A DCC MGARCH COPULA-BASED MODEL FOR MEASURING DEPENDENCE IN MULTIVARIATE FINANCIAL TIME SERIES DATA

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Abstract

In the realm of financial time series, accurately modeling dependencies among macroeconomic variables is crucial for enhancing parameter estimation and capturing nonlinear relationships, an inherent characteristic of financial time series data. This study proposes a DCC-MGARCH model integrated with copulas (Gumbel, Clayton, and t copulas) and a modified form of Kullback-Leibler (KL) divergence to improve dependency measurement and parameter estimation accuracy. The study employs a modified form of Kullback-Leibler divergence to quantify dependencies, enhancing both parameter estimation and predictive performance. The model was tested using both simulated and real-world financial data, with results indicating synchrony in volatility and covariance structures. However, the Gumbel copula did not conform well to the dependence structure, while the Clayton and t copulas provided better representations. The hybrid KL-based copula-DCC-MGARCH model outperformed the traditional DCC-MGARCH model. These findings highlight the effectiveness of KL divergence in optimizing copula selection, making the proposed model a robust tool for measuring nonlinear dependencies in multivariate financial time series.

Keywords: Kullback-Leiber Divergence; Copula; DCC-MGARCH; Dependence

1 Introduction

Financial time series data often exhibit complex dependence structures characterized by volatility clustering, autocorrelation, and non-stationarity. Dependence is an inherent characteristic of financial time series data and measures how movement in one financial variable affects movement in the others (Cuomo et al., 2022). Modeling dependence in financial time series data is crucial in various areas of finance and economics. Failure to account for dependence leads to underestimating risks and generating inaccurate forecasts of financial and economic variables.

The modeling of financial time series dependence has evolved significantly over the years owing to the limitations of traditional dependence models. Traditional dependence models are based on restrictive assumptions due to their reliance on correlation structures, which limit their ability to capture nonlinear dependencies and dynamic structures in time series data. Empirical studies highlight the limitations of these models in fully describing dynamic dependencies in financial markets. As noted by Ammann and Süss (2013) traditional methods often exhibit bias as the estimated correlation coefficient is overly simplistic and highly linear, failing to capture the complex relationship inherent in high-dimensional financial datasets. Linear correlation measures, like Pearson's correlation coefficient and linear regression, are insufficient to account for non-linear dependencies and heavy-tailed behavior prevalent in financial data.

Engle (1982) introduced the Autoregressive Conditional Heteroskedasticity (ARCH) model, which laid the groundwork for modeling volatility clustering in financial time series; later Bollerslev (1986) extended the ARCH model to Generalized Autoregressive Conditional Heteroskedasticity (GARCH).These models improved time series modeling by accounting for time-varying volatility. However, the models focused on univariate time series, prompting the need for a multivariate approach necessitating the development of multivariate GARCH models. However, despite the advancements in the MGARCH model, they are not fully sufficient in modeling high-dimensional dependence as they majorly focus on modeling volatility and linear dependency due to the correlation coefficient within their structure. Among MGARCH models, the constant conditional correlation (CCC) model (Bollerslev, 1990) assumes constant correlations over time, which is unrealistic for financial markets where dependence evolves over time. Similarly, while the BEKK model (Engle and Kroner, 1995) attempted to improve on this by ensuring a positive-definite covariance matrix and allowing for time-varying correlations, its parametric complexity and computational complexity limit its applicability in high-dimensional settings. Şerban et al. (2007) compared the performance of BEKK to a copula-based model, noting the strain of BEKK in accounting for varying levels of heavy-tailed behavior and capturing dynamic dependency observed in financial markets

The Dynamic Conditional Correlation (DCC) model introduced by Engle (2002) further improved MGARCH modeling by allowing dynamic correlations. Its flexibility and reduced computational complexity have made it garner wide use in financial modeling compared to other MGARCH models. However, despite it's benefits, the existence of a linear correlation structure within its framework limits its ability to capture the nonlinear nature inherent in financial time series data.

Copula models have emerged as a powerful alternative to overcome the limitations of correlation-based dependence measures. The copula theory, introduced in 1959 by Abe Sklar and applied for the first time in finance at the end of the twentieth century, is a powerful and flexible statistical tool for modeling. Copula makes it possible to decompose a joint distribution function for a random vector into its marginal functions and their corresponding dependence structures, allowing for the modeling of nonlinear dependencies and other high-dimensional dependence structures that are often observed in financial time series data (Chidzalo et al., 2022; Dewick and Liu, 2022) . Empirical evidence suggests that copula-MGARCH models outperform traditional MGARCH models in capturing nonlinear and other complex dependence structures (Lee and Long, 2009). While copulas offer a powerful solution, copula selection has been a challenge (Fermanian and Scaillet, 2005). Copula selection is integral to financial modeling. Different copula families capture different dependence structures; selecting the incorrect copula can bias dependence estimation and misrepresent financial forecasts.

Kullback Leiber divergence ensures that the selected copula accurately reflects the intended dependence by penalizing variability in dependence structures. Diks et al. (2013) introduced Kullback-Leibler (KL) divergence as a statistical tool to assess and identify the most suitable copula for capturing complex dependencies in financial time series.

In addition to addressing volatility clustering and autocorrelation, it is crucial to capture non-linear dependence and improve parameter estimation within the DCC-MGARCH framework owing to the complex and evolving dependence structures exhibited across financial markets. Traditional MGARCH models rely on correlation-based measures, which often fail to fully describe high dimensional dependence structures. While copula functions provide a flexible tool for modeling such relationships, their effectiveness depends on selecting and calibrating the most appropriate copula function to reflect real-world dependencies accurately.

This study aims to develop a measure of non linear dependence and enhance parameter estimation within the DCC MGARCH-Copula framework by integrating copulas with a modified form of Kullback-Leibler (KL) divergence. The KL divergence reduces the bias and inefficiencies that arise from selecting an inappropriate copula by minimizing information loss between empirical and theoretical dependence structures, ensuring that the chosen copula accurately captures the dependence structure improving model robustness in dynamic financial environments. Section 2 outlines the methodology, detailing the integration of KL divergence into the copula-based DCC MGARCH framework to form a new model that enhances nonlinear dependence modelling and parameter estimation. Section 3 presents the empirical analysis, including data description, model estimation results, and a comparison of copula selection methods. Section 6 concludes the study with a summary of key insights and recommendations for future research.

2 Methodology

This section proposes a measure of multivariate dependence suitable for variables that satisfy a GARCH process. It also introduces a method for using this measure to evaluate multivariate dependence under the assumption that the variables satisfy a GARCH process.

2.1 Proposed Model and Properties

We propose a model for measuring multivariate dependence based on a modified form of the Kullback-Leibler divergence, as follows:

$$M(X_{1:n}) = \frac{1}{N}\mathbf{1}_N^\top \log \frac{P(X_1 \le x_1, \ldots, X_n \le x_n)}{\prod_{j=1}^n P(X_j \le x_j)}, \tag{1}$$

where $\mathbf{1}_N$ is a vector of size $N \times 1$ whose elements are all ones, N is the sample size of each variable $X_i,\ i = 1, \ldots, n$, and $\top$ denotes the transpose operation. From the properties of copulas, we have:

$$P(X_1 \le x_1, \ldots, X_n \le x_n) = C(F_1(x_1), \ldots, F_n(x_n)), \tag{2}$$

where $F_i(x_i) = P(X_i \le x_i),\ i = 1, \ldots, n$.

The following theorem provides conditions under which $M(X_1, X_2, \cdots, X_n) = M(X_{i=1:n})$ indicates negative or positive dependence. We supply the proof using the definition of quadrant dependence found in Mandia et al. (2024), which states that the variables X_i exhibit negative dependence if

$$P(X_1 \le x_1, \cdots, X_n \le x_n) \le \prod_{j=1}^n P(X_j \le x_j). \tag{3}$$

The theorem is formally stated below.

Theorem 1. *Let $X_i,\ i = 1, \ldots, n$. The following facts hold:*

1. *If X_i exhibit countermonotonic dependence, $M(X_{i=1:n}) \in (-\infty, 0)$.*
2. *If X_i exhibit comonotonic dependence, $M(X_{i=1:n}) \in (0, \infty)$.*
3. *If X_i are independent, $M(X_{i=1:n}) = 0$.*

Proof. From the definition in equation (3), negative dependence is achieved when

$$P(X_1 \le x_1, \ldots, X_n \le x_n) < \prod_{i=1}^n P(X_i \le x_i).$$

Therefore,

$$0 \le P(X_1 \le x_1, \ldots, X_n \le x_n) < \prod_{i=1}^n P(X_i \le x_i),$$

$$\implies 0 < \frac{P(X_1 \le x_1, \ldots, X_n \le x_n)}{\prod_{i=1}^n P(X_i \le x_i)} < 1.$$

The value of $M(X_{1:n})$ is then given by:

$$M(X_{1:n}) = \frac{1}{N}\mathbf{1}_N^\top \log \frac{P(X_1 \le x_1, \ldots, X_n \le x_n)}{\prod_{i=1}^n P(X_i \le x_i)}.$$

For negative dependence, the numerator $P(X_1 \le x_1, \ldots, X_n \le x_n)$ is smaller than the denominator $\prod_{i=1}^n P(X_i \le x_i)$, which implies:

$$\frac{1}{N}\mathbf{1}_N^\top \log \frac{P(X_1 \le x_1, \ldots, X_n \le x_n)}{\prod_{i=1}^n P(X_i \le x_i)} < 0.$$

Similarly, for positive dependence, the numerator $P(X_1 \le x_1, \ldots, X_n \le x_n)$ exceeds the denominator $\prod_{i=1}^n P(X_i \le x_i)$, which implies:

$$\frac{1}{N}\mathbf{1}_N^\top \log \frac{P(X_1 \le x_1, \ldots, X_n \le x_n)}{\prod_{i=1}^n P(X_i \le x_i)} > 0.$$

Finally, for independence, the numerator equals the denominator, leading to:

$$\frac{1}{N}\mathbf{1}_N^\top \log \frac{P(X_1 \le x_1, \ldots, X_n \le x_n)}{\prod_{i=1}^n P(X_i \le x_i)} = 0.$$

Thus, the theorem is proven. □

Since $M(X_{1:n}) \in (-\infty, \infty)$, as stated in the theorem above, it is necessary to determine the upper and lower bounds of this measure, given that both $\prod_{i=1}^n P(X_i \le x_i)$ and $C(F_1(x_1), \ldots, F_n(x_n))$ are nonzero. The following theorem, which relies on the Frechet-Hoeffding bounds (Yan, 2023), establishes the bounds for $M(X_{1:n})$.

Theorem 2. *Suppose that for GARCH errors modeled by random variables $X_{i=1:n}$, $\prod_{i=1}^n P(X_i \le x_i) \neq 0$ and $C(F(x_1), \cdots, F(x_n)) \neq 0$. Then the following statements hold:*

1. *$M(X_{i=1:n})$ is permutation invariant.*
2. $$M(X_{i=1:n}) \le \frac{1}{N}\mathbf{1}_N \frac{\min\{F(x_i), i = 1 \cdots, n\}}{\sum_{i=1}^n F(x_i) - n + 1}.$$

Proof. The first property clearly follows because $P(X_1 \le x_1, \cdots, X_n \le x_n) = C(F(x_1), \cdots, , F(x_n))$ is invariant under definition of cupula. Similarly $\prod_{i=1}^n P(X_i \le x_i)$ is clearly invariant under a permutation. Now, considering the properties of logarithms,

$$\begin{aligned} M(X_{i=1:n}) &= \frac{1}{N}\mathbf{1}_N \log \frac{C(F(x_1), \cdots, F(x_n))}{\prod_{i=1}^n P(X_i \le x_i)} \\ &\le \frac{1}{N}\mathbf{1}_N \frac{C(F(x_1), \cdots, F(x_n))}{\prod_{i=1}^n P(X_i \le x_i)} \end{aligned}$$

Since $\prod_{i=1}^n P(X_i \le x_i) \neq 0$, then by Frechet-Hoeffding bounds,

$$\prod_{i=1}^n P(X_i \le x_i) \ge \sum_{i=1}^n F(x_i) - n + 1.$$

In addition, by Frechet-Hoeffding bounds,

$$C(F(x_1), \cdots, F(x_n)) \le \min\{F(x_i), i = 1 \cdots, n\}$$

This finishes our proof. □

In addition, we propose the use of the following to measure time filtered dependence:

$$\Lambda(X_1, X_2) = \log \frac{C_\theta(P(X_1 \le x_1), P(X_2 \le x_2))}{F(x_1)F(x_2)} \tag{4}$$

2.2 Application to GARCH Process

This subsection provides the definition of a GARCH process and introduces the proposed technique for measuring dependence in a GARCH process.

Definition 1. *The process $\{X_t\}$ is said to follow a **DCC-GARCH** process with an ARMA mean if it satisfies the following conditions:*

- *The mean equation follows the ARMA(p, q) model:*
$$X_t = \mu_t + \epsilon_t, \tag{5}$$
where
$$\mu_t = k + \sum_{i=1}^p \phi_i X_{t-i} + \sum_{j=1}^q \theta_j \epsilon_{t-j}. \tag{6}$$
- *The residuals ϵ_t are conditionally heteroskedastic and follow the DCC GARCH process, which consists of the following components:*
  1. *The conditional variance-covariance matrix is defined as:*
  $$H_t = D_t R_t D_t, \tag{7}$$
  where:
  $$D_t = diag\left(h_{11,t}^{1/2}, h_{22,t}^{1/2}, \ldots, h_{NN,t}^{1/2}\right), \tag{8}$$
  is the diagonal matrix of conditional standard deviations, and R_t is the time-varying correlation matrix.

2. *The correlation matrix R_t is updated dynamically:*

$$R_t = (1-a-b)\bar{R}+aQ_t+bR_{t-1}, \quad (9)$$

where Q_t is the covariance matrix of standardized residuals, defined as:

$$Q_t = (1-a-b)\bar{Q}+au_{t-1}u_{t-1}^\top+bQ_{t-1}, \quad (10)$$

and $\bar{R}$ and $\bar{Q}$ are the unconditional correlation and covariance matrices, respectively.

3. *The residuals are obtained as follows:*

$$\epsilon_t = H_t^{0.5}\eta_t, \quad (11)$$

where $\eta_t = u_t$ is the vector of standardized innovations and $H_t^{0.5}$ is the Cholesky decomposition of H_t.

In order to measure dependence in a GARCH model, we suggest the following methodology: Fit the GARCH model as defined in Definition 1; Extract residuals estimated from the GARCH process; Transform the residuals using the Probability Integral Transform (PIT); Fit the transformed residuals into a copula, such as Gaussian, t-copula, Gumbel, and Clayton; and we use the results from the last two steps to measure dependence using $M(X_{i=1:4})$ and Λ. The following theorem suggests why our measures are suitable for measuring dependence in GARCH process.

Theorem 3 (Synchrony of $\Lambda(X_i, X_j)$ and $h_{i,j}$)**.** *If $X_i = \epsilon_t$ and $0 < F(x_1) \leq 1$, then the sign of $h_{i,j}$ aligns with $\Lambda(X_i, X_j)$ under specific dependency conditions:*

1. *When $X_1 = X_2$, $h_{i,i} \geq 0$ and $\Lambda(X_1, X_1) \geq 0$.*
2. *When X_1 and X_2 are countermonotonic, $h_{i,j} < 0$ and $\Lambda(X_1, X_2) < 0$.*
3. *When X_1 and X_2 are comonotonic, $h_{i,j} > 0$ and $\Lambda(X_1, X_2) > 0$.*

Proof. Suppose $X_1 = X_2$. Then:

$$\frac{\partial \log |H_t^L|}{\partial \theta} = -2\sum_i \frac{1}{d_{ii,t}}\frac{\partial d_{ii,t}}{\partial \theta} + \frac{\partial \log |L_t|}{\partial \theta}, \quad \theta \in \{a, b\}. \quad (12)$$

under the same condition.Now, suppose X_1 and X_2 are countermonotonic. Then, by Theorem 1, $\Lambda_{i,j} < 0$, which is equivalent to $h_{i,j} < 0$ under the same condition. Finally, suppose X_1 and X_2 are comonotonic. Then, by Theorem 1, $\Lambda_{i,j} > 0$, which is equivalent to $h_{i,j} > 0$ under the same condition. □

2.3 Hybrid DCC-GARCH-Copula Model

This study proposes a hybrid DCC-GARCH-copula model, incorporating a novel approach to defining the roles of $\bar{R}$ and $\bar{Q}$ in the DCC-GARCH framework.

Proposition 1. *Under the conditions of Theorem 3, the roles of $\bar{R}$ and $\bar{Q}$ in the DCC-GARCH model are redefined as follows:*

$$H_t^L = \frac{1}{2\Lambda_{i,i}}L, \quad (13)$$

where the elements of L are defined as $(L)_{i,j} = \Lambda_{i,j}$. Consequently:

$$\bar{R} = \frac{1}{2\Lambda_{i,i}}L, \quad \bar{Q} = L.$$

Substituting these definitions into the DCC-GARCH framework results in the following equation:

$$R_t = (1-a-b)\left(\frac{1}{2\Lambda_{i,i}}L\right)+a\left[(1-a-b)L+au_{t-1}u_{t-1}^\top+bQ_{t-1}\right] \quad (14)$$

Assuming that the errors follow a t-distribution, the following lemma describes the maximum likelihood method for estimating a and b.

Lemma 1. *The partial derivatives of the log-likelihood function with respect to a and b are given as follows:*

$$\frac{\partial \ell}{\partial a} = \sum_{t=1}^{n}\left(-\frac{1}{2}\frac{\partial \log |H_t^L|}{\partial a} - \frac{\nu+d}{2}\frac{1}{1+f_t}\frac{\partial f_t}{\partial a}\right), \quad (15)$$

$$\frac{\partial \ell}{\partial b} = \sum_{t=1}^{n}\left(-\frac{1}{2}\frac{\partial \log |H_t^L|}{\partial b} - \frac{\nu+d}{2}\frac{1}{1+f_t}\frac{\partial f_t}{\partial b}\right), \quad (16)$$

where:

$$f_t = \frac{\epsilon_t^\top (H_t^L)^{-1}\epsilon_t}{\nu-2}, \tag{17}$$

$$\frac{\partial f_t}{\partial a} = \frac{-\epsilon_t^\top (H_t^L)^{-1}\frac{\partial H_t^L}{\partial a}(H_t^L)^{-1}\epsilon_t}{\nu-2}, \tag{18}$$

$$\frac{\partial f_t}{\partial b} = \frac{-\epsilon_t^\top (H_t^L)^{-1}\frac{\partial H_t^L}{\partial b}(H_t^L)^{-1}\epsilon_t}{\nu-2}. \tag{19}$$

The derivative of $\log|H_t^L|$ *with respect to* a *or* b *is:*

$$\frac{\partial \log|H_t^L|}{\partial\theta} = -2\sum_i \frac{1}{d_{ii,t}}\frac{\partial d_{ii,t}}{\partial\theta} + \frac{\partial \log|L_t|}{\partial\theta}, \quad \theta\in\{a,b\}. \tag{20}$$

Proof. The log-likelihood function is:

$$\ell(a,b,\nu) = \sum_{t=1}^{n}\left[\log\Gamma\left(\frac{\nu+d}{2}\right) - \log\Gamma\left(\frac{\nu}{2}\right) - \frac{d}{2}\log(\pi(\nu-2)) - \frac{1}{2}\log|H_t^L|\right] - \sum_{t=1}^{n}\frac{\nu+d}{2}\log(1+f_t). \tag{21}$$

where $f_t = \frac{\epsilon_t^\top (H_t^L)^{-1}\epsilon_t}{\nu-2}$.
The partial derivatives with respect to a and b are derived by applying the chain rule. For a, the derivative is:

$$\frac{\partial\ell}{\partial a} = \sum_{t=1}^{n}\left(-\frac{1}{2}\frac{\partial\log|H_t^L|}{\partial a} - \frac{\nu+d}{2}\frac{1}{1+f_t}\frac{\partial f_t}{\partial a}\right).$$

The term $\frac{\partial f_t}{\partial a}$ is obtained by differentiating f_t with respect to a:

$$\frac{\partial f_t}{\partial a} = \frac{-\epsilon_t^\top (H_t^L)^{-1}\frac{\partial H_t^L}{\partial a}(H_t^L)^{-1}\epsilon_t}{\nu-2}.$$

Similarly, for b, the derivative is:

$$\frac{\partial\ell}{\partial b} = \sum_{t=1}^{n}\left(-\frac{1}{2}\frac{\partial\log|H_t^L|}{\partial b} - \frac{\nu+d}{2}\frac{1}{1+f_t}\frac{\partial f_t}{\partial b}\right),$$

where:

$$\frac{\partial f_t}{\partial b} = \frac{-\epsilon_t^\top (H_t^L)^{-1}\frac{\partial H_t^L}{\partial b}(H_t^L)^{-1}\epsilon_t}{\nu-2}.$$

The derivative of $\log|H_t^L|$ with respect to $\theta\in\{a,b\}$ is:

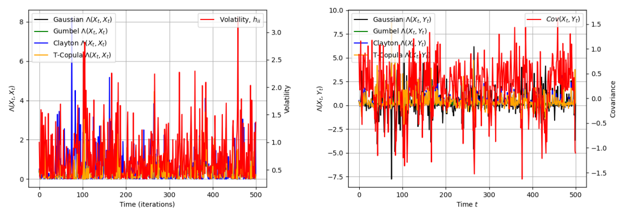
$$\frac{\partial\log|H_t^L|}{\partial\theta} = -2\sum_i\frac{1}{d_{ii,t}}\frac{\partial d_{ii,t}}{\partial\theta} + \frac{\partial\log|L_t|}{\partial\theta}.$$

This completes the proof. □

3 Results and Discussions

This section considers a DCC-MGARCH process that was simulated by choosing $a = 0.45$ and $b = 0.02$. The initial values of the random errors, R_t and Q_t are simulated from a t distribution, for example choosing $R_0 = \epsilon_0^\top\epsilon_0$, where ϵ_0 follows the t distribution. The framework in Definition 7 is then used to generate the rest of the values iteratively. The innovations are then passed to $\Lambda(X_t, Y_t)$ in Eqn. (4) to measure the pairwise dependence.

The results show that copulas of different types do not equally adhere to the postulation in Theorem 3. The values of $\Lambda(X_t, X_t)$ generated under Clayton, Gaussian, and T copulas demonstrate high synchrony with volatility (Figure 1a). In contrast, the values of $\Lambda(X_t, X_t)$ generated under the Gumbel copula do not synchronize well with volatility, as shown in Figure 1a. Furthermore, it can be observed that the values for $\Lambda(X_t, Y_t)$ generated primarily by the T-copula and Gaussian copula exhibit higher synchrony with covariance compared to those generated by the Gumbel and Clayton copulas (Figure 1b).



(a) $\Lambda_{i,i}$ versus $(H)_{i,i} = \mathbb{V}ar(X_i)$ (b) $\Lambda_{i,j}$ versus $(H)_{i,j} = \mathbb{C}ov(X_i, X_j)$, $i \neq j$

Figure 1: $\Lambda_{i,j}$ synchronizes with $(H)_{i,j} = \mathbb{C}ov(X_i, X_j)$

In addition, the global dependence from $M(X_t, Y_t)$ shows high synchrony with the usual Kendall's tau. That is, whenever Kendall's tau is positive, the global value $M(X_t, Y_t)$ is also positive. Similarly, synchrony can also be noted in cases where Kendall's tau is negative (Table 1).

Table 1: Global Dependence under $M(X_1, X_2)$

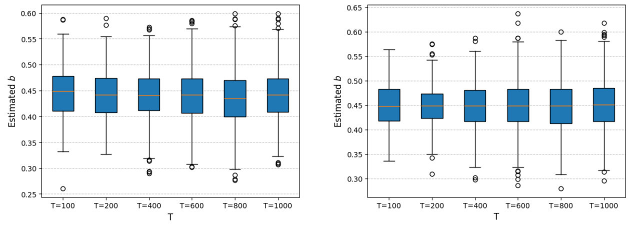
Measure	Framework	Value	Dependence Type
Kendall's Tau	Non-parametric	0.45 -0.05	Comonotonic Counter-Monotonic
$M(X_1, X_2)$	Clayton Gumbel t-Copula Gaussian	38.32 117.89 125.33 47.33	Comonotonic
	Clayton Gumbel t-Copula Gaussian	None None -49.19 -22.20	Counter-Monotonic

Given these results, the proposed hybrid DCC-GARCH-copula is fitted to the simulated data. Since errors cannot be known prior to estimating the model, copulas can be estimated using the original variable data. Thus, we assume that the errors preserve the original pairwise dependence. Thus, we re-estimate the parameters using H_t^L as well as H_t. Table 2 shows that parameter estimation improves with an increase in time only when H_t^L is used instead of H_t. The mean squared error (MSE) values are generally consistent, regardless of the increase in time. This is because H_t^L contains more information about dependence through copulas.

Table 2: Parameter Estimations under H_t and H_t^L

T	Under H_t				Under H_t^L			
	b	MSE	a	MSE	b	MSE	a	MSE
100	0.46	0.009	0.021	0.01	0.44	0.006	0.019	0.051
500	0.451	0.0019	0.0205	0.006	0.448	0.0009	0.0201	0.001
1000	0.453	0.0007	0.0198	0.004	0.447	0.0004	0.0202	0.003

For clarity of understanding, the estimation of b can be performed under different iterations for various repetitions of simulations. The box and swirl plots, generated with a sample size of 500 repetitions for each number of iterations T, show that the median of the values of b estimated under H_t^L in our hybrid model tends to be much closer to the exact value of 0.45 as the sample size increases (Figure 2b). However, the opposite trend is observed for the ordinary DCC-GARCH model (Figure 3).



(a) Under H_t (b) Under H_t^L

Figure 2: Estimating b under the ordinary and hybrid DCC-GARCH models

In order to achieve good triangulation, the model was fitted to real data of the three variables; inflation rates, exchange, and interest rates. The data featured monthly values ranging from June 2006 to January 2024. Seventy-five percent was used for training both the ordinary and hybrid models, and the rest was used for testing. Results were graphed for pictorial presentation in (Figure **??**). The figure shows that both models have same degree of accuracy especially for exchange rate. However, the points predicted from hybrid models seem to be closer to points for raw data for both testing and training parts than those predicted by ordinary GARCH. Particularly, the ordinary GARCH seems to be much further from raw data.

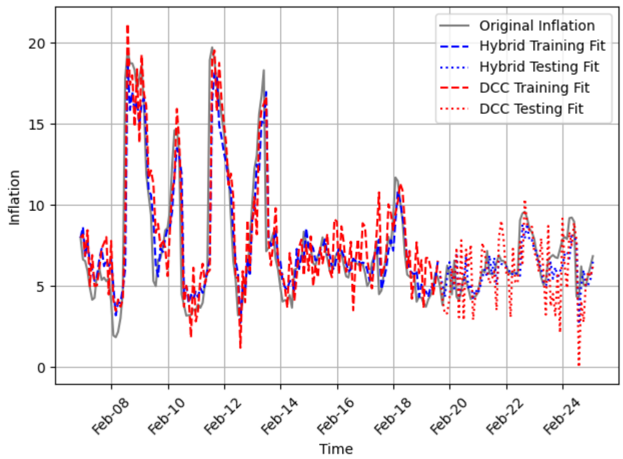


Figure 3: Inflation

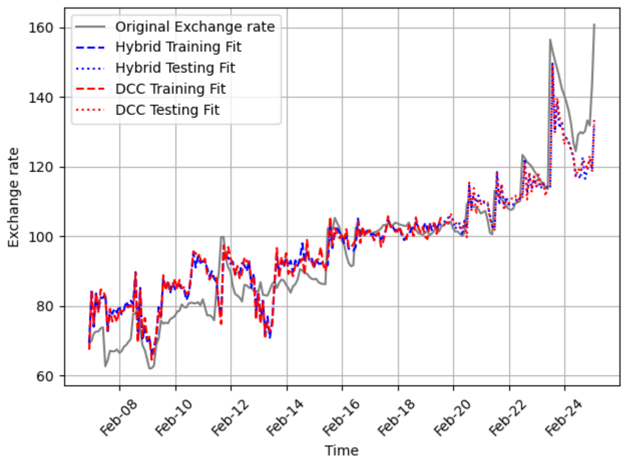


Figure 4: Exchange rate

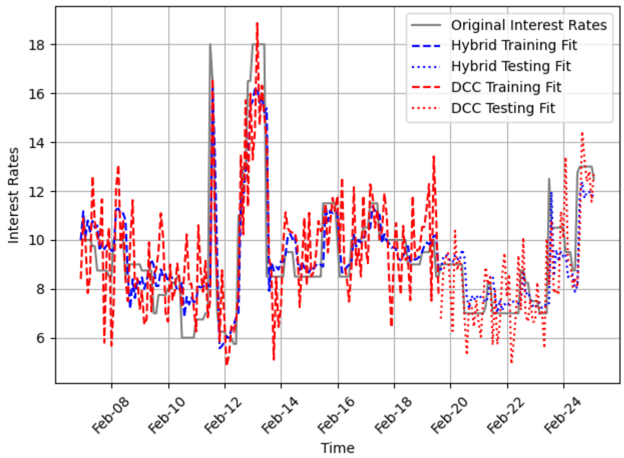


Figure 5: Interest rate

These results were confirmed with the measures of accuracy calculated . For example, the coefficient of determination is negative in for test set for inflation data. This explains why it is generally not performing well for test data. However, the hybrid GARCH model still gives a positive value of R^2. The root mean square error for both models was compared, indicating minimum evidence of overestimation or underestimation. The values of mean absolute percentage error for the hybrid model are generally lower than those of ordinary GARCH (Table 3). Both models are giving lower values of MAPE that are less than 5% for exchange rate data.

Table 3: Model Performance Metrics across Variables

Variable	Model	R^2 Train	R^2 Test	RMSE Train	RMSE Test	MAPE Train	MAPE Test
Inflation							
	Hybrid	0.764	0.437	2.15	1.11	18.9	13.4
	DCC	0.698	-0.670	2.43	1.92	27.2	26.0
Exchange Rate							
	Hybrid	0.648	0.628	7.35	9.55	7.32	4.68
	DCC	0.634	0.641	7.50	9.37	7.53	4.77
Interest Rates							
	Hybrid	0.694	0.672	1.43	1.14	9.40	7.87
	DCC	0.379	0.373	2.04	1.57	16.7	13.3

4 Conclusion

This study developed and tested a hybrid copula-DCC-MGARCH model and compared its performance against the traditional DCC-MGARCH model in measuring nonlinear dependencies using interest rates, exchange rates, and inflation rate data. By integrating the copula model into the modified KL divergence model, the study aimed to enhance parameter estimation and capture nonlinear dependencies within the DCC MGARCH framework. Empirical results show that the hybrid model outperformed the traditional DCC-MGARCH model, evidenced by lower mean squared error (MSE) in parameter estimation, indicating improved model precision. The comparative analysis of Gumbel, Clayton, and t copulas further revealed that different copulas capture distinct dependency structures. These results confirm that incorporating KL divergence into the DCC-MGARCH framework improves both dependence measurement and parameter estimation, making the hybrid model a superior alternative for multivariate financial time series analysis.

Future research could include testing the model under different economic conditions, such, financial crises, and policy shifts, which would further assess its robustness. Exploring alternative copula families, such as vine copulas, to enhance model flexibility.

References

Ammann, M. and Süss, S. (2013). Asymmetric dependence patterns in financial time series. In *Copulae and Multivariate Probability Distributions in Finance*, pages 95–111. Routledge.

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of econometrics*, 31(3):307–327.

Bollerslev, T. (1990). Modelling the coherence in short-run nominal exchange rates: a multivariate generalized arch model. *The review of economics and statistics*, pages 498–505.

Chidzalo, P., Ngare, P. O., and Mung'atu, J. K. (2022). Trivariate stochastic weather model for predicting maize yield. *Journal of Applied Mathematics*, 2022(1):3633658.

Cuomo, J., Homayouni, H., Ray, I., and Ghosh, S. (2022). Detecting temporal dependencies in data. In *Proceedings of the British International Conference on Databases*.

Dewick, P. R. and Liu, S. (2022). Copula modelling to analyse financial data. *Journal of Risk and Financial Management*, 15(3):104.

Diks, C., Panchenko, V., Sokolinskiy, O., and van Dijk, D. (2013). Comparing the accuracy of copula-based multivariate density forecasts in selected regions of support. Technical report.

Engle, R. (2002). Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models. *Journal of business & economic statistics*, 20(3):339–350.

Engle, R. F. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of united kingdom inflation. *Econometrica: Journal of the econometric society*, pages 987–1007.

Engle, R. F. and Kroner, K. F. (1995). Multivariate simultaneous generalized arch. *Econometric theory*, 11(1):122–150.

Fermanian, J.-D. and Scaillet, O. (2005). Some statistical pitfalls in copula modeling for financial applications. *Capital formation, governance and banking*, pages 57–72.

Lee, T.-H. and Long, X. (2009). Copula-based multivariate garch model with uncorrelated dependent errors. *Journal of Econometrics*, 150(2):207–218.

Mandia, A. N., Mung'atu, J. K., and Weke, P. G. (2024). Measuring multivariate negative dependence in higher dimensions using copula-based approach. *Universal Journal of Applied Mathematics*, 12(2):50–61.

Şerban, M., Brockwell, A., Lehoczky, J., and Srivastava, S. (2007). Modelling the dynamic dependence structure in multivariate financial time series. *Journal of time series analysis*, 28(5):763–782.

Yan, J. (2023). Multivariate modeling with copulas and engineering applications. *Springer handbook of engineering statistics*, pages 931–945.