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Solving Trapezoidal Intuitionistic Fuzzy Assignment Problem Using Modified Best Candidate Method

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Abstract:
The concept of an Intuitionistic Fuzzy Number (IFN) is of importance for representing an ill-known quantity. Ranking fuzzy numbers plays a very important role in the decision process, data analysis and applications. An assignment problem is a type of linear programming problem which deals with assigning various jobs to equal number of machines in such a way that the total cost is minimized by maximizing the profit. In this paper, we investigate Symmetric Trapezoidal Intuitionistic Fuzzy number by using a ranking procedure. The classical assignment problem expounded is solved by adopting Modified Best Candidate method which optimizes the cost efficiency.
Keywords: Intuitionistic Fuzzy Number, Ranking Method, Defuzzification technique.

1. Introduction:

Assignment problem is used in solving real world problems. Assignment problems is an impressive subject and is employed all the time in solving problems of engineering and management science and has been widely applied in both manufacturing and service systems. In an assignment problem, n jobs are to be performed by n machines depending on their efficiency to do the job. In an assignment problem Cij denotes the cost of assigning the j^{th} job to the i^{th} person. We assume that one machine can be assigned exactly one job. Zadeh (1965) [12] introduced the concept of fuzzy set to handle the problem of uncertainty in the evaluation of many real-life situations. Atanassov. K (1986) [1] introduced Intuitionistic fuzzy set (IFS) as an extension to the fuzzy set, where the degree of membership denoting a non-belongingness to a set is explicitly specified along with degree of membership of belongingness to the set. In 1987. Annie Varghese, Sunny Kuriakose [11] used Magnitude Ranking technique to defuzzify the Symmetric Trapezoidal Intuitionistic Fuzzy Number. [3] Hlayel Abdallah Ahmad proposed a new method called as Best Candidate Method (BCM) of electing the candidates among the other candidates and finding the solution for the optimization problems. This method increases Solution optimality comparatively than other classic methods.[1] Furthermore with slight modifications Hlayel extended the method and proposed Modified Best Candidate method which reduces the time complexity and size scalability.

In this paper we introduce the concept of solving Intuitionistic Fuzzy Assignment Problem Using Modified Best Candidate Method. The Symmetric Trapezoidal Intuitionistic Fuzzy Number (STrIFN) is defuzzified by Magnitude ranking method and the obtained classic AP is solved by Modified BCM with a numerical example.

2.Preliminaries:

2.1 Fuzzy Sets:

If X is a collection of objects denoted generically by x, then the fuzzy set A in X is a set of ordered pairs $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)), |x \in X\}$ is called the membership function of x in A that maps X to the membership space M (When M contains only the two points 0 and 1,$\tilde{A}$ is non fuzzy and $\mu_{\tilde{A}}(x)$ is identical to the characteristic function of a nonfuzzy set). The range of the membership function is a subset of the nonnegative real numbers whose supremum is finite.

2.2 Normal Fuzzy Set:

A fuzzy set A of the universe of discourse X is called a normal fuzzy set implying that there exist at least one x ε X such that μA(x) = 1.

2.3 α – cut:

The α Cut of a α level set of a fuzzy set $\tilde{A}$ is a set consisting of those elements of the universe X whose membership values exceed the threshold level α

$$\tilde{A}_\alpha = \{x \,|\mu_{\tilde{A}}(x) \geq \alpha\}$$

2.4 Intuitionistic Fuzzy set:

An Intuitionistic fuzzy set (IFS) A assigns to each element x of the universe X a membership degree A

(x) □ □0,1□ and a non-membership degree □A (x) □□0,1□ such

That $\mu_A(x)+\gamma_A(x)\le 1$. IFS is mathematically represented as $\{\langle x,\mu_A(x),\gamma_A(x)\rangle / x\in X\}$.

2.5 Intuitionistic Fuzzy number:

1. Let A□F(R) and A is normal that is there exists x□R such that A□ x□ =1 then A is called a fuzzy number.
2. Whenever □ □ □0,1□ then A□ □ □x, A□ x□ □ □□ is a closed interval denoted by

$$\lambda[A_\lambda^-, A_\lambda^+]$$

An Intuitionistic fuzzy subset $A=\{\langle x,\mu_A(x),\gamma_A(x)\rangle / x\in R\}$ of the real line is called an intuitionistic fuzzy number if $x_0, x_1\in X$ such that $\mu_A(x_0)=1, \mu_A(x_1)=1$

A is convex and μ_A is upper semi continuous and $\gamma_A$is lower continuous.

2.6 Trapezoidal Intuitionistic Fuzzy number:

A TIFN $\tilde{A}'$ is an intuitionistic fuzzy set R with the following membership function $\mu_A(x)$ and non – membership function $\gamma_A(x)$:

$$\mu_A(x)=\begin{cases}0, & for\ x<a_1\\ \dfrac{x-a_1}{a_2-a_1}, & for\ a_1\le x\le a_2\\ 1 & for\ a_2\le x\le a_3\\ \dfrac{x-a_4}{a_3-a_4}, & for\ a_3\le x\le a_4\\ 0 & a_4<x\end{cases}$$

$$\mu_A(x)=\begin{cases}0, & for\ x<b_1\\ \dfrac{x-b_1}{b_2-b_1}, & for\ b_1\le x\le b_2\\ 1 & for\ b_2\le x\le b_3\\ \dfrac{x-b_4}{b_3-b_4}, & for\ b_3\le x\le b_4\\ 0 & b_4<x\end{cases}$$

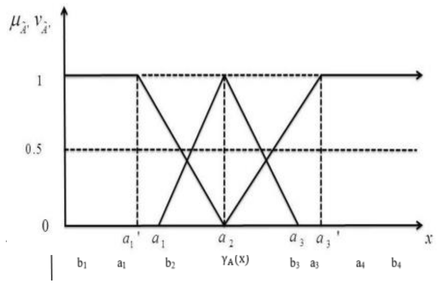

Figure 1. Membership function of Trapezoidal Intuitionistic fuzzy number

2.7 Symmetric Trapezoidal Intuitionistic Fuzzy Number:

An IFN $\tilde{A}$ in R is said to be a symmetric trapezoidal intuitionistic fuzzy number if there exists real numbers $a1$, $a2$, h, h' where $a1\le a2$, $h\le h'$ and $h, h'>0$ such that the membership and non-membership functions are as follows:

$$\mu_{\tilde{A}}(x)=\begin{cases}\dfrac{x-(a_1-h)}{h}, & for\ a_1-h\le x\le a_1\\ 1 & for\ a_1\le x\le a_2\\ \dfrac{a_2+h-x}{h}, & for\ a_2\le x\le a_2+h\\ 0 & otherwise\end{cases}$$

2.8 Magnitude Ranking function :

The magnitude of a symmetric trapezoidal intuitionistic fuzzy number,

$A=(b1, a1, a2, a2, a3, a3, a4, b4)$ is given as

$$Mag(A)=\frac{1}{12}[a_1+b_1+a_4+b_4+4(a_2+a_3)]$$

3. Problem Formulation:

3.1 Fuzzy Assignment Problem:

Let $\tilde{C}\ ij$ be the triangular fuzzy numbers cost (payment) if jth job is assigned to pth person (see table). The problem is to find an assignment xij so that the total cost for performing all the jobs is minimum.

Table 1. Fuzzy Assignment cost

Job → *Person ↓*	*Job 1*	*Job 2*	*Job k*	*Job n*
Person 1	$\tilde{C}_{11}$	$\tilde{C}_{12}$	$\tilde{C}_{1k}$	$\tilde{C}_{1n}$
Person k	$\tilde{C}_{k1}$	$\tilde{C}_{k2}$	$\tilde{C}_{kk}$	$\tilde{C}_{kn}$
Person n	$\tilde{C}_{n1}$	$\tilde{C}_{n2}$	$\tilde{C}_{nk}$	$\tilde{C}_{nn}$

The chosen Fuzzy Assignment Problem (FAP) may be formulated into the following fuzzy linear programming problem:

$$Min\ Z=\sum_{i=1}^{i=n}\sum_{j=1}^{j=n}\tilde{C}_{ij}x_{ij}$$

Subject to:

$$(AP)=\begin{cases}\displaystyle\sum_{i=1}^{i=n}x_{ij}=1, & j=1,2,\ldots\ldots\ldots\ldots\ldots n\\ \displaystyle\sum_{j=1}^{j=n}x_{ij}=1 & i=1,2,\ldots\ldots\ldots\ldots\ldots n\end{cases}$$

where $x_{ij}=$

$$\begin{cases}1 & ,if\ the\ i^{th}\ person\ assign\ the\ j^{th}\ job\\ 0 & ,otherwise\end{cases}$$

3.2 Algorithm for Solving Assignment Problem using Modified Best Candidate Method:

Our method is based on determination of the best candidates then eliminate the unwanted one in order to minimize the number of solution combinations to decide the optimal solution [Hlayel (2012)]. However, we can notice that the solution approach using this method is divided into two phases. We will describe each phase and clarify the new modifications. The first Phase, is to elect the best candidates through choosing the prime candidate and its alternative in each row depending on the objective function (maximum or minimum value) then elect one candidate for the columns that have no candidate. There are no new modifications in this phase and the solution findings with steps are as follows:

Step1: Prepare the matrix. If the matrix is unbalanced, we balance it and we would not use the added row or column candidates in our solution process.

Step2: Determination of the best candidate, it is used for minimization problems (minimum cost) or maximization problem (maximum profit): Elect the best two candidates in each row, if the candidate is repeated more than one time then elect it also. Check the columns that do not have candidates and elect one candidate from them, if the candidate repeated more than one time elect it also.

The second phase, will introduce the following steps:

a. At the end of phase one, an index matrix is produced that shows the position for each candidate.

b. Find the direct combinations and calculating the cost for each.

c. Check the unused candidates, by finding the possible candidates for them then calculate the cost for each.

d. Find the optimal solution according to the objective function.

3.3 Algorithm to solve fuzzy assignment problem with Modified BCM:

Step 1: First test whether the given fuzzy cost matrix of an fuzzy assignment problem is a balanced/unbalanced. If not change this unbalanced assignment problem by adding the dummy row (s) / column(s) and the values for the entries are zero. If it is a balanced one

then go to step 2. If it is an unbalanced one then convert it into a balanced one and then go to step2.

Step 2: Replace the cost matrix Cij with linguistic variables by triangular intuitionistic fuzzy numbers.

Step 3: Defuzzify the fuzzy cost by using Magnitude ranking method.

Step 4: Replace STrIFN by their respective ranking indices.

Step 5: Apply Modified BCM to determine the best combination to produce the lowest total weight of the costs, where elect the best two candidates in each row, if the candidate repeated more than one times elect it also. Check the columns that not have candidates and elect one candidate from them, if repeated more than once elect them.

Step 6: Construct an index matrix and find the direct combination. Calculate cost for each combination. Check for unused candidates, find the possible candidates for them and calculate cost for them also. Now find optimal solution from all the combinations.

4.Numerical Example:

The Fuzzy Assignment Problem with rows representing 4 machines M_1, M_2, M_3, M_4 and columns representing the 4 Jobs J_1, J_2, J_3, J_4. is considered. In this Fuzzy Assignment Problem, the cost matrix elements are Symmetric Trapezoidal Intuitionistic Fuzzy Numbers.

Machines	**Jobs**			
	J1	**J2**	**J3**	**J4**
M1	(5,7,1,1;5,7,3,3)	(5,6,2,2;5,6,4,4)	(11,13,8,8;11,13,10,10)	(6,8,2,2;6,8,4,4)
M2	(13,15,6,6;13,15,8,8)	(6,8,3,3;6,8,5,5)	(14,15,8,8;14,15,10,10)	(5,6,2,2;5,6,4,4)
M3	(10,11,6,6;10,11,8,8)	(6,8,2,2;6,8,4,4)	(4,5,1,1;4,5,3,3)	(6,8,1,1;6,8,3,3)
M4	(11,13,8,8;11,13,10,10)	(4,5,1,1;4,5,3,3)	(13,15,6,6;13,15,8,8)	(5,7,2,2;5,7,4,4)

Solution:

The given problem is a balanced problem. Now we have to obtain $R(\tilde{C}_{ij})$ of each $(\tilde{C}_{ij})$ using magnitude ranking function as follows,

$$Mag(A) = \frac{1}{12}[a_1 + b_1 + a_4 + b_4 + 4(a_2 + a_3)]$$

For (5,7,1,1;5,7,3,3)

$$Mag(A) = \frac{1}{12}[7 + 5 + 5 + 7 + 4(1 + 3)]$$

$$Mag(A) = \frac{1}{12}[24 + 16]$$

$$Mag(A) = 3.33$$

Similarly, the other values are determined and they are as follows,

Machines	Jobs			
	J1	J2	J3	J4
M1	3.33	3.83	10	4.33
M2	9.33	5	10.83	3.83
M3	8.17	4.33	2.83	3.67
M4	10	2.83	9.33	4

In Conformation to model the fuzzy assignment problem can be formulated as:

$$\text{Min}\{R(3.33)x_{11} + R(3.83)x_{12} + R(10)x_{13} + R(4.33)x_{14} + R(9.33)x_{21} + R(5)x_{22} + R(10.83)x_{23} + R(3.83)x_{24} + R(8.17)x_{31} + R(4.33)x_{32} + R(2.83)x_{33} + R(3.67)x_{34} + R(10)x_{41} + R(2.83)x_{42} + R(9.33)x_{43} + R(4)x_{44}$$

Subject to

$$x_{11} + x_{12} + x_{13} + x_{14} = 1 \qquad x_{11} + x_{21} + x_{31} + x_{41} = 1$$

$$x_{21} + x_{22} + x_{23} + x_{24} = 1 \qquad x_{12} + x_{22} + x_{32} + x_{42} = 1$$

$$x_{31} + x_{32} + x_{33} + x_{34} = 1 \qquad x_{13} + x_{23} + x_{33} + x_{43} = 1$$

$$x_{41} + x_{42} + x_{43} + x_{44} = 1 \qquad x_{14} + x_{24} + x_{34} + x_{44} = 1$$

where $x_{ij} \in [0,1]$

We solve it by modified best candidate method to get the following optimal solution.

Phase 1: Election of Candidates

Step 1: The matrix is Balanced, where the number of rows is equal to the number of columns as shown in table 1.

Table 1: Person-Job assignment Profit matrix after balance

Machines	Jobs			
	J1	J2	J3	J4
M1	3.33	3.83	10	4.33
M2	9.33	5	10.83	3.83
M3	8.17	4.33	2.83	3.67
M4	10	2.83	9.33	4

Step 2: Elect the best Candidates as shown in table 2

Table 2: Best Candidates Determination Matrix

Machines	Jobs			
	J1	J2	J3	J4
M1	3.33	3.83	10	4.33
M2	9.33	5	10.83	3.83
M3	8.17	4.33	2.83	3.67
M4	10	2.83	9.33	4

Phase 2: Obtain the BCM Combinations.

a. Draw the following index matrix (Table 3) showing the position of each candidate

Table 3: Best Candidate Combination Position Matrix.

Machines	Jobs			
	J1	J2	J3	J4
M1	A1	A2	-	-
M2	-	B2	-	B4
M3	-	-	C3	C4
M4	-	D2	-	D4

From the above table we obtain the solution set {A2, A3, B2, B3, C1, C3}.

b. The direct combinations for all the candidates from the solution set and calculate the cost for each:

Combination 1: {A2, B2, C3, D4} = 3.33+5+2.83+4 = 15.16

Combination 2: {A1, B4, C3, D2} = 3.33+3.83+2.83+2.83 = 12.82

c. Check for unused candidates in the solution set{A2}, then find the possible combinations:

Combination 3: {A2, B4, C3} then we add to them D1 and become

{A2, B4, C3, D1} = 3.83+3.83+2.83+10 = 20.49

d. Find the optimal solution according to the objective function (maximum of minimum cost):

In our case it is combination number 2 (modified-BCM solution).

Machine M1 → Job J1
Machine M2 → Job J4
Machine M3 → Job J3
Machine M4 → Job J2

5.Conclusion:

In this paper, the fuzzy assignment costs are taken to be as Symmetric Trapezoidal Intuitionistic Fuzzy Number. The membership and non-membership functions of a STrIFN are defuzzified using a magnitude ranking function. Further the assignment problem is solved by using Modified Best Candidate Method for its time reducibility and optimal solution.

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