

A Study on Mathematical Model of Entropy Change of Bosonic Field of Black Holes of Spin Parameters ($a^* = +3, -3$) in XRBs and AGN

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Abstract

The present work is concerned to discuss a model for the change in entropy of spinning black holes w.r.t. the mass(Mahto and Kumari 2018 & Mahto et al. 2020) of spin parameters $a^* = +3$ & -3 and concludes that the bosonic field of black holes of co-rotation decreases the entropy and the same of black holes of counter rotation increases the entropy. The present model also predicts the emission of energy due to loss of mass in the form of Hawking radiation and dark matter and energy.

Keywords: Spin parameter, Hawking radiation & Entropy

1. Introduction:

The entropy is the intrinsic property of any system, which measures the disorderness of the system. There are so many works had been done regarding the entropy, mass, spin and the other properties of black holes by Bardeen et al.(1973), Hawking (1974, 1975), Transchen (2000), Wald (2001), Narayan (2005), Dabholkar (2005), Bekenstein (2008), Nozari K and Behnaz Fazlpour (2008), Mahto et al. (2012), Dolan (2012) etc. A theoretical model for the change in entropy of black holes due to change in mass for unit spin parameter and angular momentum in XRBs is studied(Mahto and Kumari, 2018). The change in entropy of Bosonic field of black holes w. r. t. the mass of spin parameters $a^* = +1$ and -1 is studied (Mahto et al. 2020). The same work for Fermionic fields of black holes w.r.t. the mass is also studied (Mahto et al. 2020).

In the present research work, we have discussed a model for the change in entropy of uncharged bosonic field of spinning black holes w.r.t. the mass (M) of maximum co-rotation and counter rotation spin($a^* = +3$ & -3) of unit angular velocity (Ω)

represented

by

$$\delta S / \delta M = 8\pi M (1 - 2\Omega M a^* + a^{*2} / 2 - M \Omega a^{*3})$$

where a^* and κ are spin parameter and surface gravity of black holes respectively.

2. Theoretical discussion:

The change in entropy due to corresponding change in mass (M) and angular momentum (J) of spinning black holes is also dependent on surface gravity (κ) and given by the following equation (Mahto & Kumari, 2018, Mahto et al. 2020).

$$\delta S / \delta M = 8\pi M (1 - 2\Omega M a^* + a^{*2} / 2 - M \Omega a^{*3}) \quad (1)$$

The equation (1) gives a model for the change in entropy with mass of the spinning black holes in terms of the mass (M), angular velocity (Ω) and spin parameter (a^*).

Now the model represented by the equation (1) is applied for the $+3$ and -3 spin parameters (Tayal 1990, Yash 2022) of unit angular velocity, the following equation is obtained

$$\left(\frac{dS}{dM}\right)_{+3} = -44\pi M(6M - 1) \tag{2}$$

$$\left(\frac{dS}{dM}\right)_{-3} = 44\pi M(6M + 1) \tag{3}$$

For maximum/minimum change in entropy w.r.t. mass change, the following condition must be satisfied.

$$\left(\frac{\delta S}{\delta M}\right)_{\pm 3} = 0 \tag{4}$$

Using the above equations (4) into equations (2) & (3) as required, we have

$$M(6M - 1) = 0 \tag{5}$$

$$M(6M + 1) = 0 \tag{6}$$

The solution of all equations (5) and (6) gives

$$M=0 \text{ or } M=1/6. \tag{7}$$

$$M=0 \text{ or } M=-1/6 \tag{8}$$

As per all solutions, the masses are 0, 1/6, -1/6. Now the model (2) & (3) are used for spin parameter $a^*=3$ and -3 and listed in the table 1 & 2.

3. Table:

Table 1: The change in entropy w.r.t. the change in mass of bosonic field of black holes of spin parameters +3 & -3 with unit angular velocity in XRBs.			
Sl. No	Mass (M) [Solar Mass (M_{\odot})]	$\left(\frac{dS}{dM}\right)_{+3} = -44\pi M(6M - 1)$ [Joule/Kelvin/kg]	$\left(\frac{dS}{dM}\right)_{-3} = 44\pi M(6M + 1)$ [Joule/Kelvin/kg]
1	5 M_{\odot}	-0.82896 x 10 ⁶⁵	0.82896 x 10 ⁶⁵
2	6 M_{\odot}	-1.1937 x 10 ⁶⁵	1.1937 x 10 ⁶⁵
3	7 M_{\odot}	-1.62476 x 10 ⁶⁵	1.62476 x 10 ⁶⁵
4	8 M_{\odot}	-2.12214 x 10 ⁶⁵	2.12214 x 10 ⁶⁵
5	9 M_{\odot}	-2.68583 x 10 ⁶⁵	2.68583 x 10 ⁶⁵
6	10 M_{\odot}	-3.31584 x 10 ⁶⁵	3.31584 x 10 ⁶⁵
7	11 M_{\odot}	-4.01217 x 10 ⁶⁵	4.01217 x 10 ⁶⁵
8	12 M_{\odot}	-4.77481 x 10 ⁶⁵	4.77481 x 10 ⁶⁵
9	13 M_{\odot}	-5.60377 x 10 ⁶⁵	5.60377 x 10 ⁶⁵
10	14 M_{\odot}	-6.49905 x 10 ⁶⁵	6.49905 x 10 ⁶⁵
11	15 M_{\odot}	-7.46064 x 10 ⁶⁵	7.46064 x 10 ⁶⁵

12	16 M _⊙	-8.48855 x 10 ⁶⁵	8.48855 x 10 ⁶⁵
13	17 M _⊙	-9.58278 x 10 ⁶⁵	9.58278 x 10 ⁶⁵
14	18 M _⊙	-10.7433 x 10 ⁶⁵	10.7433 x 10 ⁶⁵
15	19 M _⊙	-11.9702 x 10 ⁶⁵	11.9702 x 10 ⁶⁵
16	20 M _⊙	-13.2634 x 10 ⁶⁵	13.2634 x 10 ⁶⁵

Table 2:

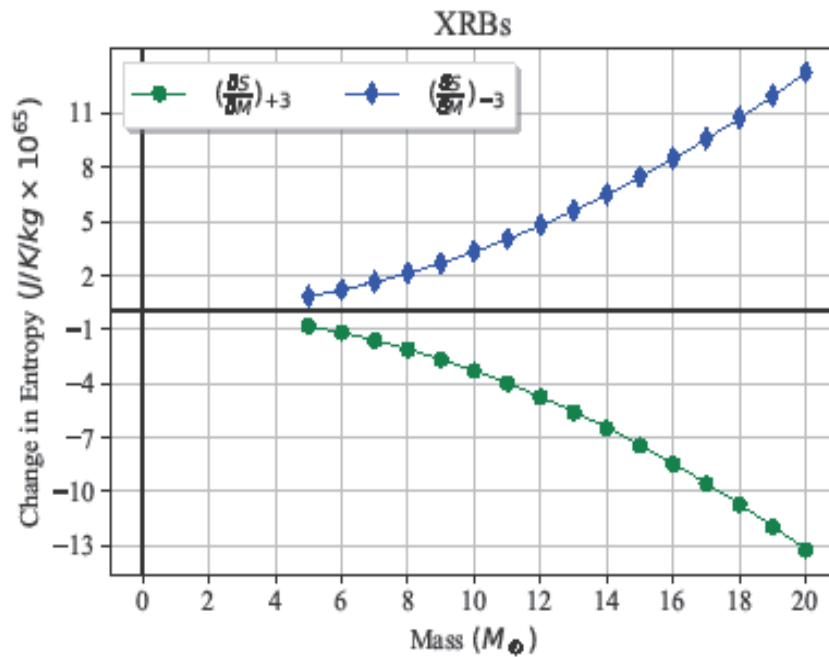
The entropy change w.r.t. the change in mass of bosonic field of BHs of spin +3 & -3 with unit angular velocity in AGN.

S. No.	Mass of BH _s (M) in solar masses	Mass of BHs in terms of 10 ⁷ M _⊙	$\left(\frac{dS}{dM}\right)_{+3} = -44\pi M(6M - 1)$ [Joule/Kelvin/kg] × 10 ⁷⁷	$\left(\frac{dS}{dM}\right)_{-3} = 44\pi M(6M + 1)$ [Joule/Kelvin/kg] × 10 ⁷⁷
1	1 x 10 ⁶ M _⊙	.1	-0.0331584 x 10 ⁷⁷	0.0331584 x 10 ⁷⁷
2	2 x 10 ⁶ M _⊙	.2	-0.132634 x 10 ⁷⁷	0.132634 x 10 ⁷⁷
3	3 x 10 ⁶ M _⊙	.3	-0.298426 x 10 ⁷⁷	0.298426 x 10 ⁷⁷
4	4 x 10 ⁶ M _⊙	.4	-0.530534 x 10 ⁷⁷	0.530534 x 10 ⁷⁷
5	5 x 10 ⁶ M _⊙	.5	-0.82896 x 10 ⁷⁷	0.82896 x 10 ⁷⁷
6	6 x 10 ⁶ M _⊙	.6	-1.1937 x 10 ⁷⁷	1.1937 x 10 ⁷⁷
7	7 x 10 ⁶ M _⊙	.7	-1.62476 x 10 ⁷⁷	1.62476 x 10 ⁷⁷
8	8 x 10 ⁶ M _⊙	.8	-2.12214 x 10 ⁷⁷	2.12214 x 10 ⁷⁷
9	9 x 10 ⁶ M _⊙	.9	-2.68583 x 10 ⁷⁷	2.68583 x 10 ⁷⁷
10	1 x 10 ⁷ M _⊙	1	-3.31584 x 10 ⁷⁷	3.31584 x 10 ⁷⁷
11	2 x 10 ⁷ M _⊙	2	-13.2634 x 10 ⁷⁷	13.2634 x 10 ⁷⁷
12	3 x 10 ⁷ M _⊙	3	-29.8426 x 10 ⁷⁷	29.8426 x 10 ⁷⁷
13	4 x 10 ⁷ M _⊙	4	-53.0534 x 10 ⁷⁷	53.0534 x 10 ⁷⁷
14	5 x 10 ⁷ M _⊙	5	-82.896 x 10 ⁷⁷	82.896 x 10 ⁷⁷
15	6 x 10 ⁷ M _⊙	6	-119.37 x 10 ⁷⁷	119.37 x 10 ⁷⁷
16	7 x 10 ⁷ M _⊙	7	-162.476 x 10 ⁷⁷	162.476 x 10 ⁷⁷

17	$8 \times 10^7 M_{\odot}$	8	-212.214×10^{77}	212.214×10^{77}
18	$9 \times 10^7 M_{\odot}$	9	-268.583×10^{77}	268.583×10^{77}
19	$1 \times 10^8 M_{\odot}$	10	-331.584×10^{77}	331.584×10^{77}
20	$2 \times 10^8 M_{\odot}$	20	-1326.34×10^{77}	1326.34×10^{77}
21	$3 \times 10^8 M_{\odot}$	30	-2984.26×10^{77}	2984.26×10^{77}
22	$4 \times 10^8 M_{\odot}$	40	-5305.34×10^{77}	5305.34×10^{77}
23	$5 \times 10^8 M_{\odot}$	50	-8289.6×10^{77}	8289.6×10^{77}
24	$6 \times 10^8 M_{\odot}$	60	-11937.0×10^{77}	11937.0×10^{77}
25	$7 \times 10^8 M_{\odot}$	70	-16247.6×10^{77}	16247.6×10^{77}
26	$8 \times 10^8 M_{\odot}$	80	-21221.4×10^{77}	21221.4×10^{77}
27	$9 \times 10^8 M_{\odot}$	90	-26858.3×10^{77}	26858.3×10^{77}
28	$1 \times 10^9 M_{\odot}$	100	-33158.4×10^{77}	33158.4×10^{77}

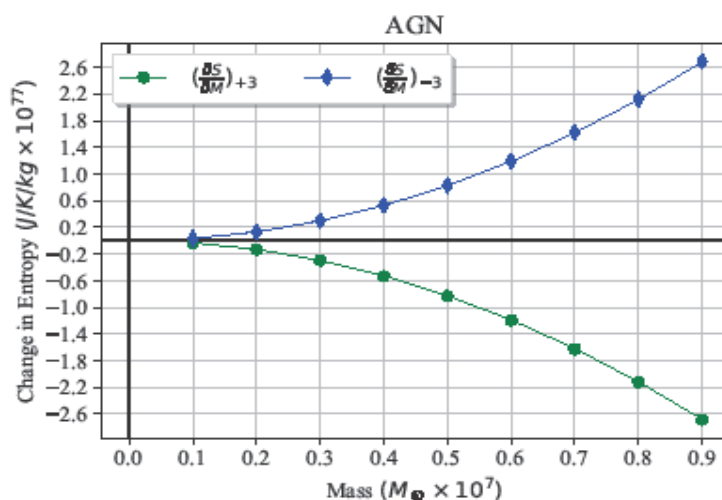
4. Figure:

Figure: 1



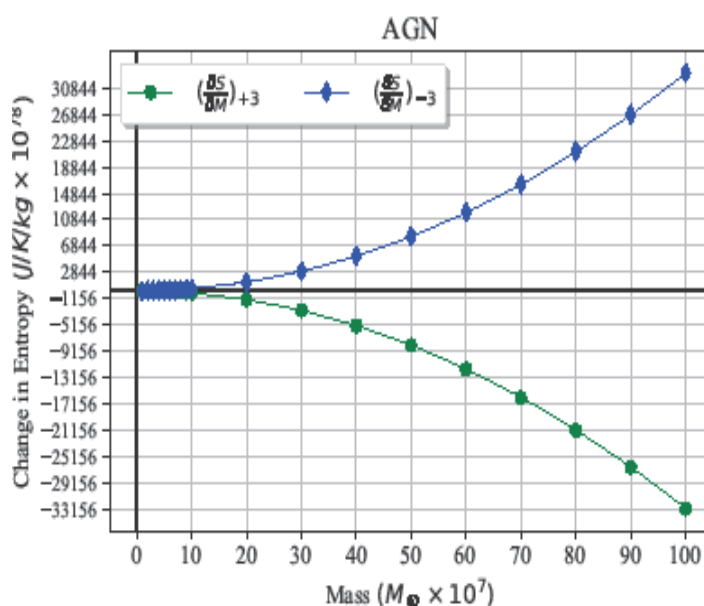
The figure 1 shows the change in entropy w.r.t. the change in mass of the black holes of spin parameter +3 and -3 with unit angular velocity in XRBS.

Figure:2a.



The figure 6a shows the change in entropy w.r.t. the change in mass of the black holes of mass range from .1 to .9 solar mass of spin parameter +3 and -3 with unit angular velocity in AGN.

Figure: 2b.



The figure 2b shows the change in entropy w.r.t. the change in mass of the black holes of mass range from .1 to .9 solar mass of spin parameter +3 and -3 with unit angular velocity in AGN.

5. Results and Discussion:

The present work is concerned with the extension of work of Mahto and Kumari (2018) and Mahto et al. (2020) for a model for the change in entropy w.r.t. mass in terms of M , Ω and a^* is given by the following equation

$$\delta S / \delta M = 8\pi M (1 - 2\Omega M a^* + a^{*2} / 2 - M \Omega a^{*3})$$

From the above equation, we observe that dS/dM depends on: (a) The angular velocity (Ω), (b) spin parameters (a^*) and (c) mass (M) of black holes.

This model is extended for the spin parameters ($a^*=+3$), we see that the change in entropy w.r.t mass provides the negative values to show decreasing entropy and models are represented by equations (2). When the same model is further extended for

the spin parameters ($a^* = -3$), we see that the change in entropy w.r.t mass provides the positive values to show increasing entropy and models are represented by equations (3). This concludes that the dS/dM increases the entropy change for counter rotation and decreases the entropy change for co-rotation w.r.t. mass regarding the bosonic field of black holes.

The physical interpretation of the equations (7) and (8) show that due to rotation of black holes, the mass is decreased from its initial original mass. This loss is converted into energy in the form of Hawking radiation and detected as the gravitational waves from the collision of binary black holes for which three American scientists named Rainer Weiss, Bary Barish and Kip Thorne were awarded by Nobel Prize in 2017.

Three types of masses are theoretically observed in this work zero, positive and negative masses. These values have their own significances: The positive, negative and zero masses justify the **laws of gravity, naked singularity along with dark matter & energy and general relativity & quantum theory** respectively.

6. Conclusion:

1. The dS/dM increases the entropy change for counter rotation and decreases the entropy change for co-rotation w.r.t. mass of the bosonic field of black holes.

2. Our model justifies the laws of gravity, naked singularity of black holes, dark matter & energy, general relativity and quantum theory.

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