Journal of Harbin Engineering University Vol 44 No. 7
ISSN: 1006-7043 July 2023

A New Novel Gaussian Kernel-Induced Particle Swarm Optimization Based Spatial FCM Algorithm for Image Segmentation

Anju Bala[1]*, Aman Kumar Sharma[2]
Research Scholar[1], Professor[2]
Computer Science Department, H.P.university
[1]anjubala.hpu@gmail.com
[2]sharmaas1@gmail.com

Abstract—The advancement in image processing has witnessed massive progress in image segmentation technologies in medical as well as other fields. However, these advancements in technologies have not only made image analyst's life easy but also given them more challenging goals in image segmentation. Fuzzy c-means an unsupervised clustering algorithm plays a crucial role in the field of image segmentation. But suffer from many problems, such as center initialization, trapping into local optima and noise sensitivity. There are many algorithms which try to solve these problems. But no single approach is there which solves all these problems all alone. Hence, this paper proposed a novel Gaussian Kernel-induced PSO-based spatial FCM (GKPSOSFCM) which uses Gaussian Kernel Based Distance instead of Euclidean distance to increase efficiency, including spatial information to reduce noise sensitivity and Particle Swarm optimization that helps in finding global solutions. The result of the proposed method is compared with conventional Fuzzy C-means to demonstrate the efficiency of the proposed method. Experiments show that the proposed method outperforms the standard FCM algorithm and gives better-segmented results.

Keywords— Particle Swarm Optimization, Fuzzy c means, image segmentation, Gaussian Radial base Kernel function, membership function, spatial function.

I. Introduction

Image segmentation is one of the emerging fields of image processing. To process or analyse an image it is necessary to segment it into objects. Image segmentation is a process that classifies images into different regions or objects according to the continuity or discontinuity of intensities of pixels. The group of pixels having similar properties such as color, texture, intensity etc. is classified into an object. There are various image segmentation techniques present in the literature, but the most widely used technique is unsupervised clustering.

Clustering divides the image into clusters where pixels belong to different clusters which shows the similarity in pixels of the same cluster and dissimilarity in pixels of different clusters [1]. Clustering is generally of two types: hierarchical and partitioning. Hierarchical clustering works in a nested hierarchy manner by taking an input and generating the corresponding sequence of clustering as an output. It is not able to isolate overlapping clusters due to lack of information about the shape or size of the clusters but it does not require cluster number prior. On the other hand, the partitioning clustering method is based on the objective function for partitioning the input into a set of fixed number of clusters to produce the output. It requires the cluster number to be specified in advance as well as the initialization of cluster centers. This clustering process starts with a random cluster partition and gradually refines the partition for the optimization of the objective function value [2]. Again, partitioning clustering can be classified into either hard clusters or soft clusters. In hard clustering, each pixel belongs specifically to one cluster, with the nature of "either-or". But it is not possible for complex images and to overcome this condition soft clustering is introduced. In soft clustering "and also" concept is introduced which means a pixel can be part of more than one clusters [3]. Soft clustering is also known as fuzzy clustering which takes care of overlapping clusters successfully.

The fuzzy set theory was introduced by Zadeh in 1965 [4]. Based on this theory Bezdek proposed fuzzy c-means clustering that is widely used in image segmentation because of its information-retaining capability [5]. In Fuzzy c-means, a pixel can be a part of more than one cluster by representing a degree of membership to the corresponding cluster. It makes FCM better than hard clustering or K-means clustering [6] [7]. Along with advantages it also has many disadvantages: i) FCM is sensitive to noise or artefacts present in the image which leads to over or under-segmentation, ii) the cluster number should be known prior which requires prior knowledge of the input image, iii) the initial cluster center determines the accuracy or efficiency of FCM segmentation, if the initial cluster centers are not good enough the FCM can be trapped in local optimum conditions [8], iv) the Euclidean distance used in the objective function of the FCM good enough only for spherical shapes.

To overcome these shortcomings of the FCM algorithm many improved versions of FCM were proposed in literature.

To provide better cluster separability, more recently, great research efforts emphasis on kernel-based clustering[9] The Kernel distance metric is a well-known clustering technique and can effectively identify non-linearly separable structures [10]. Kernel-based clustering algorithms usually reformulate the distance measure of data samples with kernel functions [11] and have been proven to achieve better performance in comparison to their original fuzzy algorithm [12]. There are two major branches of kernel-based fuzzy clustering: one with prototypes retained in the original feature space (KFCM-F), and the other with prototypes defined in kernel space (KFCM-K) [11]. The kernel provides the capability of capturing the different structures of data. Introducing a new kernel-induced metric with a spatial constraint, Chen and Zhang proposed a noise-robust FCM algorithm for medical image segmentation [13].

The optimization techniques used for cluster number estimation and center initialization such as Genetic algorithm (GA) [14], teaching learning-based optimization (TLBO) [2], Ant Colony Optimization (ACO) [15], particle swarm optimization(PSO) [16]. The GA and ACO could obtain efficient FCM with an automatically initialized center and better clustering performance. The GA algorithms have many parameters to set up and are not good for complex problems. The GA sometimes failed to find out the global optimal solution [17]. The ACO has its advantages but is not as good as PSO for fuzzy clustering [18]. So, the PSO is considered for this research because of its simplicity and bigger optimization capability over GA and ACO.

This paper proposes an FCM algorithm having Kernel-induced metric with spatial constraints to increase the noise immunity and better outlier rejection than standard FCM. The PSO optimization technique is also introduced to obtain the global optimum results. This study formulates the Gaussian-based kernel distance function as a fitness function for PSO to initialize cluster centers. The cluster centers obtained from PSO are given as input to the kernel-induced spatial membership function of FCM and results show improved performance. The FCM-PSO generates efficient segmentation results and is robust to noise and intensity inhomogeneity. The results of new novel Gaussian Kernel-Induced Particle Swarm Optimization based spatial FCM (GKPSOSFCM) are analyzed and validated using different metrics.

The rest of the paper is organized as follows: In section 2 includes related work on FCM and PSO algorithms, section 3 presents a new proposed method, section 4 describes experimental results and Section 5 will give concluding remarks.

II. Methods

The FCM is an unsupervised clustering algorithm that produces a very good segmentation result efficiently. But it is sensitive to noise which is since it does not include spatial information. It also uses the Euclidean distance function as the objective function which leads to trapping in the local optima. These two factors hinder the overall performance of the FCM. This study tried to solve these problems by adding spatial constraints and replacing Euclidean distance by kernel Induced objective function. This leads to an increase in computation time and thus to optimize the overall process Particle

Swarm Optimization is used. This study aims to give a segmentation method that is robust to noise and gives global results. The proposed method is implemented using the Matlab 2013a tool on five test images taken from Berkeley's database. These are colored images and no pre-processing is done to enhance the quality of images. The FCM is improvised and all the parameters are tuned automatically to reduce the interventions of humans. The cluster numbers are generated from the method present in the literature and cluster centers are initialized using PSO. The PSO has some of the parameters fixed values such as population is 50 and maximum iterations are also 50 whereas other parameters are calculated during the execution of the program and depend upon the image feature values which makes the process auto-adjustable according to the type of image helps in finding out good segmentation results. The main aim of this study is to propose a method that requires less human intervention and gives good segmentation results.

III. Related work:

Image segmentation is considered one of the crucial steps in image processing and FCM clustering has been a commonly used method for segmentation.

Fuzzy c means is considered as simplest clustering algorithm used for image segmentation. As there are some drawbacks of FCM the literature has many studies considering the drawbacks and proposed various methods to overcome them. Tolias and Panas (1998) introduced spatial constraints in the membership function of fuzzy c-means. This also solves the cluster number problem with the adaptation of cluster prototypes to the image characteristics [19]. Chen and Zhang (2004) found that the inclusion of spatial information leads to noise insensitivity to some extent but fails to noise robustness and outlier rejection. For that purpose, Kernel-Induced membership function is used [13]. Chen et al. (2007) proposed a fast and robust method that incorporates spatial information in it. This leads to noise robustness and fast convergence but fails to consider the center initialization problem which leads to trapping in local optima [20]. Graves and Pedrycz (2010) presented a Kernel-based FCM and found that it gives effective segmentation results whereas sensitive to values of kernel parameters [11]. Memon et al. (2018) found that FCM with local information makes FCM noise robust to some extent, however for high noise-contaminated images Kernel Possibilistic FCM performs better [21]. Hu and du (2019) realized that the kernel FCM is more robust to noise still the neighborhood pixels affect the segmentation results. To improve the segmentation results spatial constraints are used along with kernel FCM and found it more efficient [22].

The study concludes that the FCM with Kernel distance metrics and spatial constraints makes the process more robust to noise and realizes efficient segmentation. But this leads to increased computational cost which is then a topic of great concern.

To reduce the computational cost of various improved versions of FCM many optimization techniques are introduced. The optimization techniques not only optimize the process but also help in improving the segmentation results. Ding and Fu (2016) proposed a Kernel FCM based on Genetic algorithms to improve the clustering performance. In this paper genetic algorithm is used to initialize the cluster centers which found better segmentation results [23]. But the run time increases and GA is not considered to optimize the computational cost. Venkatesan and Parthiban (2017) introduces kernelized FCM hybridized on QPSO to obtain more robust segmentation results and have low computation cost. But applies only to MRI medical images [24]. As the study find out there are very few research papers on this topic and this can be considered as a research concern.

It is also found from the literature survey that a maximum of algorithms are applied to medical images and avoid color images whereas, with the advancement of technology, color images are in trend. The other thing found in the literature survey is that the optimization is applied to solve only one problem which is initialization cluster centers and ignores others such as intensity inhomogeneity, spatial information, noise sensitivity etc.

IV. Proposed Algorithm:

This paper proposes a new novel Gaussian Kernel-induced PSO-based spatial FCM (GKPSOSFCM) algorithm that does not use optimization just to initialize the cluster centers but optimizes the whole FCM process. The kernel-induced objective function is used to make the process noise robust and spatial constraints are included to make it more effective. This makes the proposed method of great importance and different from the various methods present in the literature. The proposed method works in two main steps that are:

1. The first step using PSO to initialize the cluster centers which also has a twist that it does not use the Euclidean distance Metric as a fitness function to find out the initial cluster centers. But the Gaussian Kernel function is used as an objective function.

2. The second step is the membership function that includes spatial constraints and kernel metric as distance function to make the results more accurate.

There are no pre-processing steps to enhance the image so that the results are obtained on the original image.

The detail of each step is discussed in the following sub-sections.

GKPSOSFCM

The PSO is a population-based metaheuristic optimization technique. It can be used to search the global optimal solutions in a large and complex search space. In the search for an optimal solution, the particles move from one location to another with a certain velocity and change their position until they reach an optimal solution. The Particle Swarm Optimization uses a fitness function that is to be minimized and particles continue their journey until the fitness function is optimized and find the optimal solution.

The quality of all swarm positions is determined by incorporating decision variables into the objective function. The proposed method does not include Euclidean distance as a fitness function because it will not take into account the geometrical shapes. The fitness function is calculated using the Gaussian Radial basis Kernel Function. The kernel functions can project the data into multi-dimensional space where the data could be better constructed using kernel trick [25]. The kernel trick is very useful in bridging linearity to non-linearity using dot products between two vectors [26]. The Gaussian radial kernel function replaces Euclidean distance to make clustering more robust to noise [27].

The proposed algorithm shows accurate segmentation results and produces homogenous clusters.

Algorithm

1. Initialization: for each particle i=1…Np, where Np represents the population and it is 20, normally.

do

(a) Initialize the particle's position with a uniform distribution as Pi(0) ~U(LB, UB), where UB and LB represent the upper bound and lower bound respectively and calculated as:

LB= minimum value of image data

UB= maximum value of image data

kappa = 1;

phi1 = 2.05;

phi2 = 2.05;

phi = phi1 + phi2;

chi = 2*kappa/abs(2-phi-sqrt(phi^2-4*phi));

w = chi;

wdamp = 1;

c1 = chi*phi1;

c2 = chi*phi2;

c1 and c2 are constriction factors to the particle's velocity-update rule to avoid the unlimited growth of the velocity of the particles [28], w represents inertia weight, wdamp is the damping ratio to set the inertia weight according to a time-decreasing function to have an algorithm that initially explores the search space as the large value of w is appropriate for global search and small value good for local search that later focuses on the most promising regions [29].

(b) Initialize pbest to its initial positions: pbest(I,0)=Pi(0)

(c) Initialize gbest to minimal value of the swarm: gbest(0)=argmin f[pi(0)]

(d) Initialize velocity:

MaxVelocity = 0.2*(VarMax-VarMin);

MinVelocity = -MaxVelocity;

2. Repeat until a termination criterion is met i.e. number of iterations 50

For each particle i=1,….Np, do

(a) Pick random numbers: r1,r2 U(0,1).

(b) Update the particle's velocity.

Velocity=w*particle(i).Velocity+c1*rand(VarSize).*(particle(i).Best.Position-particle(i).Position)+c2*rand(VarSize).*(GlobalBest.Position - particle(i).Position);

(c) Update particle position

Position = particle(i).Position + particle(i).Velocity;

3. Evaluate the fitness function

The Gaussian based Radial kernel is used as an objective function and calculated as:

$$\|\emptyset(xi) - \emptyset(vj)\|^2 = 2(1 - K(xi, vj)) \quad (1)$$

$\|\emptyset(xi) - \emptyset(vj)\|^2$ is distance metrics which replace Euclidean distance metrics.

Whereas K is the kernel function defined as:

$$K(x_i, v_j) = exp\left(-\frac{\|x_i - v_j\|^2}{2\sigma^2}\right) \quad (2)$$

The choice of Kernel width σ is still a problem as if its value is small it is sensitive to noise. So, in this paper, it is taken from image feature values.

4. If f[Pi(t)]<f[pbest(I,t)], do

(a) Update the best-known position of particle i: pbest (I,t)= Pi(t).

(b) If f[Pi(t)]<f[gbest(t)], update the swarm's best-known position: gbest(t)=Pi(t).

(c) T=(t+1);

5. Output gbest(t) that holds the global best optimal solution.

1. The membership function consisting of spatial constraints is applied. Now, the membership function is:

$$u'_{ij} = \frac{u_{ij}^p h_{ij}^q}{\sum_{k=1}^{c} u_{ij}^p h_{kj}^q} \quad (3)$$

Where hij is the spatial function defined as:

$$h_{ij} = \sum_{k \epsilon NB(x_j)} u_{ik} \quad (4)$$

Where NB(xj) represents a square window (3*3) centered on pixel xj in the spatial domain.

2. Thus the GKPSOSFCM segmented image was obtained.

V. Results and Discussion:

To investigate and evaluate the performance of the proposed method it is being compared with standard FCM. The 5 colored images from the Berkeley's dataset are considered test images and experimentation is done using Matlab2013a.

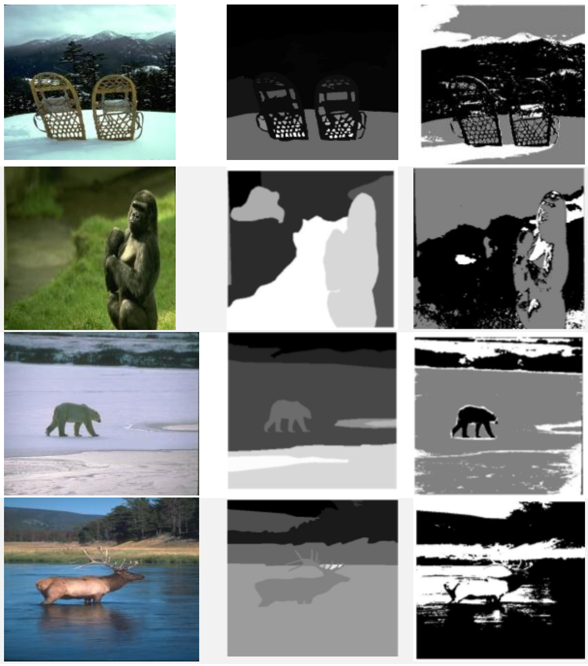

Fig. 1. Shows (1) the test image (2) the ground truth Image (3) The segmented image using the proposed method

The Fig. 1 indicates the quality and performance of the proposed method. The proposed method for each test image gives very good segmentation results visibly. These results are compared using performance metrics which show the effectiveness of the proposed method.

To calculate the cluster number for these test images the method proposed in [30] is used. Then the clustering results are validated using clustering validity indices called Partition Coefficient and Partition Entropy. The PC determines the belongingness of data or can say that it measures the amount of overlap between clusters [31-33]. PE determines the entropy measurement or fuzziness of clusters [34].

$$PC(c) = \frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{c} u_{ij}^{2} \quad (5)$$

$$PE(c) = -\frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{c} u_{ij}\, log_2\, u_{ij} \quad (6)$$

Where c is the number of clusters, N is the number of pixels, uij is the membership degree of ith pixel for jth cluster. The idea of these validity functions is that the partition with fuzziness means better performance. As a result, the best clustering is achieved when the value of PC is maximum and PE is minimum.

Table 1. Shows the Validity Indices PE and PC values for standard FCM and GKPSOSFCM

Images	**Cluster Number**	GKPSOSFCM		**FCM**	
		PC	**PE**	**PC**	**PE**
Image1	3	0.9699	0.0682	0.8313	0.3161
Image2	3	0.9632	0.4656	0.7437	0.4656
Image3	3	0.9682	0.0534	0.8261	0.3229
Image4	2	0.9515	0.0826	0.749	0.4015
Image5	5	0.9278	0.1326	0.6443	0.4268

Table 1. Shows the validity indices values for standard FCM and the proposed method, and it indicates that the proposed method gives better results. For each image the values of the proposed method for PC are high and PE is lower than the standard FCM. For example, if we take image 5 which has cluster number 5 and PC and PE values 0.9278 and 0.1326 for the proposed method whereas 0.6443 and 0.4268 for standard FCM. It shows the better clustering indices for the proposed method as we know high PC and low PE value indicates good segmentation results.

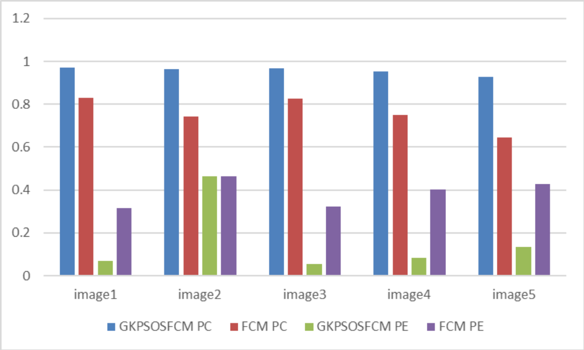

Fig. 2 shows the graphical comparison of standard FCM and the proposed method for PC and PE values

Fig. 2 is the graphical representation of Table 1. It represents the graphical comparison and indicates the superiority of the proposed method. The graph shows the significant difference between the PC and PE values of the proposed and standard FCM methods.

The other performance metrics are used to further evaluate the efficiency of the proposed method. The Precision, recall and Accuracy are considered as performance metrics. Precision tells the probability of valid results whereas recall tells about the probability of detection of ground truth value [35]. Accuracy is another measure that is used to measure the accuracy of segmentation results. It is defined as the ratio between the correctly predicted observations to the total observations.

Precision = Matched(A,B)/|A| (7)

Recall = Matched(B,A)/|B| (8)

$$Accuracy = \frac{\text{number of correct predicted pixels}}{\text{Total number of predicted pixels}} \quad (9)$$

Suppose, A is the segmentation result and B is the ground truth of the same image, the precision is proportional to the segment of edges from A that matches with the B ground truth image whereas recall is the segment of edges from B for which a suitable match is found in A.

Table 2 Accuracy, Precision and Recall values for standard FCM and Proposed GKPSOSFCM

IMAGES	**ACCURACY**		**PRECISION**		**RECALL**	
	FCM	GKPSOSFCM	FCM	GKPSOSFCM	FCM	GKPSOSFCM
Image1	0.5052	0.7604	0.7154	0.8169	0.6322	0.8377
Image2	0.4686	0.5036	0.8954	0.9348	0.555	0.6214
Image3	0.7561	0.7941	0.8495	0.9051	0.8061	0.8587
Image4	0.7208	0.7614	0.9799	0.9999	0.7069	0.7495
Image5	0.5462	0.6426	0.7401	0.7701	0.6758	0.795

The high values of Precision, Recall and Accuracy indicate good clustering results. As we can see in Table 2 the values of these performance metrics of proposed GKPSOSFCM are higher than the proposed method. If we consider image 3 accuracy, precision and recall values for the proposed method are 0.7941, 0.9051 and 0.8587 respectively whereas for standard FCM the values are 0.7561, 0.8495 and 0.8061 respectively. This indicates the vital difference between the values of the two and confirms the good segmentation results of the proposed method.

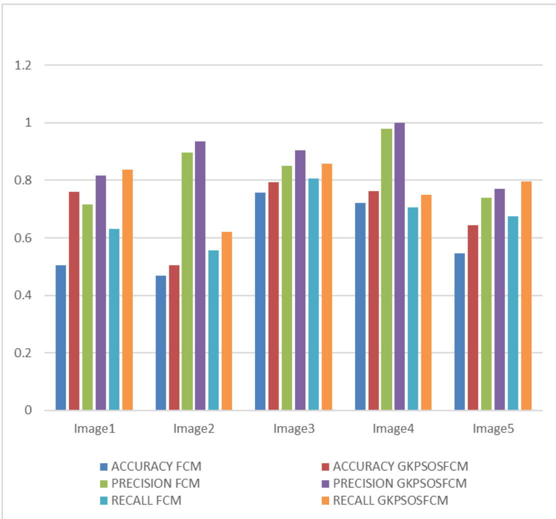

Fig. 3 Accuracy, Precision and Recall Value of standard FCM and Proposed GKPSOSFCM

The fig. 3 graphically represents the comparison of Accuracy, Precision and Recall Value of standard FCM and Proposed GKPSOSFCM for each image and indicates the significant performance of the proposed method.

VI. Conclusions:

The image segmentation is a very crucial step in image analysis and FCM is a popular image segmentation algorithm. It is inexpensive and efficient but suffers from some problems which are considered in this paper. The trapping into local optima and noise sensitivity problems are solved using spatial membership function along with Kernel induced distance function instead of Euclidean distance as well as initialization problem is solved using Particle Swarm Optimization method having kernel based objective function which makes the process more robust. Here the optimization algorithm is not only used to initialize the cluster centers but also uses the kernel metric to make the process noise insensitive. The results are evaluated using cluster validity indices PC and PE and also with performance metrics Accuracy, precision and recall. The experimental results justify that the proposed method works better than the standard method. In future, we can also check the noise robustness of the proposed method.

List of Abbreviations

FCM Fuzzy c-Means

PSO Particle Swarm Optimization

PC Partition Coefficient

PE Partition Entropy

GKPSOSFCM Gaussian kernel induced PSO based Spatial Fuzzy C-Means

Declarations

Availability of data and materials

Data sharing applies to this article as Berkeley's datasets were used during the study.

Competing interests

The authors declare that they have no competing interests.

Funding

No funding was received for conducting this study.

Authors' contributions

The first draft of the manuscript was written by AB. experimentation part was done by AB and AKS. AB and AKS performed the conceptualization of the research idea, participated in the interpretation of the results, and reviewed the edited manuscript. All authors have made a substantial contribution to the manuscript. The authors read and approved the final manuscript.

Acknowledgements

Not Applicable

References

1. Nayak, J., Naik, B., Kanungo, D., & Behera, H. (2018). A hybrid elicit teaching learning-based optimization with fuzzy c-means (ETLBO-FCM) algorithm for data clustering. Ain Shams Engineering Journal, 9(3), 379-393.
2. Zou, K.-q., Wang, Z.-p., Pei, S.-j., & Hu, M. (2009). A New Initialization Method for Fuzzy c-Means Algorithm Based on Density Fuzzy Information and Engineering (pp. 547-553): Springer.
3. Zadeh, L. A. (1965). Fuzzy sets. Information and control, 8(3), 338-353.
4. Bezdek, J. C. (2013). Pattern recognition with fuzzy objective function algorithms: Springer Science & Business Media
5. Bala, A., & Sharma, A. K. A Review of Challenges in Clustering Techniques for Image Segmentation. IJRASET, 8(12), 496-502
6. Ghosh, S., & Dubey, S. K. (2013). Comparative analysis of k-means and fuzzy c-means algorithms. International Journal of Advanced Computer Science and Applications, 4(4)

7. Ma, L., Li, Y., Fan, S., & Fan, R. (2015). A Hybrid Method for Image Segmentation Based on Artificial Fish Swarm Algorithm and Fuzzy-Means Clustering. Computational and Mathematical Methods in Medicine. 2015.
8. Nayak, J., Nanda, M., Nayak, K., Naik, B., & Behera, H. S. (2014). An improved firefly fuzzy c-means (FAFCM) algorithm for clustering real-world data sets *Advanced Computing, Networking and Informatics-Volume 1* (pp. 339-348):
9. Bodyanskiy, Y. V., Deineko, A., & Eze, F. (2018). Kernel fuzzy Kohonen's clustering neural network and its recursive learning. Automatic Control and Computer Sciences, 52(3), 166-174.
10. Marin, D., Tang, M., Ayed, I. B., & Boykov, Y. (2017). Kernel clustering: density biases and solutions. IEEE Transactions on Pattern Analysis and Machine Intelligence, 41(1), 136-147.
11. Graves, D., & Pedrycz, W. (2010). Kernel-based fuzzy clustering and fuzzy clustering: A comparative experimental study. Fuzzy sets and systems, 161(4), 522-543.
12. Abu, A., & Diamant, R. (2019). Enhanced fuzzy-based local information algorithm for sonar image segmentation. IEEE Transactions on Image Processing, 29, 445-460.
13. Chen, S., & Zhang, D. (2004). Robust image segmentation using FCM with spatial constraints based on new kernel-induced distance measure. IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics), 34(4), 1907-1916.
14. Balafar, M., Saripan, M. I., Mahmud, R., Mashohor, S., & Balafar, H. (2008). MRI segmentation of medical images using FCM with initialized class centres via genetic algorithm. Paper presented at the 2008 International Symposium on Information Technology.
15. Kanade, P. M., & Hall, L. O. (2004). Fuzzy ant clustering by centroid positioning. Paper presented at the 2004 IEEE International Conference on Fuzzy Systems (IEEE Cat. No. 04CH37542).
16. Siringoringo, R., & Jamaluddin, J. (2019). Initializing the Fuzzy C-Means Cluster Center With Particle Swarm Optimization for Sentiment Clustering. Paper presented at the Journal of Physics: Conference Series.
17. Kole, D. K., & Halder, A. (2010). An efficient dynamic image segmentation algorithm using a hybrid technique based on particle swarm optimization and genetic algorithm. Paper presented at the 2010 international conference on Advances in computer engineering. 252-255: IEEE.
18. Selvi, V., & Umarani, R. (2010). Comparative analysis of ant colony and particle swarm optimization techniques. International Journal of Computer Applications, 5(4), 1-6.
19. Tolias, Y. A., & Panas, S. M. (1998). Image segmentation by a fuzzy clustering algorithm using adaptive spatially constrained membership functions. IEEE Transactions on Systems, Man, and Cybernetics-Part A: Systems and Humans, 28(3), 359-369.
20. Cai, W., Chen, S., & Zhang, D. (2007). Fast and robust fuzzy c-means clustering algorithms incorporating local information for image segmentation. Pattern Recognition, 40(3), 825-838.
21. Memon, K. H., Memon, S., Qureshi, M. A., Alvi, M. B., Kumar, D., & Shah, R. A. (2019). Kernel possibilistic fuzzy c-means clustering with local information for image segmentation. International Journal of Fuzzy Systems, 21(1), 321-332.
22. Hu, G., & Du, Z. (2019). Adaptive kernel-based fuzzy c-means clustering with spatial constraints for image segmentation. International Journal of Pattern Recognition and Artificial Intelligence, 33(01), 1954003.
23. Ding, Y., & Fu, X. (2016). Kernel-based fuzzy c-means clustering algorithm based on genetic algorithm. Neurocomputing, 188, 233-238.
24. Venkatesan, A., & Parthiban, L. (2017). Medical Image Segmentation With Fuzzy C-Means and Kernelized Fuzzy C-Means Hybridized on PSO and QPSO. International Arab Journal of Information Technology (IAJIT), 14(1).
25. Hofmann, T., Schölkopf, B., & Smola, A. J. (2008). Kernel methods in machine

learning. The annals of statistics, 1171-1220.

26. Souza, C. R. (2010). Kernel functions for machine learning applications. Creative Commons Attribution-Noncommercial-Share Alike, 3, 29.
27. Elazab, A., Wang, C., Jia, F., Wu, J., Li, G., & Hu, Q. (2015). Segmentation of brain tissues from magnetic resonance images using adaptively regularized kernel-based fuzzy-means clustering. Computational and Mathematical Methods in Medicine, 2015.
28. De Oca, M. A. M., Stutzle, T., Birattari, M., & Dorigo, M. (2009). Frankenstein's PSO: a composite particle swarm optimization algorithm. IEEE Transactions on Evolutionary Computation, 13(5), 1120-1132.
29. Suresh, K., Ghosh, S., Kundu, D., Sen, A., Das, S., & Abraham, A. (2008). Inertia-adaptive particle swarm optimizer for improved global search. Paper presented at the 2008 eighth international conference on intelligent systems design and applications.2. 253-258.IEEE.
30. A. Bala and A. K. Sharma, "Image Segmentation: Automatic Cluster Number Generation in K-Means," in *2019 Fifth International Conference on Image Information Processing (ICIIP)*, 2019, pp. 199-204: IEEE.
31. J. C. Bezdek, "Cluster validity with fuzzy sets," 1973.
32. N. Zahid, M. Limouri, and A. Essaid, "A new cluster-validity for fuzzy clustering," *Pattern Recognition,* vol. 32, no. 7, pp. 1089-1097, 1999.
33. W. Wang and Y. Zhang, "On fuzzy cluster validity indices," *Fuzzy sets and systems,* vol. 158, no. 19, pp. 2095-2117, 2007.
34. J. C. Bezdek, "Mathematical models for systematics and taxonomy," in *Proceedings of the 8th International Conference on Numerical Taxonomy, 1975*, 1975: Freeman.
35. D. R. Martin, J. Malik, and D. Patterson, *An empirical approach to grouping and segmentation*. Computer Science Division, University of California, 2003.