z,s) = 0,$

(32)
$\frac{\partial}{\partial x}\overline{R}^{(1)}(x,z,s) + \big(s + \lambda - \lambda z + \beta_1(x)\big)\overline{R}^{(1)}(x,z,s) = 0,$
(33)
$\frac{\partial}{\partial x}\overline{R}^{(2)}(x,z,s) + \big(s + \lambda - \lambda z + \beta_2(x)\big)\overline{R}^{(2)}(x,z,s) = 0,$
(34)
$z\overline{P}(0,z,s) = \Big(1 - s\overline{Q}(s)\Big) + \lambda(z-1)\overline{Q}(s) + \int_0^\infty \overline{R}^{(2)}(x,z,s)\beta_2(x)dx,$
(35)
$\overline{R}^{(1)}(0,z,s) = \alpha z\int_0^\infty \overline{P}(x,z,s)dx,$ (36)
$\overline{R}^{(2)}(0,z,s) = \int_0^\infty \overline{R}^{(1)}(x,z,s)\beta_1(x)dx,$
(37)

Next we integrate equations (32) – (34) between limits 0 and x and obtain

$\overline{P}(x,z,s) = \overline{P}(0,z,s)e^{-(s+\lambda-\lambda z+\alpha)x - \int_0^x \mu(t)dt},$
(38)
$\overline{R}^{(1)}(x,z,s) = \overline{R}^{(1)}(0,z,s)e^{-(s+\lambda-\lambda z)x - \int_0^x \beta_1(t)dt},$
(39)
$\overline{R}^{(2)}(x,z,s) = \overline{R}^{(2)}(0,z,s)e^{-(s+\lambda-\lambda z)x - \int_0^x \beta_2(t)dt}.$
(40)

We again integrate equations (38) – (40) with respect to x by parts to obtain

$\overline{P}(z,s) = \overline{P}(0,z,s)\left[\frac{1-\overline{B}(S+\lambda-\lambda z+\alpha)}{s+\lambda-\lambda z+\alpha}\right]$ (41)

Where $\overline{B}(s + \lambda - \lambda z + \alpha) = \int_0^\infty e^{-(s+\lambda-\lambda z+\alpha)x}\,\mathrm{dB(x)}$ is the Laplace – Steiljes transform of service time.

$\overline{R}^{(1)}(z,s) = \overline{R}^{(1)}(0,z,s)\left[\frac{1-\overline{R_1}(S+\lambda-\lambda z)}{s+\lambda-\lambda z}\right]$
(42)

Where $\overline{R_1}(s + \lambda - \lambda z) = \int_0^\infty e^{-(s+\lambda-\lambda z)x}\,\mathrm{dR_1(x)}$ is the Laplace – Steiltjes transform of the phase 1 repair time.

$\overline{R}^{(2)}(z,s) = \overline{R}^{(2)}(0,z,s)\left[\frac{1-\overline{R_2}(S+\lambda-\lambda z)}{s+\lambda-\lambda z}\right]$
(43)

Where $\overline{R_2}(s + \lambda - \lambda z) = \int_0^\infty e^{-(s+\lambda-\lambda z)x}\,\mathrm{dR_2(x)}$ is the Laplace – Steiltjes transform of the phase 2 repair time.

Now we shall determine the following integrals $\int_0^\infty \overline{P}(x,z,s)\mu(x)dx\,, \int_0^\infty \overline{R}^{(1)}(x,z,s)\beta_1(x)dx$ and $\int_0^\infty \overline{R}^{(2)}(x,z,s)\beta_2(x)dx.$

For this purpose, we multiply equations (38) – (40) by $\mu(x), \beta_1(x)$ and $\beta_2(x)$, integrate with respect to x and obtain

$\int_0^\infty \overline{P}(x,z,s)\mu(x)dx = \overline{P}(0,z,s)\overline{B}(s + \lambda - \lambda z + \alpha).$

(44)
$\int_0^\infty \overline{R}^{(1)}(x,z,s)\beta_1(x)dx = \overline{R}^{(1)}(0,z,s)\overline{R_1}(s + \lambda - \lambda z)$

(45)
$\int_0^\infty \overline{R}^{(2)}(x,z,s)\beta_2(x)dx = \overline{R}^{(2)}(0,z,s)\overline{R_2}(s + \lambda - \lambda z)\,.$

(46)

Now using equations (38) – (40) into equations (35) – (37), we get on simplifying

$z\,\overline{P}(0,z,s) = \Big(1 - s\overline{Q}(s)\Big) + \lambda(z-1)\overline{Q}(s) + \overline{R}^{(2)}(0,z,s)\overline{R_2}(s + \lambda - \lambda z)$ (47)

$\overline{R}^{(1)}(0,z,s) = \alpha z\int_0^\infty \overline{P}(x,z,s)dx = \alpha z\overline{P}(z,s),$ (48)

$\overline{R}^{(2)}(0,z,s) = \alpha z\overline{P}(z,s)\overline{R_1}(s + \lambda - \lambda z)\,.$ (49)

Now using equation (43) in equation (47) and then simplifying we get

$\overline{P}(0,z,s) = \left[\frac{(1-s\overline{Q}(s))+\lambda(z-1)\overline{Q}(s)}{\overline{D}(z,s)}\right].$ (50)

where

$\overline{D}(z,s) = z - \alpha z\overline{R_1}(s + \lambda - \lambda z)\,\overline{R_2}(s + \lambda - \lambda z)$

$$[1-\overline{B}(s+\lambda-\lambda z+\alpha)]. \quad (51)$$

Using equations (50) and (51) in equation (41) we get

$$\overline{P}(z,s)=\left[\frac{\overline{N_1}(z,s)}{\overline{D_1}(z,s)}\right], \quad (52)$$

Where

$$\overline{N_1}(z,s)=\left(1-s\overline{Q}(s)\right)+\lambda(z-1)\overline{Q}(s)$$
$$[1-\overline{B}(s+\lambda-\lambda z+\alpha)], \quad (53)$$

$$\overline{D}_1(z,s)=(s+\lambda-\lambda z+\alpha)z-\alpha z\overline{R_1}(s+\lambda-\lambda z)$$
$$\overline{R_2}(s+\lambda-\lambda z)\left[\frac{1-\overline{B}(s+\lambda-\lambda z+\alpha)}{(s+\lambda-\lambda z+\alpha)}\right] \quad (54)$$

Using equations (48) and (49) into equations (42) and (43), we get

$$\overline{R}^{(1)}(z,s)=\alpha z\overline{P}(z,s)\left[\frac{1-\overline{R_1}(s+\lambda-\lambda z)}{s+\lambda-\lambda z}\right].$$

(55)

$$\overline{R}^{(2)}(z,s)=\alpha z\overline{P}(z,s)\overline{R_1}(s+\lambda-\lambda z)\left[\frac{1-\overline{R_2}(s+\lambda-\lambda z)}{s+\lambda-\lambda z}\right],$$

(56)

5. The Steady State Results

In this section, we shall derive the steady state probability distribution for our queueing model. To define the steady state probabilities, we suppress the argument t wherever it appears in the time-dependent analysis. This can be obtained by applying the well-known Tauberian property,

$$\lim_{n\to 0} s\,\overline{f}(s)=\lim_{t\to\infty} f(t) \quad (57)$$

In order to determine $\overline{P}(z,s)$, $\overline{R}^{(1)}(z,s)$ and $\overline{R}^{(2)}(z,s)$ completely, we have yet to determine the unknown Q. For that purpose, we shall use the normalizing condition.

$$P(1)+R^{(1)}(1)+R^{(2)}(1)+Q=1$$

(58)

Thus, multiplying both sides of equations (52), (55) and (56) by s, taking limit as $s\to 0$, applying property (57) and simplifying we have

$$P(z)=\frac{\lambda(z-1)Q[1-\overline{B}(\lambda-\lambda z+\alpha)]}{D(z)},$$

(59)

$$R^{(1)}(z)=\frac{\alpha Q[1-\overline{B}(\lambda-\lambda z+\alpha)][\overline{R_1}(\lambda-\lambda z)-1]}{D(z)},$$

(60)

$$R^{(2)}(z)=\frac{\alpha Q[1-\overline{B}(\lambda-\lambda z+\alpha)]\overline{R_1}(\lambda-\lambda z)[\overline{R_2}(\lambda-\lambda z)-1]}{D(z)},$$

(61)

where

$$D(z)=(\lambda-\lambda z+\alpha)z$$
$$-\alpha z\overline{R_1}(\lambda-\lambda z)\overline{R_2}(\lambda-\lambda z)\left[1-\overline{B}(\lambda-\lambda z+\alpha)\right].$$

(62)

We see that for $z=1, P(z), R^{(1)}(z)$ and $R^{(2)}(z)$ in equations (59) - (61) are indeterminate of the $\frac{0}{0}$ form. Therefore, we apply L'Hopital's rule using the fact that

$$\overline{B}(0)=1, -\overline{B}'(0)=\frac{1}{\mu},$$
$\overline{R_1}(0)=1, -\overline{R_1}'(0)=\frac{1}{\beta_1}=E(r_1)$ and
$$\overline{R_2}(0)=1, -\overline{R_2}'(0)=\frac{1}{\beta_2}=E(r_2).$$

Thus on simplifying we have

$$P(1)=\frac{[1-\overline{B}(\alpha)]\lambda Q}{D(1)}, \quad (63)$$

$$R^{(1)}(1)=\frac{\alpha\lambda Q[1-\overline{B}(\alpha)]E(r_1)}{D(1)},$$

(64)

$$R^{(2)}(1)=\frac{\alpha\lambda Q[1-\overline{B}(\alpha)]E(r_2)}{D(1)}.$$

(65)

Where

$$D(1)=\alpha\overline{B}(\alpha)-\lambda\left[1-\overline{B}(\alpha)\right]+\alpha\lambda\overline{B}(\alpha)\overline{R_1}'(0)$$
$$+\alpha\lambda\overline{B}(\alpha)\overline{R_2}'(0). \quad (66)$$

Now since $P(1)+R^{(1)}(1)+R^{(2)}(1)+Q=1$, we have

$$Q=1-\frac{\lambda}{\alpha\overline{B}(\alpha)}\left\{\begin{matrix}\left[1-\overline{B}(\alpha)\right]-\alpha\left[1-\overline{B}(\alpha)\right]\overline{R_1}'(0)\\ -\alpha\left[1-\overline{B}(\alpha)\right]\overline{R_2}'(0)\end{matrix}\right\}.$$

(67)

Equation (67) gives the steady state probability that there is no customer in the system and the server is idle. Also from equation (67), we obtain ρ, the utilization factor of the system as

$$\rho=1-Q=\frac{\lambda}{\alpha\overline{B}(\alpha)}\left\{\begin{matrix}\left[1-\overline{B}(\alpha)\right]-\alpha\left[1-\overline{B}(\alpha)\right]\overline{R_1}'(0)\\ -\alpha\left[1-\overline{B}(\alpha)\right]\overline{R_2}'(0)\end{matrix}\right\}<1.$$

(68)

5. Conclusion

We have studied a single server M/G/1 queueing system subject to random breakdowns and two phases of repairs. We have found the time-dependent probability generating functions in terms of their Laplace transforms using supplementary variable technique and have derived explicitly the corresponding steady state results.

Acknowledgement
The author thanks the management of Sri Sivasubramaniya Nadar College of Engineering for providing the necessary requirements during the preparation of this paper.

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