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A Study of Mathematical Model on Entropy Change of Bosonic Field of Black Holes of Spin Parameters a* = +2 & -2 in XRBs and AGN

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Abstract

A Mathematical model for the change in entropy of Bosonic fields of black holes w.r.t. the mass as discussed by Mahto et al (2020) is studied for spin parameters a* =+2 & -2 in XRBs and AGN categories of black holes with calculations of their values for different masses provides the concept of the naked singularity as well as dark matter and dark energy and concludes also that this field of black holes of co-rotation decreases the entropy and counter rotation increases the entropy.

1. Introduction:

The quantum theory predicts that the black hole should radiate energy like a black body with a temperature proportional to the surface gravity of the black hole **(Hawking,1974).** The black hole entropy has been also studied on the basis of quantum and string theory **(Dabholkar, 2005).** The black holes should have a finite, non-zero temperature and entropy **(Bekenstein, 2008).** A model for both the changes in energy and entropy has been proposed and their values have been calculated for different test non-spinning black holes in XRBs and AGN **(Mahto et al. 2012).** The relation between the entropy and surface area of the black holes(S=A/4) has been given as basically proposed by Stephen Hawking **(Mahto et al. 2013).** A theoretical model for the change in entropy of spinning black holes due to change in mass is proposed to use the first law of black hole mechanics for unit spinning parameter and angular momentum in XRBs **(Mahto and Kumari, 2018)**. The change in entropy of Fermionic field of black holes with respect to the mass of spin parameters a* = +1 and -1 is studied **(Mahto et al. 2020).** The same work for Bosonic fields of black holes w.r.t. the mass for maximum co-rotation & counter rotation is also studied **(Mahto et al. 2020).**

The present work deals with the change in entropy of the spinning black holes due to the corresponding change in mass with unit angular velocity for -2 and +2 spin parameter of bosonic field of black holes.

2. Theoretical Model:

The change in entropy due to corresponding change in mass (M) and angular momentum (J) of spinning black holes is also dependent on surface gravity (κ) and given by the following equation **(Mahto & Kumari, 2018, Mahto et al. 2020).**.

$$\delta S / \delta M = 8\pi M (1 - 2\Omega M a^* + a^{*2} / 2 - M \Omega a^{*3}) \quad (1)$$

When the model represented by the equation (1) is applied for the +2 and -2 spin parameters of black holes **(Tayal 1990, Yash 2022)** of unit angular velocity, we obtain.

$$\left(\frac{dS}{dM} \right)_{+2} = -24\pi M (4M - 1) \quad .(2)$$

$$\left(\frac{dS}{dM}\right)_{-2} = 24\pi M(M+1) \quad (3)$$

For max/min change in entropy w.r.t. mass, the following condition must be satisfied.

$$\left(\frac{\delta S}{\delta M}\right)_{\pm 2} = 0 \quad (4)$$

using the above equations (2), (3) and (4) as required , we have

$$M(4M-1) = 0 \quad (5)$$

$$M(4M+1) = 0 \quad (6)$$

The solution of equations (5) and (6) gives

M=0 or M=1/4 (7)

M=0 or M=-1/4. (8)

As per all solutions, the possible masses of black holes are 0, 1/4 and -1/4. We have now calculated $\delta S / \delta M$ in XRBs and AGN of different test black holes for co-rotational and counter-rotational spin parameter a*= 2 and and a*=-2 and then listed in the table 1 & 2,

3. Table:

Table 1: The change in entropy w.r.t. the change in mass of bosonic field of black holes of spin parameters +2 & -2 with unit angular velocity in XRBs.

Sl. No	**Mass (M)** **[Solar Mass ($M_\odot$)]**	$\left(\frac{dS}{dM}\right)_{+2} = -24\pi M(4M-1)$ [Joule/Kelvin/kg]	$\left(\frac{dS}{dM}\right)_{-2} = 24\pi M(M+1)$ [Joule/Kelvin/kg]
1	$5M_\odot$	-3.0144 x 10^{64}	3.0144 x 10^{64}
2	6 $M_\odot$	-4.34074 x 10^{64}	4.34074 x 10^{64}
3	7 $M_\odot$	-5.90822 x 10^{64}	5.90822 x 10^{64}
4	8 $M_\odot$	-7.71686 x 10^{64}	7.71686 x 10^{64}
5	9 $M_\odot$	-9.76666 x 10^{64}	9.76666 x 10^{64}
6	10 $M_\odot$	-12.0576 x 10^{64}	12.0576 x 10^{64}
7	11 $M_\odot$	-14.5897 x 10^{64}	14.5897 x 10^{64}
8	12 $M_\odot$	-1.73629 x 10^{64}	1.73629 x 10^{64}
9	13 $M_\odot$	-20.3773 x 10^{64}	20.3773 x 10^{64}
10	14 $M_\odot$	-23.6329 x 10^{64}	23.6329 x 10^{64}
11	15 $M_\odot$	-27.1296 x 10^{64}	27.1296 x 10^{64}
12	16 $M_\odot$	-30.8675 x 10^{64}	30.8675 x 10^{64}

13	17 $M_\odot$	-3.48465 x 10^{64}	3.48465 x 10^{64}
14	18 $M_\odot$	-39.0666 x 10^{64}	39.0666 x 10^{64}
15	19 $M_\odot$	-4.35279 x 10^{64}	4.35279 x 10^{64}
16	20 $M_\odot$	-48.2304 x 10^{64}	48.2304 x 10^{64}

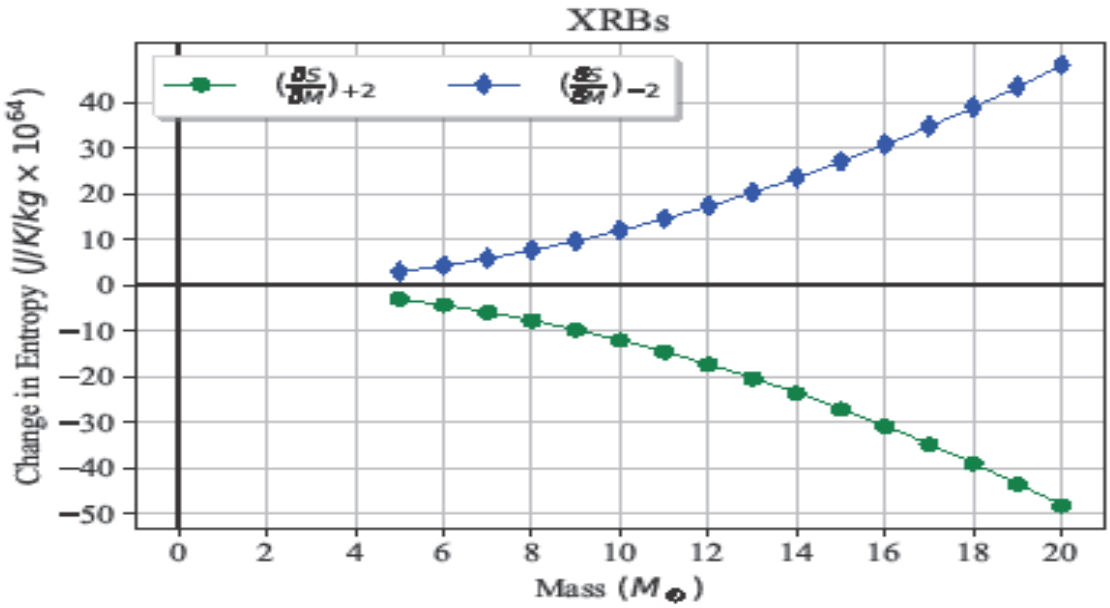
Table2: The change in entropy w.r.t. the change in mass of bosonic field of black holes of spin parameters +2 & -2 with unit angular velocity in AGN.

S. No.	Mass of BH_s (M) in solar masses	Mass of BHs in terms of $10^7 M_\odot$	$\left(\frac{dS}{dM}\right)_{+2} = -24\pi M(4M-1)$ [Joule/Kelvin/kg]× 10^{77}	$\left(\frac{dS}{dM}\right)_{-2} = 24\pi M(M+1)$ [Joule/Kelvin/kg]× 10^{77}
1	1 x 10^6 $M_\odot$	.1	-0.0120576 x 10^{77}	0.0120576 x 10^{77}
2	2 x 10^6 $M_\odot$	.2	-0.0482304 x 10^{77}	0.0482304 x 10^{77}
3	3 x 10^6 $M_\odot$	.3	-0.108518 x 10^{77}	0.108518 x 10^{77}
4	4 x 10^6 $M_\odot$	.4	-0.192922 x 10^{77}	0.192922 x 10^{77}
5	5 x 10^6 $M_\odot$	.5	-0.30144 x 10^{77}	0.30144 x 10^{77}
6	6 x 10^6 $M_\odot$	.6	-0.434074 x 10^{77}	0.434074 x 10^{77}
7	7 x 10^6 $M_\odot$	.7	-0.590822 x 10^{77}	0.590822 x 10^{77}
8	8 x 10^6 $M_\odot$	.8	-0.771686 x 10^{77}	0.771686 x 10^{77}
9	9 x 10^6 $M_\odot$	.9	-0.976666 x 10^{77}	0.976666 x 10^{77}
10	1 x 10^7 $M_\odot$	1	-1.20576 x 10^{77}	1.20576 x 10^{77}
11	2 x 10^7 $M_\odot$	2	-4.82304 x 10^{77}	4.82304 x 10^{77}
12	3 x 10^7 $M_\odot$	3	-10.8518 x 10^{77}	10.8518 x 10^{77}
13	4 x 10^7 $M_\odot$	4	-19.2922 x 10^{77}	19.2922 x 10^{77}
14	5 x 10^7 $M_\odot$	5	-30.144 x 10^{77}	30.144 x 10^{77}
15	6 x 10^7 $M_\odot$	6	-43.4074 x 10^{77}	43.4074 x 10^{77}
16	7 x 10^7 $M_\odot$	7	-59.0822 x 10^{77}	59.0822 x 10^{77}
17	8x 10^7 $M_\odot$	8	-77.1686 x 10^{77}	77.1686 x 10^{77}

18	9×10^7 $M_\odot$	9	-97.6666×10^{77}	97.6666×10^{77}
19	1×10^8 $M_\odot$	10	-123.0576×10^{77}	123.0576×10^{77}
20	2×10^8 $M_\odot$	20	-482.304×10^{77}	482.304×10^{77}
21	3×10^8 $M_\odot$	30	-1085.18×10^{77}	1085.18×10^{77}
22	4×10^8 $M_\odot$	40	-1929.22×10^{77}	1929.22×10^{77}
23	5×10^8 $M_\odot$	50	-3014.4×10^{77}	3014.4×10^{77}
24	6×10^8 $M_\odot$	60	-4340.74×10^{77}	4340.74×10^{77}
25	7×10^8 $M_\odot$	70	-5908.22×10^{77}	5908.22×10^{77}
26	8×10^8 $M_\odot$	80	-7716.86×10^{77}	7716.86×10^{77}
27	9×10^8 $M_\odot$	90	-9766.66×10^{77}	9766.66×10^{77}
28	1×10^9 $M_\odot$	100	-12057.6×10^{77}	12057.6×10^{77}

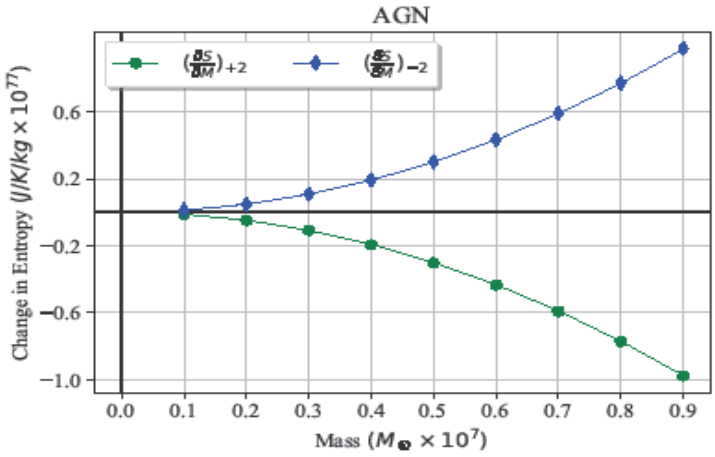
4. Figure:

Figure:1.



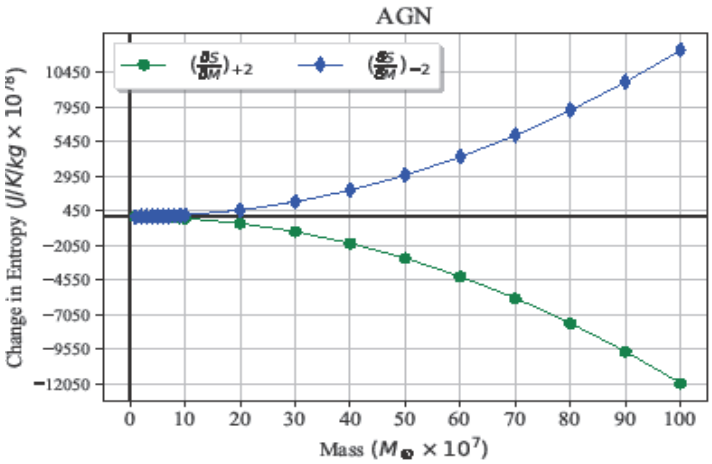
The figure 1 shows the change in entropy w.r.t. the change in mass of the black holes of spin parameter +2 and -2 with unit angular velocity in XRBs.

Figure: 2a.



The figure 2a shows the change in entropy w.r.t. the change in mass of the black holes of mass range from .1 to .9 solar mass of spin parameter +2 and -2 with unit angular velocity in AGN.

Figure: 2b.



The figure 2b shows the change in entropy w.r.t. the change in mass of the black holes of mass range 1 to 100 solar mass of spin parameter +2 and -2 with unit angular velocity in AGN.

5. Results and Discussion:

The present paper is the extension of work of Mahto and Kumari (2018) and Mahto et al. (2020) for the relation $\delta S = 2\pi[\delta M - \Omega\delta J]/\kappa$ with application of the relation J = a*M^2 (Narayan 2005) for unit angular velocity for spin parameters a* +2 & a* -2 I XRBs and AGN. With proper mathematical operations, the required model for the change in entropy w.r.t. the mass of black holes is obtained represented by the equations (2) & (3). For the spin parameters (a*=+2), we see that the change in entropy w.r.t mass provides the negative values which shows the decrease entropy change, while for the spin parameters (a*=-2,), the change in entropy w.r.t mass provides the positive values to show increasing entropy. This concludes that the dS/dM decreases the entropy change w.r.t. mass for co-rotation and increases for counter rotation of bosonic field of black holes.

The clear observations for the study of dS/dM, the graphs have been plotted between the mass on X axis and dS/dM on the Y axis in XRBs and AGN. The nature of graph in both categories of black holes is the same in symmetrical manners for co-rotation and counter rotation shown in the figure 1 for XRBs and 2(a) & (b) for AGN black holes.

To obtain the maximum or minimum entropy change, the condition (4) is applied to the equations (2) & (3) to give the three possible values of mass like 0, 1/4 and -1/4. These three values of the mass of black holes have their own significances.

The zero mass of any object is explained as per classical theory that the body has zero momentum and zero energy and will be remain at the rest and also exerts no force on other objects. Hence there is no existence of zero mass in nature, but quantum theory is not agreed to this ideas and states that the light particle is massles and it has energy and postulates that the radiation (light) consists of large number of packets of energy, each packet is called quanta or photon, which has energy hv, where h and v be the Planck constant and frequency of radiation respectively. The mass of photon is also zero. The general theory of relativity also contradicts the concept of zero mass as proposed by classical theory and says that the light bends when passing near the strong gravitational fields like black holes and other massive/super-massive objects in our universe. This means that a body which has strong gravity exerts a force on zero mass particles.

The positive sign of mass gives the general concept of mass as proposed in Newtonian mechanics well known for the concept of gravity and hence justifies the laws of gravity.

The negative sign of mass gives the concept of negative mass and it leads the idea of naked singularity of black holes as explained by Schwarzschild in 1916 during solving the Einstein field equation. The negative mass also leads the concept of dark energy and dark matter.

6. Conclusion:

1. The bosonic field of black holes of co-rotation decreases the change in entropy w.r.t. the mass of black holes.

2. The bosonic field of black holes of counter rotation increases the change in entropy w.r.t. the mass of black holes.

3. The negative mass leads the idea of naked singularity of black holes concept of dark energy and dark matter.

4. The positive justifies the laws of gravity.

5. The zero mass of any object is justified by quantum theory and general relativity.

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