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Theoretical Analysis Of Undamped Dynamic VibrationAbsorber Subjected To Harmonic Excitation

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Abstract

In the design and analysis of undamped dynamic vibration absorber the absorber mass and spring stiffness of absorber plays important role. Here effect of absorber mass and spring stiffness on displacement amplitude of the main mass will be done and verification of same will be done through MATLAB. For this analysis vibration absorber system is excited by harmonic excitation.

Key Words

Undamped Vibration, Dynamic Vibration, Harmonix Excitation, MTLAB.

Introduction

Adding a tuned spring-mass absorber to a system to produce anti-resonance at a resonance of the original system is known as vibration absorption. The natural frequencies of the resulting system are separated from the stimulation frequency by the design of the dynamic vibration absorber. If a force acting on a machine or system has an excitation frequency that is close to the machine's inherent frequency, the machine or system may vibrate excessively. In these situations, a dynamic vibration absorber or vibration neutralizer—basically, another spring mass system—can be used to lessen the machine's vibration.

The amplitude of the vibration can become very high in an undamped or mildly damped system when the excitation frequency approaches the natural frequency. Resonance is the term for this phenomenon. In a mechanical system, resonance can be extremely detrimental and ultimately result in system failure. Consequently, predicting when resonance might occur and figuring out how to avoid it are two of the main goals of vibration analysis. The size of the response can be minimized by shifting the natural frequency away from the forcing frequency by adjusting the system's stiffness or mass. An additional degree of freedom can be provided by attaching an auxiliary mass to the system via a spring and/or damper known as a vibration absorber, so that the system's natural frequency differs from the excitation frequency. The auxiliary mass, when coupled to the system via an elastic element (spring), known as an undamped vibration absorber, decreases the system response by keeping the system's natural frequency apart from the stimulation frequency. Undamped vibration absorbers are also known as Frahm's vibration absorbers. It is the simplest kind of vibration absorber.

Mathematical Modeling of Vibration Absorber

Undamped dynamic vibration absorber is shown in Figure 1.

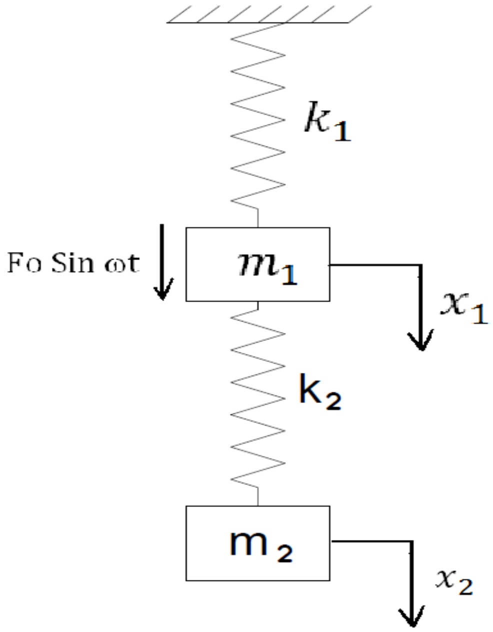


Figure 1 Undamped Dynamic Vibration Absorber

Where,

m_1= Main mass of system.

m_2= Absorber mass of system.

k_1= Spring stiffness of main system.

k_2= Spring stiffness of absorber system.

Equations of motions are obtained as follows,

$$m_2\ddot{x}_2 = -k_2(x_2 - x_1) \qquad 3.2$$

After rearranging we get above equations in dimensionless form as follows

$$\frac{X_1}{X_{st}} = \frac{1-\frac{\omega}{\omega_2{}^2}^{2}}{\frac{\omega^4}{\omega_1{}^2\omega_2{}^2}-\left[(1+\mu)\frac{\omega}{\omega_1{}^2}^{2}+\frac{\omega}{\omega_2{}^2}^{2}\right]+1} \qquad 3.3$$

$$\frac{X_2}{X_{st}} = \frac{1}{\frac{\omega^4}{\omega_1{}^2\omega_2{}^2}-\left[(1+\mu)\frac{\omega}{\omega_1{}^2}^{2}+\frac{\omega}{\omega_2{}^2}^{2}\right]+1} \qquad 3.4$$

$m_1\ddot{x}_1 + Fo\ Sin\ mt = -k_1x_1 + k_2(x_2 - x_1)$ 3.

Theoretical Analysis of Undamped Dynamic Vibration Absorber

In order to carry out vibrational analysis of undamped dynamic vibration absorber it is necessary to find out various parameters of undamped dynamic vibration absorber.

While designing vibration absorber following assumptions are made.

- Excitation Frequency is 70.71 rad/sec.
- Mass ratio is 0.25.

The following notations are typically used:

Permissible shear stress = 0.3 * S_{ut}

=0.30* 1250

=375 N/mm^2.

S_{ut} =1250 N/mm^2 for oil hardened and tempered steel wire.

Spring index c=4 to 12. (Richard G. Budynas and J. Keith Nisbett,2011)

$$K = \frac{4C-1}{4c-4} + \frac{0.615}{c}$$

Where K= Wahl's stress factor

$$= \frac{(4*11-1)}{(4*11-4)} + \frac{0.615}{11}$$

= 1.132

We have,

Since maximum load on spring will not be more than 49.05 N. So it decided todesign the spring for this load,

$$\tau = k\left(\frac{8PC}{d^2}\right)$$

$$375 = (1.1840)\left\{\frac{8*(49.05)*11}{d2}\right\}$$

∴ d = 2.034 mm (say d = 2 mm)As D = Cd

D = 22 mm

Therefore, spring stiffness is given by,

$$k = \frac{Gd^4}{8D^3N}$$

Where N= Total No. of turns of spring

The mass and spring are main component of the undamped dynamic vibration absorber. As masses can be prepared of any dimensions so first design of spring is carried out.

A mechanical spring is an elastic body that deflects or distorts under force and returns to its original shape once the load is removed. The long-standing compression spring design theory involves oversimplifying the stress distribution within the wire. It is based on the idea that an element of an axially loaded helical spring behaves basically like a straight bar in pure torsion.

By putting G = 84 GPa (Modulus of Rigitdity)We get k=2629.60 N/m

Since we are using two sprigs in parallel for main system combined stiffnesbecomes (Refer-Appendix III-CAD Drawing of Experimental Setup),

k = 5259.2 N/m

As we know, ω=70.71 rad/sec

$$\omega = \omega_n = \sqrt{\frac{K}{m}}$$

m = 1.05184 kg.

So parameters of main systems are found out, similarly parameters of absorber system are found out. Table 3.1 shows various parameters of undamped dynamicvibration absorber.

Table 1 Parameters of Undamped Dynamic Vibration Absorber

Main Mass (m_1)	1.05184 Kg
Absorber Mass (m_2)	0.26296 Kg
Main Spring Stiffness (k_1)	5259.2 N/m
Absorber Spring Stiffness (k_2)	1314.8 N/m

3.2.2 Effect of Absorber Mass

In order to study the effect of absorber mass on displacement amplitude of the main mass Eq.3.3 and 3.4 are modeled in MATLAB. By using MATLAB, the variationsof vibration amplitudes of the main and absorber masses of vibration absorber are plotted as function of the frequency ratio.

Figure 2 shows variations of vibration amplitudes of the main and absorber masses of vibration absorber plotted as function of the frequency ratio, for mass ratio = µ = 0.05.

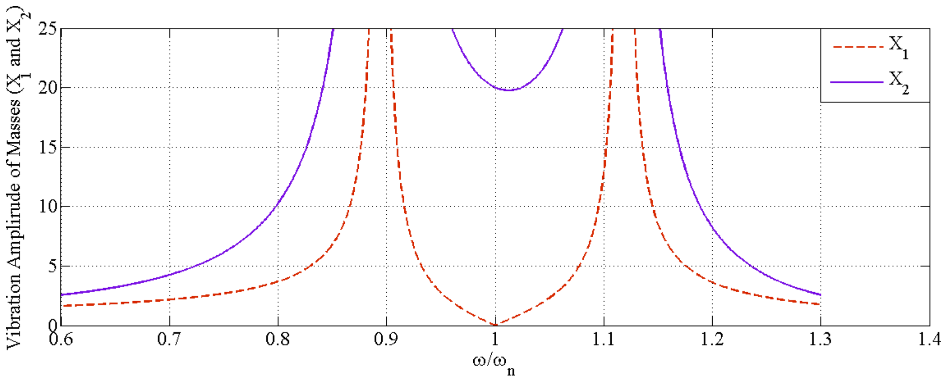


Figure 2 Variations of Vibration Amplitudes of the Main and AbsorberMasses of Vibration Absorber for µ = 0.05

Figure 3 shows variations of vibration amplitudes of the main and absorber masses of vibration absorber are plotted as function of the frequency ratio, for mass ratio = μ = 0.10

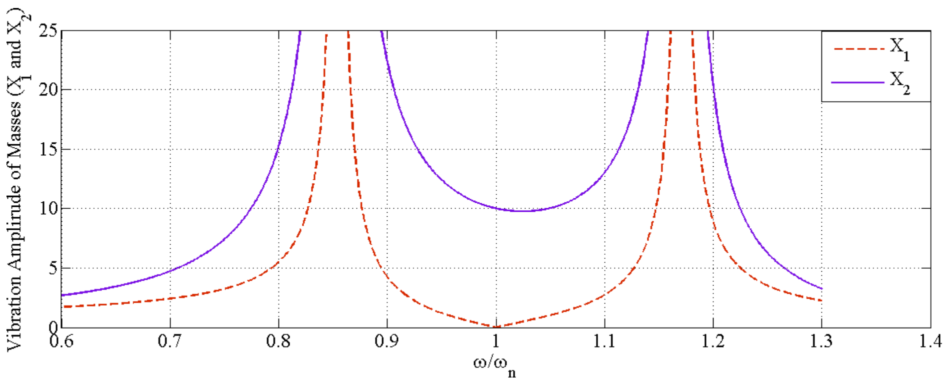
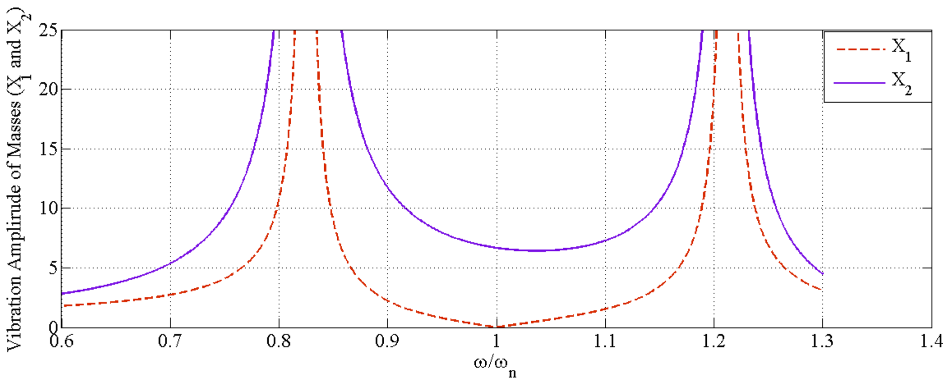


Figure 3 Variations of Vibration Amplitudes of the Main and Absorber Masses of Vibration Absorber for μ = 0.10

Figure 4 shows variations of vibration amplitudes of the main and absorber masses of vibration absorber plotted as function of the frequency ratio, for mass ratio = μ = 0.15.

Figure 4 Variations of Vibration Amplitudes of the Main and Absorber Masses of Vibration Absorber for μ =



0.15

Figure 5 shows variations of vibration amplitudes of the main and absorber masses of vibration absorber plotted as function of the frequency ratio, for mass ratio = μ = 0.20.

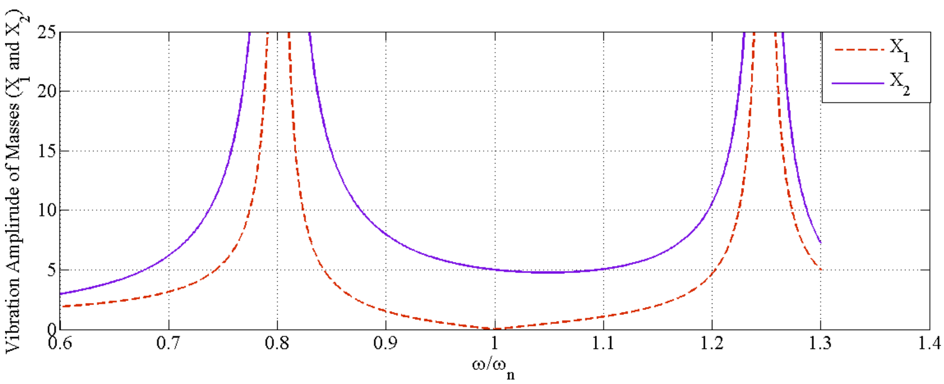
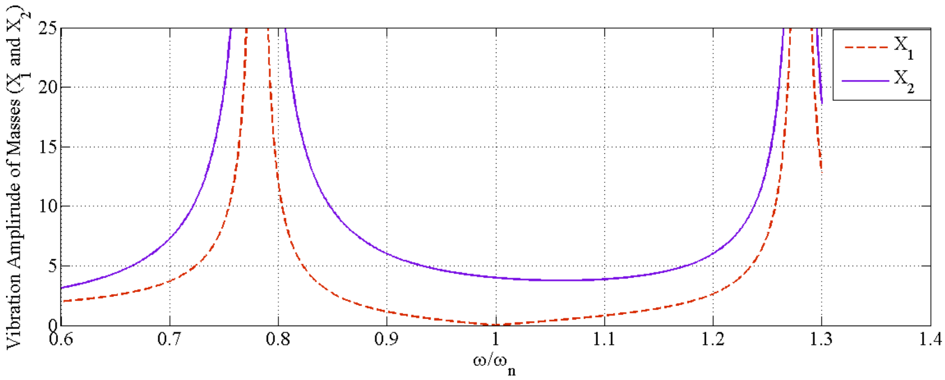


Figure 5 Variations of Vibration Amplitudes of the Main and Absorber Massesof Vibration Absorber for μ = 0.20

Figure 6 shows variations of vibration amplitudes of the main and absorber masses of vibration absorber plotted as function of the frequency ratio, for mass ratio = μ = 0.25.

Figure 6 Variations of Vibration Amplitudes of the Main and Absorber Masses of Vibration Absorber for μ = 0.25



Resonant Frequencies

The dynamic vibration absorber eliminates vibration at the known impressed frequency, as seen in the above figures. , introduces two resonant frequencies 1 and 2, where the amplitude of machine is infinite. In actual, frequencies 1 and 2 must be avoided by the operating frequency . The values of 1 and 2 can be found by equating denominator of Eq.3.3 to zero, noting that

$$\frac{k_2}{k_1} = \frac{k_2}{m_2}\frac{m_2}{m_1}\frac{m_1}{k_1} = \frac{m_2}{m_1}\left(\frac{\omega_2}{\omega_1}\right)^2 \qquad 3.5$$

And setting denominator of Eq.3 to zero leads to

$$\left(\frac{\omega}{\omega_2}\right)^4\left(\frac{\omega_2}{\omega_1}\right)^2 - \left(\frac{\omega}{\omega_2}\right)^2\left[1 + \left(1 + \frac{m_2}{m_1}\right)\left(\frac{\omega_2}{\omega_1}\right)^2\right] + 1 = 0$$

The roots of this equation are given by

$$\left(\frac{\Omega_1}{\omega_2}\right)^2, \left(\frac{\Omega_2}{\omega_2}\right)^2 = \frac{\left\{\left[1+\left(1+\frac{m_2}{m_1}\right)\left(\frac{\omega_2}{\omega_1}\right)^2\right] \pm \left\{\left[1+\left(1+\frac{m_2}{m_1}\right)\left(\frac{\omega_2}{\omega_1}\right)^2\right]^2 - 4\left(\frac{\omega_2}{\omega_1}\right)^2\right\}^{1/2}\right\}}{2\left(\frac{\omega_2}{\omega_1}\right)^2} \qquad 3.6$$

This appears to be functions of (m_2/m_1) and (⍰$_2$/⍰$_1$).

In order to obtain variations of the resonant frequency ratios given by Eq.3.6 with mass ratio, (m_2/m_1). MATLAB program is used. The ratios of ⍰1) and ⍰2) (ω (ω are

2 2

plotted for ⍰2 = 0.5, 1.0 and 2 over the range of m2 = 0 to 1.

⍰1 m1

⍰2 = 0.5.

⍰1

Figure 7 shows variations of the resonant frequency ratios with mass ratio for

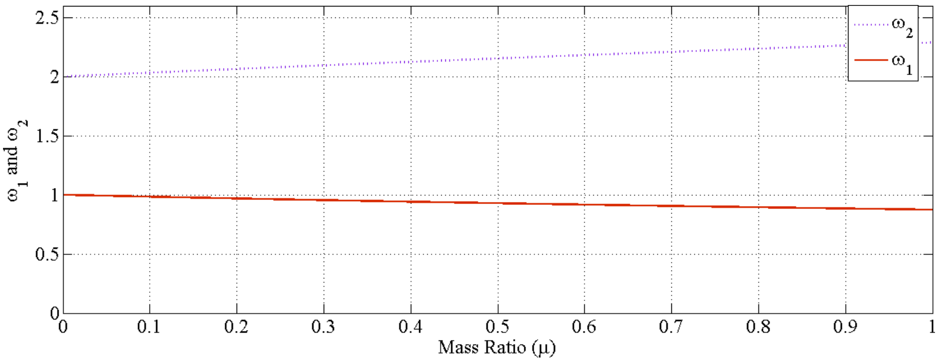


Figure 7 Variations of the Resonant Frequency Ratios with Mass Ratiofor ⍰2 = 0.5

⍰1

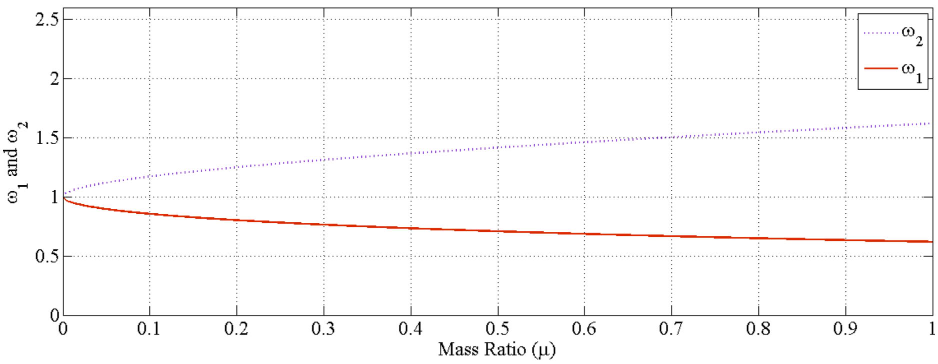


Figure 8 shows variations of the resonant frequency ratios with mass ratio for

⍰2 = 1.0.

⍰1

⍰2 = 2.0.

⍰1

Figure 8 Variations of the Resonant Frequency Ratios with Mass Ratiofor ⍰2 = 1.0

⍰1 Figure 9 shows variations of the resonant frequency ratios with mass ratio for

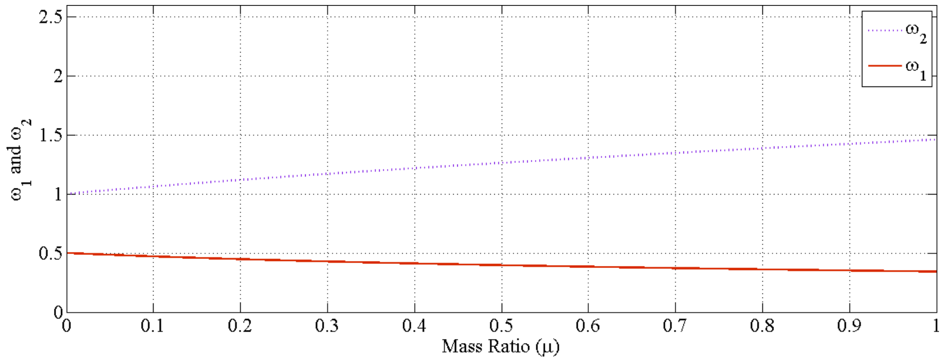


Figure 9 Variations of the Resonant Frequency Ratios with Mass Ratio

for ⍰2 = 2.0

⍰1

<u>Effect of Absorber Mass and Spring Stiffness</u>

To study the effect of absorber mass and spring stiffness on displacement of main mass, MATLAB/Simulink model is developed. Linear Undamped Dynamic Vibration Absorber is modeled in MATLAB/Simulink. Figure 3.10 shows main mass displacement amplitude of linear undamped dynamic vibration absorber.

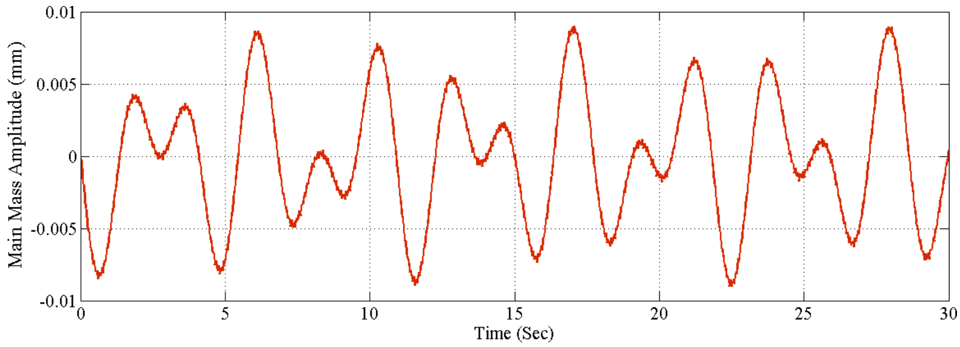


Figure 10 Main Mass Displacement Amplitude vs. Time for LinearUndamped Dynamic Vibration Absorber

Figure 10 shows absorber mass displacement amplitude of linear undamped dynamic vibration absorber.

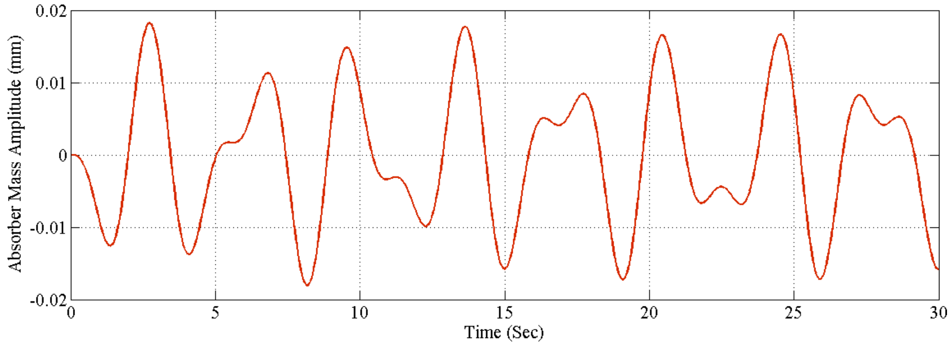


Figure 11 Absorber Mass Displacement Amplitude vs. Time for Linear Undamped Dynamic Vibration

Absorber

Table 2 shows comparison between main mass and absorber mass displacement amplitude for linear undamped dynamic vibration absorber.

Table 2 Comparison between Main Mass and Absorber Mass Displacement Amplitude for Linear Undamped Dynamic Vibration Absorber

Sr.No.	Main Mass Displacement Amplitude (mm)	Absorber Mass Displacement Amplitude (mm)	Percentage Variation (%)
1	0.009000	0.0184	102.28

Discussion of Results

From Figure 2 to Figure 6 it can be concluded that the mass ratio determines how much the main and absorber masses of the vibration absorber vibrate. For example, the amplitude decreases as the mass ratio grows. Also, it is verified that by adding absorber system we have introduced another resonant point in the system. It is also verified that greater the mass ratio, greater is the spread between the resonant frequencies. From Table 1 it is observed that effectiveness of the undamped dynamic vibration absorber as percentage variation in main mass and absorber mass displacement amplitude is 102.22%.

Conclusion

The undamped dynamic vibration absorber system under harmonic excitation is theoretically analyzed in this work. Additionally, MATLAB is used to verify the theoretical analysis of how absorber mass and spring stiffness affect the main mass's displacement amplitude.

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