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Hall Effects on Thermal Oscillatory Boundary Layer Flow

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Abstract:
In the present study, we have discussed the Hall effects on temperature in oscillatory MHD laminar layer flow when the fluid is injected from outside to the flow field and its direction is kept normal to the flow of direction. The expression for velocity field and skin friction are obtained and studied for wide range of injection parameter variation. The effect of small and large frequencies on flow field has been also discussed by graphically. In the graph, it is shown the fluctuation in injection velocity increases and the amplitude of the skin friction decreases and phase angle increases.
Keywords: Oscillatory boundary layer flow, MHD, Skin friction, Velocity field, Injection parameter, Phase angle, Injection velocity.

Introduction:
The effects of free stream oscillations on the laminar boundary layer flow past horizontal bodies were studied Light hill [1954] and Lin [1957]. Ong and Nicholls [1959] discussed the flow of a viscous incompressible and electrically conducting fluid past in the presence of transverse magnetic field when the plate executes a simple harmonic motion parallel to itself. The effect of transversely applied magnetic field on Stuart [1955] problem was studied by Rao [1962]. Messiha [1966] studied the effect of variable suction on laminar boundary layer in oscillatory flow along an infinite flat plate. He assumed that the stream velocity as well as the suction velocity normal to the plate varies periodically with the same frequency about a non-zero constant mean. The MHD aspect of the Messisha problem was studied by Pop [1967] Kumar et.al [2007] and Lal [1972]. Pop has discussed the hydromagnetic laminar periodic boundary layer flow for the suction velocity of the form $\bar{v} = v_0\left(1+\varepsilon A e^{i\varpi t}\right)$ .The non-dimensional expression for the velocity field in the boundary layer flow has been considered. As $u(y,t) = (1-f_1(y)) + \varepsilon e^{i\omega t}(1-f_2(y))$ and the stream velocity as $U(t) = 1 + e^{i\omega t}$ extending the investigation of Pop and Lal has shown that if more harmonic terms are considered in the expression for velocity distribution , its value decreases considerably. Chung and Kassemi [1980], Gordan et al[2003], Nag [1980], Soundalgekar [1979] and Soundalgekar and Bhat [1990] studied heat transfer problem for laminar flow over a plate of varing temperature. Das et.al [2013] studied on hall effect oscillatory MHD coquette flow .The thermal boundary condition is either that the fluid has the same temperature as the well, which is prescribed or the heat flow through the surface is prescribed. The boundary conditions for the fluid at a surface are the no slip condition; the tangential velocity component is zero. The study of MHD viscous flow with current gives qualitative insight to the problem of MHD power generators and Hall accelerators. Hall Effect as the steady hydro magnetic flow of viscous and incompressible fluid through parallel plate has been studied.

Many applications in Turbo- machinery such as fans, turbines and compressor pump are designed on the assumption that the flow passage is steady. But actually it is not so rather the assumed steady state is the time average of the actual flow which fluctuates periodically around the above mean state.

Mathematical Formulation:
We consider the two dimensional oscillatory flow of an electrically conducting,viscous, incompressible fluid past an infinite horizontal

porous flate plate, x axis is taken along the plate in the direction of flow and y axis is taken perpendicular to the plate. A uniformly distributed the plate. The flow is assumed to be at small magnetic Reynolds number so that induced effect constant magnetic field B_0 is acting perpendicularto

may be neglected. Under the MHD boundary layer assumption, flow is governed by the equations.

The governing equations are

$$\frac{\partial \nu}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + \nu\frac{\partial u}{\partial y} = -\frac{1}{p}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} - \sigma\frac{B_0{}^2}{\rho}u \tag{2}$$

Since we have assumed time dependent injection velocity, therefore integrating equation (1), we have

$$\nu = \nu_0\left(1 + \varepsilon A e^{i\varpi t}\right) \tag{3}$$

Notations have their usual meanings.

At the outer edge of the boundary layer , we get

$$\frac{dU}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial x} - \sigma\frac{B_0{}^2}{\rho}U \tag{4}$$

Eliminating $\frac{\partial p}{\partial x}$ from equation (2) with the help of equation (4) and substituting in equation (3), we get

$$\frac{\partial u}{\partial t} + \nu_0\left(1 + \varepsilon A e^{i\varpi t}\right)\frac{\partial u}{\partial y} = \frac{\partial U}{\partial t} + \nu\frac{\partial^2 u}{\partial y^2} + \frac{\sigma B_0{}^2}{\rho}(U - u) \tag{5}$$

The boundary conditions are

$$y = 0;\ u = 0 \ \& \ y \to \infty\ ;\ u = U(t) \tag{6}$$

By virtue of the non-dimensional transformations, the equations (5) and (6) reduce to

$$\frac{\partial^2 u}{\partial y^2} = \left(1 + \varepsilon A e^{i\varpi t}\right)\frac{\partial u}{\partial y} + \frac{M}{4}(U - u)a + \frac{1}{4}\frac{dU}{dt} - \frac{1}{4}\frac{\partial u}{\partial t} = 0 \tag{7}$$

$$\& \ y = 0;\ u = 0 \ \& \ y \to \infty\ ;\ u = U(t) \tag{8}$$

Solution:

The solution of equation (7) in this form

$$u(y,t) = [1 - \phi_0(y)] + \sum_{r=1}^{n}\varepsilon^r e^{i\omega t}[1 - \phi_r(y)] \tag{9}$$

$$U(t) = 1 + \sum_{r=1}^{n}\varepsilon^r e^{ir\omega t}, \qquad r = 1,2,3,\ldots\ldots\ldots n \tag{10}$$

Now substituting equation (9) and (10) in equation (7) and equating the coefficients of non-harmonic and harmonic terms, we have the solutions of the form

$$\phi_0'' = \phi_0' - \frac{M}{4}\phi_0 = 0$$

$$\phi_1'' = \phi_1' - \frac{1}{4}(M + i\omega)\phi_1 = A\phi_0',$$

$$\phi_2'' = \phi_2' - \frac{1}{4}(M + 2i\omega)\phi_2 = A\phi_1',$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots, \quad (11)$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots,$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots,$$

$$\phi_n'' = \phi_n' - \frac{1}{4}(M + in\omega)\phi_n = A\phi_{n-1}',$$

Where prime denotes differentiations withrespect to y .

The reduced boundary conditions are

$$\begin{aligned} y = 0; \quad & \phi_0 = \phi_1 = \phi_2 = .. = \ldots\ldots\ldots\ldots = \phi_n = 1 \\ y \to \infty; \quad & \phi_0 = \phi_1 = \phi_2 = .. = .. \ldots\ldots\ldots\ldots = \phi_n = 0 \end{aligned} \quad (12)$$

Integrating the equation (11) and using boundaryconditions (12) , we get

$$\phi_0(y) = e^{-L_0 y}$$

$$\phi_1(y) = e^{-L_1 y} + \frac{4iAL_0}{\omega}\left(e^{-L_1 y} - e^{-L_0 y}\right)$$

$$\phi_2(y) = e^{-L_2 y} + \frac{8A^2 L_0^2}{\omega^2}\left(e^{-L_2 y} - e^{-L_0 y}\right) \quad (13)$$

$$+\left(\frac{4iAL_1}{\omega} - \frac{16A^2 L_0 L_1}{\omega^2}\right)\left(e^{-L_2 y} - e^{-L_1 y}\right)$$

$$L_0 = \frac{1}{2}\left[(1+M)^{\frac{1}{2}} - 1\right],$$

Where $L_1 = \frac{1}{2}\left[(1 + M + i\omega)^{\frac{1}{2}} - 1\right] = L_{1r} + iL_{1i},$

$$L_2 = \frac{1}{2}\left[(1 + M + 2i\omega)^{\frac{1}{2}} - 1\right] = L_{2r} + iL_{2i}$$

Substituting the values of $\phi_0, \phi_1, \phi_2 \ldots\ldots\ldots\ldots$ in equation (9), we get

$$u(y,t) = 1 - e^{-L_0 y} + \varepsilon e^{i\omega t}\left[1 - e^{-L_1 y} - \frac{4iAL_0}{\omega}\left(e^{-L_1 Y} - e^{-L_0 y}\right)\right] + \varepsilon^2 e^{2i\omega t}\left[\begin{array}{l} 1 - e^{-L_2 y} - \frac{8A^2 L_0{}^2}{\omega^2}\left(e^{-L_1 y} - e^{-L_0 y}\right) \\ -\left(\frac{4iAL_1}{\omega} - \frac{16A^2 L_0 L_1}{\omega^2}\right)\left(e^{-L_2 y} - e^{-L_1 y}\right)\end{array}\right] \tag{14}$$

The non- dimensional expression for the skin friction is obtained as

$$\tau = \left(\frac{du}{dy}\right)_{y=0} = L_0 + \varepsilon e^{i\omega t}\left[1 + \left(\frac{4iAL_0}{\omega}\right) - \frac{4iAL_0{}^2}{\omega}\right] + \varepsilon^2 e^{2i\omega t}\left[L_2 + \frac{8A^2 L_0{}^2}{\omega^2}\left(L_2 - L_0\right) + \left(\frac{4iAL_1}{\omega} - \frac{16A^2 L_0 L_1}{\omega^2}\right)\left(L_0 - L_1\right)\right]$$

(15)

On separating real part of (14) , we get

$$\left(u(y,t) = 1 - e^{-L_{0y}} + \varepsilon\left(R_1 \cos\omega t - R_2 \sin\omega t\right) + \varepsilon^2\left(R_3 \cos 2\omega t - R_4 \sin 2\omega t\right) + \ldots\ldots..\right.$$

(16)

Where

$$R_1 = 1 - e^{-L_{1ry}}\left(\frac{4AL_0}{\omega}\sin L_{1i} y + \cos L_{ii} y\right),$$

$$R_2 = \frac{4AL_0}{\omega} e^{-L_0 y} - e^{-L_{1y}}\left(\frac{4AL_0}{\omega}\cos L_{1i} y - \sin L_{1i} y\right),$$

$$R_3 = 1 + \frac{8A^2 L_0{}^2}{\omega^2} e^{-L_{0y}} - \left(1 + \frac{8A^2 L_0{}^2}{\omega^2}\right) e^{-L_{2y}} \cos L_{2I} y$$
$$- \left(\frac{4AL_{1i}}{\omega} + \frac{16A^2 L_0 L_{1r}}{\omega^2}\right)\left(e^{-L_{1rY}} \cos L_{1i} y - e^{-L_{2ry}} \cos L_{2i} y\right)$$
$$+ \left(\frac{4AL_{1r}}{\omega} - \frac{16A^2 L_0 L_{1i}}{\omega^2}\right)\left(e^{-L_{1ry}} \sin L_{1r} y - e^{-L_{2ry}} \sin L_{2i} y\right),$$

$$R_4 = \left(1 + \frac{8A^2 L_0{}^2}{\omega^2}\right) e^{-L_{2ry}} \sin L_{2i} y + \left(\frac{4AL_{1r}}{\omega} - \frac{16A^2 L_0 L_{1I}}{\omega^2}\right)\left(e^{-L_{1ry}} \sin L_{1i} y - e^{-L_{2ry}} \sin L_{2i} y\right)$$

$$L_{1r} = -\frac{1}{2} + \frac{1}{2}\left[\frac{1}{2}\left\{\left(1+M^2\right)+\omega^2\right\}^{\frac{1}{2}} + \frac{1}{2}(1+M)\right]^{\frac{1}{2}},$$

$$L_{1i} = \frac{1}{2} + \frac{1}{2}\left[\frac{1}{2}\left\{\left(1+M^2\right)+\omega^2\right\}^{\frac{1}{2}} - \frac{1}{2}(1+M)\right]^{\frac{1}{2}},$$

$$L_{2r} = -\frac{1}{2} + \frac{1}{2}\left[\frac{1}{2}\left\{\left(1+M^2\right)+4\omega^2\right\}^{\frac{1}{2}} + \frac{1}{2}(1+M)\right]^{\frac{1}{2}},$$

$$L_{2i} = \frac{1}{2} + \frac{1}{2}\left[\frac{1}{2}\left\{\left(1+M^2\right)+4\omega^2\right\}^{\frac{1}{2}} - \frac{1}{2}(1+M)\right]^{\frac{1}{2}},$$

On separating real part of equation (15), we get

$$\tau = L_0 + \varepsilon|P|\cos(\omega t + \lambda) + \varepsilon^2|Q|\cos(2\omega t + \delta) + \ldots\ldots\ldots\ldots \quad (17)$$

Where

$$|P| = \left(P_r^{\ 2} + P_i^{\ 2}\right), \quad |Q| = \left(Q_r^{\ 2} + Q_i^{\ 2}\right), \quad \lambda = \tan^{-1}\left(\frac{P_i}{P_r}\right), \quad \delta = \tan^{-1}\left(\frac{Q_i}{Q_r}\right)$$

$$P_r = L_{1r} - \frac{4AL_0}{\omega}L_{1i}, \quad P_i = L_{1i} + \frac{4AL_0}{\omega}\left(L_{1r} - L_0\right)$$

$$Q_r = -\frac{8A^2L_0^{\ 3}}{\omega^2} + L_{2r}\left(1 - \frac{4AL_{1i}}{\omega} + \frac{8A^2L_0^{\ 2}}{\omega^2} - \frac{16A^2L_0L_{1r}}{\omega^2}\right) + L_{2i}\left(\frac{-4AL_{1i}}{\omega} + \frac{16A^2L_0L_{1i}}{\omega^2}\right) + L_{1r}\left(-\frac{4AL_{1i}}{\omega} + \frac{16A^2L_0L_{1r}}{\omega^2}\right) + L_{1i}\left(\frac{-4AL_{1r}}{\omega} + \frac{16A^2L_0L_{1i}}{\omega^2}\right)$$

$$Q_I = L_{2r}\left(\frac{4AL_{1r}}{\omega} - \frac{16A^2L_0L_{1i}}{\omega^2}\right) + L_{2i}\left(1 - \frac{4AL_{1i}}{\omega} + \frac{8A^2L_0^{\ 2}}{\omega^2} - \frac{16A^2L_0L_{1r}}{\omega^2}\right) + L_{1r}\left(\frac{-4AL_{1r}}{\omega} + \frac{16A^2L_0L_{1i}}{\omega^2}\right) + L_{1i}\left(\frac{-4AL_{1i}}{\omega} + \frac{16A^2L_0L_{1r}}{\omega^2}\right)$$

Results and Discussion:

By considering first and second harmonic terms we have discussed the effect of small and large frequencies on velocity field for wide range of injection parameter variation. The amplitude and phase of the first harmonic fluctuations of skin friction have been also studied. Putting $M = 3$ and $\omega = 1,2,3$, we have calculated the velocity field for $A = 0.0, 0.1, 0.2\ldots\ldots\ldots 1.0$ and illustrated graphically. Graph 1, 2 and 3 show the variation of u when $\omega t = \frac{\pi}{2}, \varepsilon = \frac{1}{2}, y = 1$ and $R_3 = 0$ or $R_3 \neq 0$ where, $u = u_1$, if

$R_3 = 0$ and $u = u_2$ if $R_3 \neq 0$ .Graphs 1 and 2 show that with increase in A, u_1 decreases slowly at small ω and fastly at large ω. Graph 3 shows that u_2 is negative at large ω and its magnitude increases with increase in A.From this we conclude that the reverse flow may occur due to the consideration of second harmonic term in the expression of velocity field.

For small and large frequencies, the variation of amplitude and phase of the skin friction are shown in the graph 4 , 5 and 6. Graph 4 shows that increase in A leads to a decrease in skin friction amplitude $|P|$. From graph 6, it can be seen that at large ω, phase λ of skin friction increases rapidly with the increase in A.

Table -1: Velocity Field Relative to A for Fixed Frequency Parameter

A	u_1		
	$\omega = 1$	$\omega = 2$	$\omega = 3$
0.0	0.2907	0.3680	0.3260
0.2	0.2669	0.3460	0.3066
0.4	0.2480	0.3274	0.2880
0.6	0.2293	0.3110	0.2770
0.8	0.2134	0.2960	0.2565
1.0	0.2025	0.2810	0.2456

Table -2: Velocity Field Relative to A for Fixed Value of M

A	u_1
0.0	0.2275
0.2	0.1919
0.4	0.1559
0.6	0.1250
0.8	0.0934
1.0	0.0618

Table -3: Variation of Velocity Field u_2 Relative to A

A	u_2
0.0	0.0688
0.2	0.1289
0.4	0.1852
0.6	0.2395
0.8	0.2900
1.0	0.3374

Table -4: Amplitude Variation of Skin Friction $|P|$ Relative to A

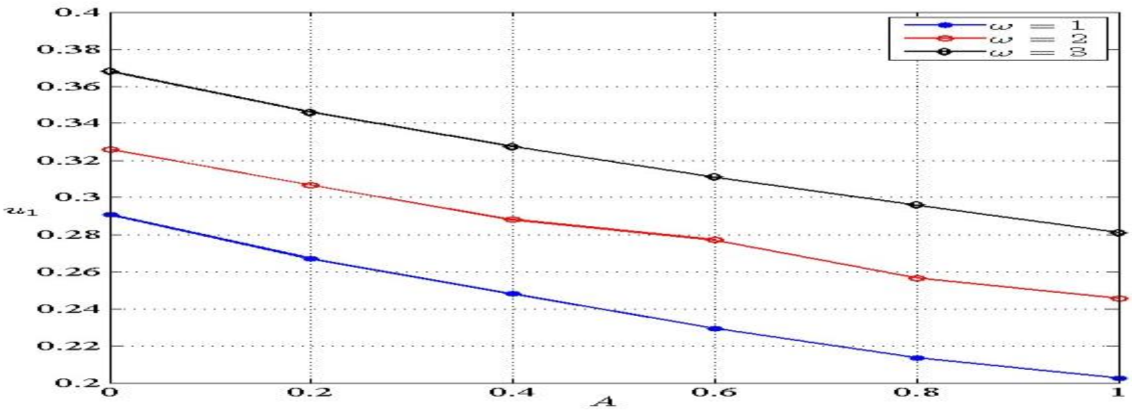
A	$\lvert P \rvert$		
	$\omega = 1$	$\omega = 2$	$\omega = 3$
0.0	0.5200	1.250	0.673
0.2	0.4230	1.165	0.608
0.4	0.3442	1.073	0.546
0.6	0.2660	1.008	0.480
0.8	0.2000	0.946	0.427
1.0	0.1270	0.892	0.381

Table -5: PhaseVariation of Skin Friction $\tan\lambda$ Relative to A for Different Frequency Parameter

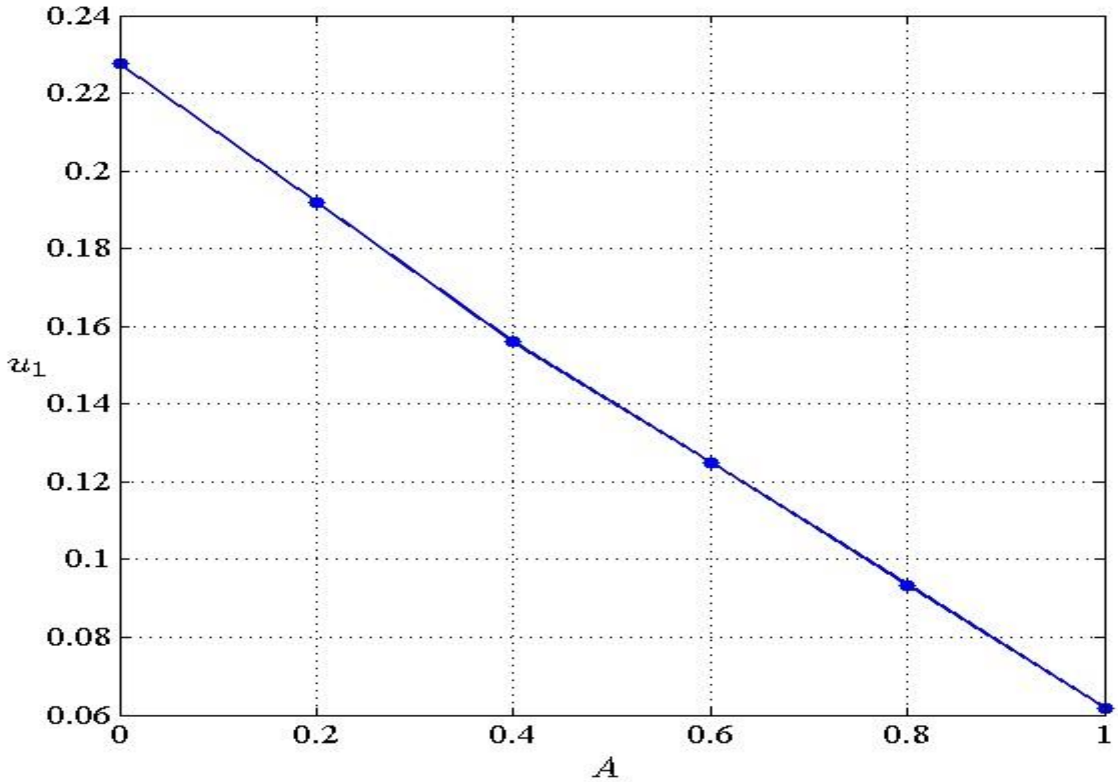
A	$\tan\lambda$		
	$\omega = 1$	$\omega = 2$	$\omega = 3$
0.0	0.2470	0.6520	0.4460
0.2	0.2760	0.6850	0.4870
0.4	0.3120	0.7380	0.5290
0.6	0.3400	0.7970	0.5570
0.8	0.3820	0.8520	0.6160
1.0	0.4510	0.9670	0.7490

Table -6: PhaseVariation of Skin Friction $\tan\lambda$ Relative to A for Fixed Frequency Parameter

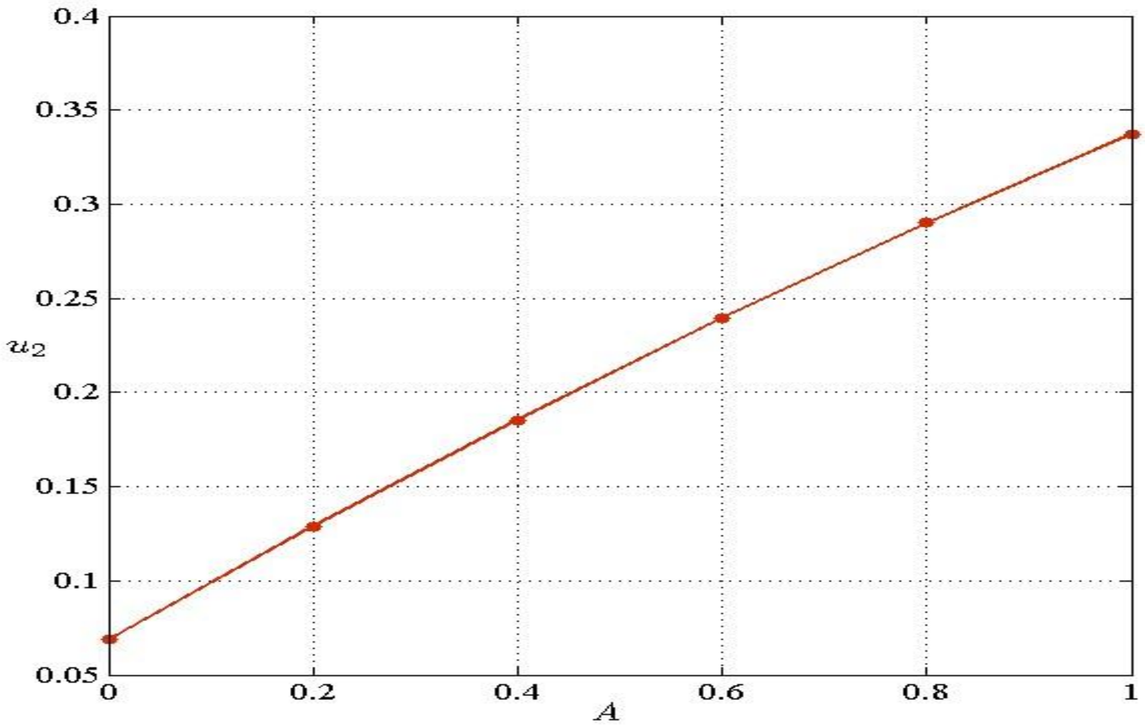
A	$\tan\lambda\ (\omega = 10)$
0.0	1.075
0.2	1.12
0.4	1.17
0.6	1.223
0.8	1.284
1.0	1.365



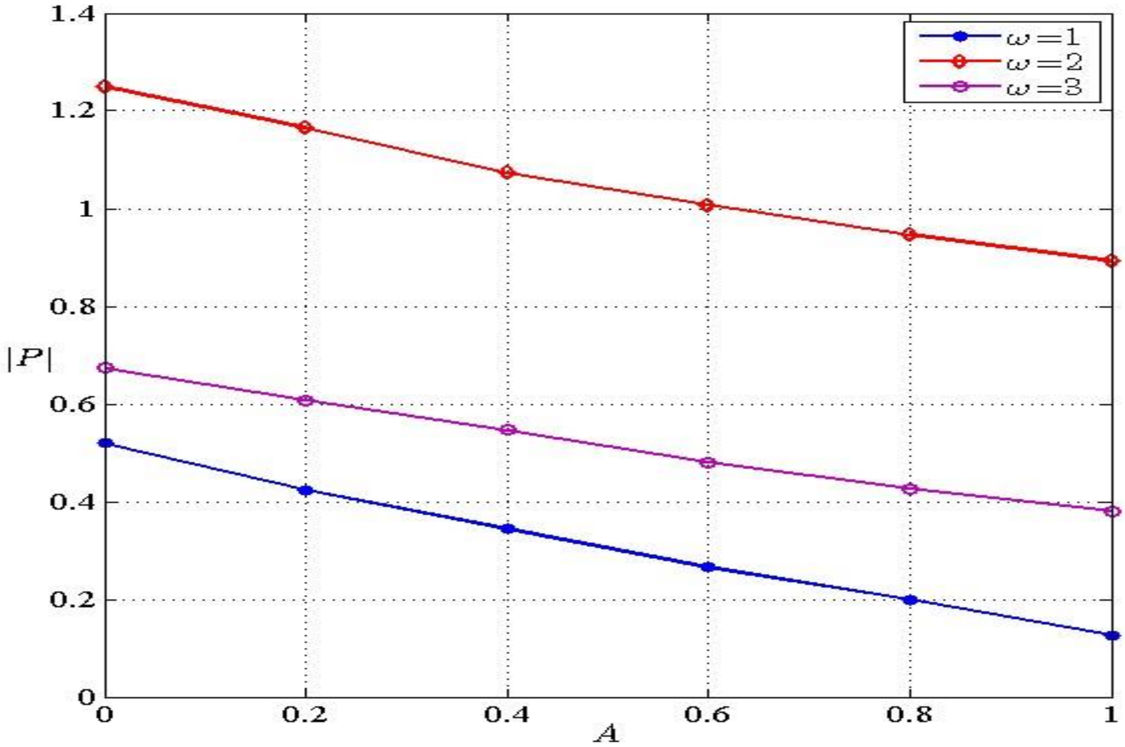
Graph-1: Velocity Field u_1 Relative to A for Fixed Frequency Parameter



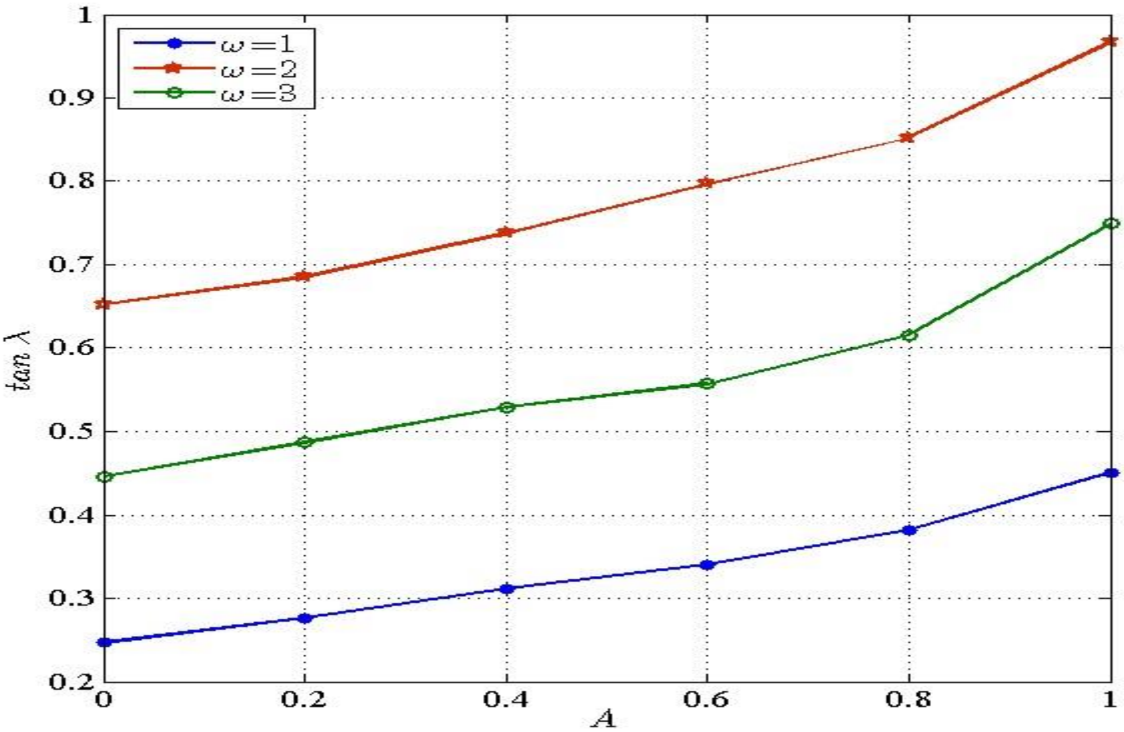
Graph-2: Velocity Field u_1 Relative to A for Fixed Value of M



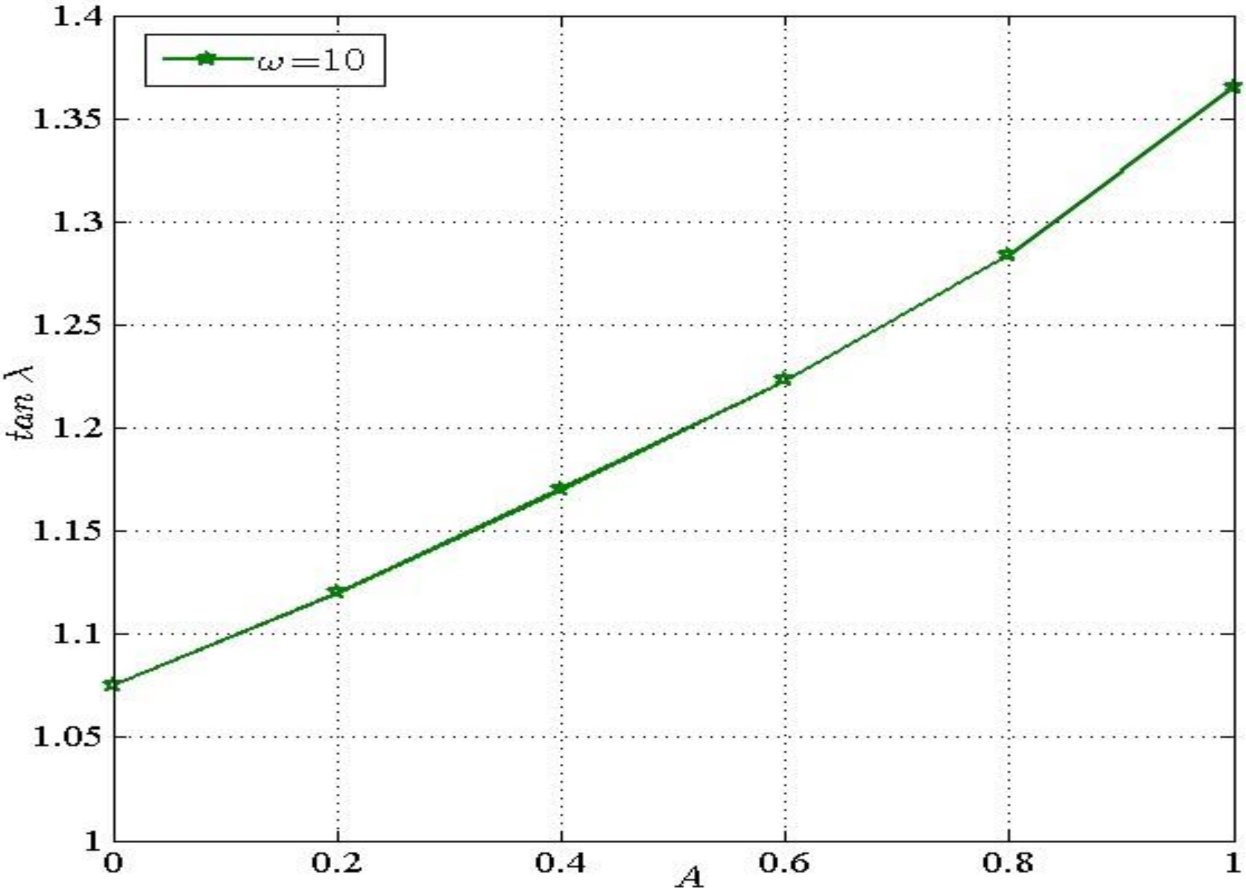
Graph-3: Variation of Velocity Field u_2 Relative to A



Graph-4: Amplitude Variation of Skin Friction $|P|$ Relative to A



Graph-5: Phase Variation of Skin Friction $\tan\lambda$ Relative to A for different Frequency Parameter



Graph - 6: Phase Variation of Skin Friction $\tan\lambda$ Relative to A for fixed FrequencyParameter

References:

(1) Lighthill, M.J(1954): The Response of Laminar Skin Friction and Heat Transfer to fluctuations in the stream Velocity''. Proc. Roy. Soc. Land. Vol. A (224), pp. 1-23.

(2) Stuart, J.T (1955): Proc. Roy. Soc., Vol. A (231), p 116.

(3) Lin, C.C (1957): Proc 9th Intl. Congress Appl. Mech., Vol. 4, p. 155.

(4) Ong, R.S and Nicholls, J.A (1959): J. Aero/ Space Sci., Vol.26, p-313.

(5) Rao, U.S (1962): ZAMM, Vol.42, p.133.

(6) Pop, I. (1967): Buletinul Inst. Politechnic, Din. IASI Serie Nova Tomil, XII(XVII), pp-3-4.

(7) Lal, K. (1972): Ind. J. Physics., Vol.46,p-393.

(8) Soundalgekar, V.M (1979): "Effect of Free Convection Currents on the Oscillatory Flow of Electrically Conducting Fluid Past an Infinite Vertical Porous Plate with Constant Suction and Heat Flux.''ZAMM (Germany), Vol.59, No.1, p-69.

(9) Soundalgekar, V.M.and Bhat, J.P (1990): Bull. Cal. Math. Soc., Vol.82, pp.93-98.

(10) SanatanDas, Nirmalandu Gara and Rabindranath Jana (2013): Hall Effect Oscillatory MHD Coquette flow Under Boundary Layer Approximation. Journal of Nature Science and Sustainable Technology, Vol.7 No.3, pp. 291-308.