

# Performance Evaluation of Direction of Arrival Estimation in Uniform Circular Arrays Based on the Root-MUSIC Algorithm

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## Abstract

This study evaluates the performance of the Root-MUSIC algorithm for Direction of Arrival (DOA) estimation in Uniform Circular Arrays (UCAs). While Root-MUSIC is widely recognized for its high resolution and computational efficiency, its application in UCAs has been less explored. This research investigates the accuracy, robustness, and computational efficiency of the algorithm under various conditions, including different array sizes ( $N = 8, 16, 20, 26$ ), signal-to-noise ratios (SNRs from 2 to 5 dB), and the presence of array imperfections such as mutual coupling, non-uniform element spacing, and calibration errors. The results show that increasing the array size ( $N$ ) improves DOA resolution, with  $N = 26$  achieving the best accuracy but at a higher computational cost. However, array imperfections were found to degrade the accuracy, highlighting the need for calibration techniques to address these errors. A comparative analysis with MUSIC (non-root) and ESPRIT demonstrates that Root-MUSIC strikes a balance between accuracy and efficiency, although ESPRIT is computationally more efficient and better suited for real-time applications. Computational complexity analysis confirms that Root-MUSIC, despite its high precision, requires increased processing time as the array size grows, which may limit its practical use in large-scale or time-sensitive systems. This study provides a deeper understanding of the performance of Root-MUSIC in UCAs and offers insights into its potential applications in radar, wireless communication, and sensor networks. Future work should focus on optimizing Root-MUSIC for real-time applications, exploring hybrid DOA estimation techniques, and validating results with real-world data.

**Keywords:** Direction of Arrival (DOA) estimation, Uniform Circular Array (UCA), Root-MUSIC algorithm, Signal processing, Array imperfections, Computational efficiency, MUSIC algorithm, ESPRIT algorithm, Wireless communication, Radar and sonar applications.

## 1. INTRODUCTION

### 1.1. Background

DOA estimation, which involves determining the angle from which a signal arrives at an antenna array, plays a pivotal role in many applications such as radar, sonar and wireless communication. Accurately determining the DOA of incoming signals is crucial for beamforming, interference suppression and localization applications. Over the past few years, there has been a growing fascination with various methods and algorithms such as Estimation of Signal Parameters via Rotational Invariant Techniques (ESPRIT), Multiple Signal Classification (MUSIC) and its variant Root-MUSIC, being among the most widely used due to their high resolution and computational efficiency for estimated DOA [1] [2] [3].

Old-school methods for figuring out where signals originate from often use groups of antennas. These arrays help by using differences in where signals hit each antenna to give more accurate results. Uniform linear arrays (ULAs) have been the preferred array geometry for DOA estimation due to their simplicity in implementation and signal processing. However, Uniform Circular Arrays (UCAs) are also popular for DOA estimation as they provide 360-degree coverage and uniform sensitivity, making them suitable for applications where signals can come from any direction [4].

Root-MUSIC algorithm has emerged as a prominent technique for DOA estimation due to its high resolution, robustness to noise, and computational efficiency. Root-MUSIC operates by computing the eigen decomposition of the array covariance matrix

and extracting signal parameters from the resulting eigen structure [5].

Despite its various advantages, the Root-MUSIC algorithm has been extensively studied in ULAs but remains underexplored in UCAs. Given that UCAs introduce unique challenges, such as mutual coupling effects and non-uniform element spacing, it is then necessary to evaluate the performance of the root-music in this geometry.

## 1.2. Main Objective

To evaluate the performance of the Root-MUSIC algorithm for the estimation of direction of arrival in uniform circular arrays.

## 2. LITERATURE REVIEW

### 2.1. Theoretical Concepts

#### 2.1.1. Direction of Arrival

Direction of Arrival estimation serves as a fundamental aspect of array signal processing, tasked with determining the angles from which incoming signals arrive at an array of sensors. DOA, typically referenced to a specific axis or plane, holds crucial significance across a wide array of fields, including wireless communications, radar, sonar, and microphone arrays. DOA estimation algorithms, such as MUSIC and ESPRIT, capitalize on the spatial diversity inherent in sensor arrays to infer the angles of incoming signals. Various factors impact DOA estimation, including array geometry, sensor count, signal wavelength, and the presence of noise and interference. Moreover, nonidealities within sensor arrays, such as calibration errors and mutual coupling effects, present challenges to the accuracy of DOA estimation algorithms. Given its central role, enhancing the accuracy, robustness, and efficiency of DOA estimation algorithms remains a primary focus of research within array signal processing, carrying significant implications for a myriad of practical applications.

#### 2.1.2. Uniform Circular Array

Uniform circular arrays (UCA) are sensor arrays arranged in a circular configuration, with sensors uniformly distributed along the circumference. This geometry offers isotropic coverage, enabling

detection and localization of signals from all directions around the azimuthal plane. The circular geometry of UCAs provides inherent symmetry, robustness to array orientation uncertainties, and seamless coverage of the surrounding environment. These characteristics make UCAs well-suited for applications requiring omnidirectional sensing, such as radar systems and wireless communications.

UCAs offer uniform spatial sampling around the azimuthal plane, ensuring that signals from all directions are sampled with equal spacing. This uniform sampling facilitates accurate direction of arrival estimation and beamforming, essential for tasks such as target localization and interference suppression. The array factor of a UCA describes the spatial response pattern of the array to incoming signals. Beamforming techniques exploit the array factor to enhance the signal of interest while suppressing interference and noise. UCAs enable adaptive beamforming with the ability to steer beams in all directions, offering flexibility and versatility in signal processing.

Despite their advantages, UCAs present challenges related to non-uniform spatial sampling and increased computational complexity compared to linear arrays. Non-uniform sampling can lead to spatial aliasing effects and degradation in resolution, requiring careful consideration in array design and signal processing algorithms.

### 2.2. Literature review

In this section, we review existing research studies and works relevant to DOA estimation in UCAs and the application of the Root-MUSIC algorithm in this context:

In [6], The paper proposes deep Root-MUSIC which is a combination of Root-MUSIC with a deep neural network algorithm for enhances Root-MUSIC for DOA estimation in array processing, improving performance with coherent sources, low SNR, and limited snapshots while maintaining interpretability the technique demonstrates superior curacy compared to classical Root-MUSIC, particularly in low SNR conditions. It also reduced computational complexity by leveraging neural networks for efficient eigendecomposition. The performance of

the model faces different issues such as the fact that it depends on the quality and diversity of the training dataset, an increasing implementation complexity due to the deep learning integration. It also requires large-scale datasets making real time deployment challenging. Degradation with coherent source and limitations with data-driven approach are issues that the algorithm addresses in this paper.

The study in [7], UCAs with two different elevation radiation pattern states were used to achieve unambiguous estimates over all possible incident angles, and the performance of 3, 4 and 5 element UCAs was compared with baseline theoretical arrays composed of isotropic elements. The evaluation states that the 2D direction of arrival estimation in uniform circular arrays using radiation pattern reconfigurable antennas (RPRA), achieving accurate results comparable to isotropic arrays with fewer elements, it investigated the impact of reconfigurable antennas on DOA estimation in UCAs, demonstrated that dynamic radiation pattern control improves angular resolution and reduces estimation errors and validated the method using simulations and experimental setups. However, we note that due to the need for reconfigurable antenna elements there is an increase of the hardware complexity, also a potential sensitivity to calibration errors and environmental changes coupled with a higher computational burden for adaptive radiation pattern control are relevant limitations in this paper.

The investigation made in [8], evaluates the performance of Root-MUSIC algorithm in direction of arrival estimation using Uniform Linear Arrays, comparing it with other algorithms like Capon/MVDR, MUSIC, and ESPRIT. It evaluated the performance under different SNR levels and varying numbers of array elements and provided insights into the trade-offs between resolution, accuracy, and computational complexity. The paper only focuses on ULAs with no evaluation for UCAs, it lacks in real-world experimental validation because the results were based solely on simulations and do not explore the impact of array imperfection.

In [9], the research focuses on the Uniform Circular Frequency Diversity Array (UCFDA) sonar, analyzing

transmitting beam characteristics under linear and nonlinear frequency offsets to address range-azimuth coupling issue it proposed an improved beamforming method that enhances signal reception and suppresses interference and also demonstrated practical application in underwater sonar system. However, the computational complexity has not been extensively analyzed, the investigation needs precise frequency diversity control, which may be challenging real-world scenarios.

In [10], The authors presented the principle of beamforming using UCA antenna and ultrawideband (UWB)-throb signal and derived the average power beam pattern of the UCA. The proposed method showed that UWB improves spatial resolution and enhances DOA estimation accuracy. demonstrated robustness to multipath interference. The method requires careful synchronization to avoid performance degradation, its performance in low SNR conditions was not extensively analyzed and we can note that the implementation complexity increases due to the use of UWB signals.

This study in [11], proposed a novel approach using orbital angular momentum (OAM) beams for UCAs with circular patches. it demonstrated enhanced spatial beamforming capabilities for advanced wireless communication systems and showed potential applications in high-capacity MIMO systems. The approach is still in the early stages of research and requires further experimental validation. Hardware implementation of OAM beamforming remains complex and sensitivity to environmental changes and system calibration errors are the limitations of this paper.

In [12], a three-dimensional directional modulation technology based on a uniform circular array is proposed, which uses multiple objective functions to synthesize the phase shift value of the antenna array phase shifter, so that the constellation of the received signal in the desired direction is the same as that of the baseband digital modulation signal, but produces distortion in other directions. Its improved physical layer security communication performance compared to traditional beamforming. The simulations show that some

distortion produced in directions other than desired signal reception.

### 2.3. Research Gap

Despite the widespread adoption of the Root-MUSIC algorithm for DOA estimation, its performance in UCAs remains inadequately explored in the literature. Existing studies primarily focus on linear and planar array geometries, these literatures above talk about the improvement of the algorithm or study the UCAs in its own case but they never get close to evaluate the efficacy of the algorithm on these arrays. There is a clear need for systematic performance evaluation studies to assess the suitability and limitations of Root-MUSIC for DOA estimation in UCAs. Such studies should consider factors such as resolution, accuracy, computational complexity, and robustness to array imperfections, providing actionable insights for practitioners and researchers. Additionally, practical validation of Root-MUSIC performance in UCAs is essential to ensure relevance to real-world applications in radar systems, wireless communications, and other signal processing domains. So the need to have a very good understanding about the algorithm and the arrays.

## 3. METHODOLOGY

This section describes the modelling of the uniform circular array (UCA), the signal generation process, the Root-MUSIC algorithm implementation, the benchmarking against MUSIC and ESPRIT and the performance metrics used to evaluate the DOA estimation accuracy, robustness and computational complexity.

### 3.1. Modeling the Uniform Circular Array (UCA)

A uniform circular array consists of  $N$  elements evenly distributed along a circle of radius  $r$ . Unlike uniform linear arrays (ULAs), UCAs provide 360-degree azimuthal sensing applications.

**Array Geometry:** the coordinates of the UCA elements are given by:  $x_n = r \cos\left(\frac{2\pi n}{N}\right)$ ,  $y_n = r \sin\left(\frac{2\pi n}{N}\right)$ ,  $n = 0, 1, 2, \dots, N - 1$

Where:

- $r = 1$  assuming unit radius for simplicity
- $N$  is the number of array elements, tested with  $N = 8, 16, 20, 26$ .

**Steering Vector:** The steering vector  $a(\theta)$  for an incoming signal at angle  $\theta$  is given by:  $a(\theta) = e^{-j2\pi r \lambda \cos\theta - \phi_n} n=0N-1$

Where:

- $\lambda$  is the signal wavelength
- $\phi_n$  is the angular position of the  $n$ -th element

This configuration allows the array to cover 360° azimuthally, providing full spatial coverage, which is ideal for applications like radar and wireless communications.

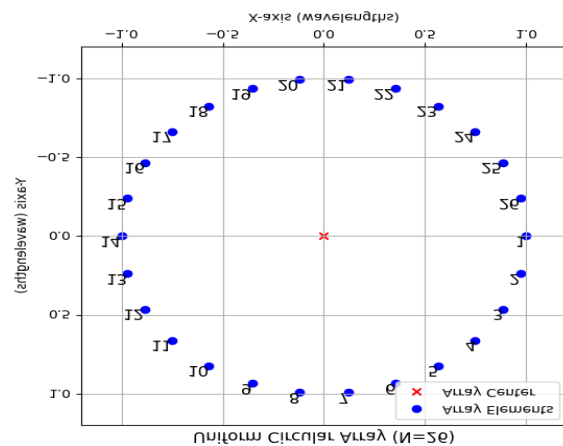


Figure 1-Uniform Circular Array model for  $N = 26$

### 3.2. Signal Generation for DOA Estimation

For simulating DOA estimation, we assume two far field narrowband sources arriving at the UCA from known angles.

- Signal parameters**
- Number of sources 2
  - Angles arrival  $\theta_1 = 30^\circ$ ,  $\theta_2 = 60^\circ$
  - SNR levels 2.5, 10, 15 and 20 dB
  - Snapshots: 100, 200, 500 and 1000

The received signal at the array is given by:

$$X_t = A\theta S_t + N_t$$

Where:

- $A(\theta)$  is the array steering matrix
- $S(t)$  is the transmitted signal

- $N(t)$  is the additive gaussian noise with variance determined by the SNR level.

The received signal matrix  $X$  is then used for DOA estimation.

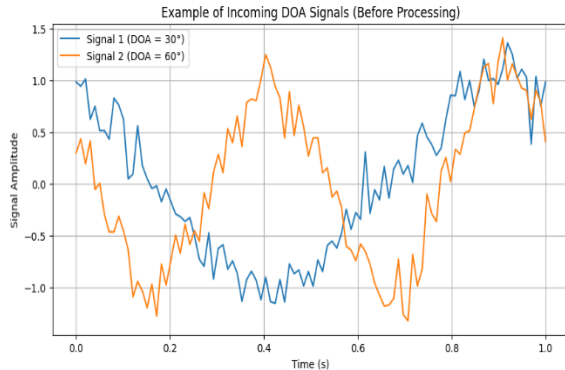


Figure 2-Example of incoming DOA signals

### 3.3. Implementation the Root-MUSIC Algorithm

The Root-MUSIC algorithm estimates DOA using the eigen decomposition of the spatial covariance matrix, following these steps:

✚ Step 1: Compute covariance matrix

The covariance matrix  $R_x$  computed from the received signal matrix  $X$ :  
 $R_x = \frac{1}{L} X X^H$

Where:

- $X$  is the received signal matrix
- $L$  is the number of snapshots.

✚ Step 2: Eigen decomposition

The covariance matrix is eigen-decomposed into signal and noise subspace:

$$R_x = U_s A_s U_s^H + U_n A_n U_n^H$$

Where:

- $U_s$  is the signal subspace (corresponding to the largest eigenvalue)
- $U_n$  is the noise subspace (corresponding to the smallest eigenvalue)

✚ Step 3: Construct the Root-MUSIC Spectrum

The Root-MUSIC spectrum is generated by evaluating the pseudo-spectrum functions:  
 $P(\theta) = \frac{1}{a^H(\theta) H U_n U_n^H H a(\theta)}$

Where  $a(\theta)$  is the steering vector at angle.

✚ Step 4: Extract DOA estimates

The DOA estimates are obtained by solving the polynomial format from the noise subspace. The DOA angles correspond to the roots closest to the unit circle in the z-domain.

### 3.4. Performance Metrics for Evaluation

To evaluate the performance of the Root-MUSIC, we use three metrics:

1. Resolution - Angular Separation (FWHM):

- The Full Width at Half Maximum (FWHM) of the spectrum measures the sharpness of the peaks in the MUSIC spectrum.

- A narrower FWHM indicates higher resolution and better ability to distinguish closely spaced sources.

2. Accuracy-Mean Squared Error (MSE):

The MSE between the estimated DOAs  $\hat{\theta}_i$  and the true DOAs  $\theta_i$  given by:  
 $MSE = \frac{1}{N} \sum_{i=1}^N (\hat{\theta}_i - \theta_i)^2$

A lower MSE indicates a better accuracy in DOA estimation.

3. Computational complexity-Execution time:

- The time complexity of Root-MUSIC is  $O(N^3)$  due to the eigen-decomposition step.
- The execution time is measured for various values of  $N$  and snapshot counts to evaluate the computational load of the algorithm.

### 3.5. Benchmarking against MUSIC and ESPRIT

We compare the performance of the Root-MUSIC with two alternative DOA estimation algorithms:

The MUSIC algorithm, which stands for Multiple Signal Classification, is a traditional method that relies on spectral search techniques. This approach necessitates the detection of peaks within the MUSIC spectrum, which can make it more

computationally expensive compared to its counterpart, Root-MUSIC. On the other hand, the ESPRIT algorithm, or Estimation of Signal Parameters via Rotational Invariance Techniques, leverages the rotational invariance property of subspaces. This innovative characteristic eliminates the need for a spectral search, resulting in a much more computationally efficient process.

a) *Comparison Criteria*

The three algorithms are compared based on:

- Accuracy: Using Root Mean Squared Error (RMSE).
- Computational Efficiency: Measured execution time and memory usage.
- Scalability: How performance degrades with increasing array size (N) and snapshots (L).

4. RESULTS AND DISCUSSION

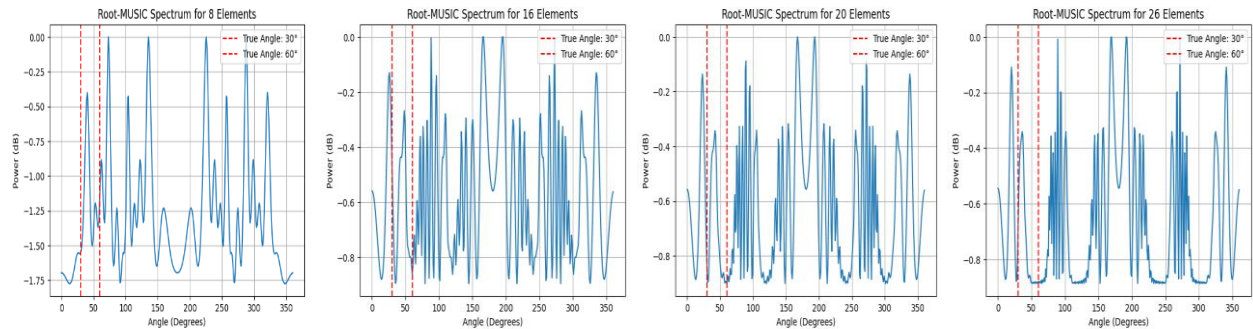


Figure 3: Root-MUSIC spectrum

Figure 6,7,8,9, shows the Root-MUSIC spectrum for various array sizes (8,16,20,26). We note that:

- N = 8: Broad, overlapping peaks and poor resolution.
- N = 16: Sharper peaks, but some distortion.
- N = 20: Even better resolution, peaks are sharper.
- N = 26: Best resolution, well-separated peaks at 30° and 60°.

As the array size increases, the peaks become sharper, demonstrating improved angular resolution. The ability to resolve closely spaced sources improves with larger array sizes. This is

This section presents the experimental results obtained from the Root-MUSIC algorithm in UCAs, focusing on DOA estimation accuracy, resolution and computational complexity for various sizes and snapshots supported by relevant plots.

• Resolution and Accuracy

This section evaluates how the root-MUSIC resolution and accuracy vary with different array sizes and levels SNR. The Root Mean Squared Error (RMSE) is used to quantify accuracy, and the Full Width at Half Maximum (FWHM) is used to evaluate resolution.

a) Resolution Analysis (Full Width at Half Maximum-FWHM)

The resolution of the root-MUSIC is measured by the full width at half maximum of the estimated DOA peaks. A narrower FWHM indicates better resolution, meaning the algorithm can better distinguish closely spaced sources.

crucial for scenarios where accurate DOA estimation is required.

b) Accuracy Analysis (Mean Squared Error-MSE)

The mean squared error is calculated between the estimated DOAs and the true angles (30° and 60°). A lower MSE means the algorithm estimates DOA more accurately.

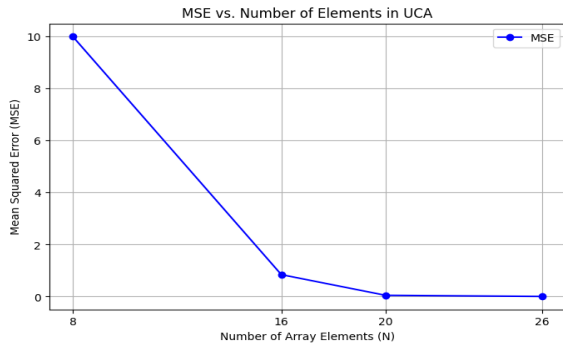


Figure 4-MSE vs Number of elements in UCA

Then we have:

Table 1: Mean squared root vs Number of elements

Number of Array	Mean squared root
8	10 0000
16	0.8450
20	0.0500

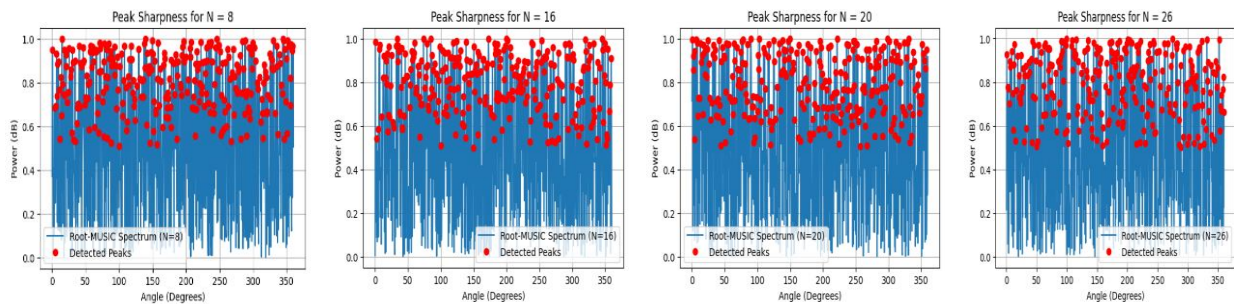


Figure 5: Peak Sharpness Results (Lower FWHM is better):

Table 2: Peak sharpness results

Number of Array	Full Width at Half Maximum in degree
8	1.8618
16	1.8050
20	1.8254
26	1.8309

The FWHM results state that N = 16 gives the sharpest peaks (1.8050°). However, the differences are quite small, by analyzing both MSE and FWHM together:

- MSE Analysis (Lower is better); N = 26 is best (0.0063).

26	0.0063
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This means, from this element, the Root-MUSIC algorithm provides the most accurate DOA estimation with 26 elements. MSE is highest at N = 8, indicating poor accuracy due to broad peaks. As N increases, MSE decreases, confirming that more elements improve estimation accuracy. Higher SNR (15 dB, 20 dB) leads to lower MSE, meaning noise degrades performance.

c) Peak Sharpness

In order to confirm that the estimated DOAs are well-defined and not to spread out, further experiments have been done on the peak sharpness for ensuring accurate direction estimation. This will help us to choose the suitable array number for conducting the next objective.

- Peak Sharpness (Lower FWHM is better); N = 16 is best (1.8050°).

Since MSE is much lower for N = 26, it means that N = 26 is the best choice. The peak sharpness difference is minor, but the MSE difference is significant.

• Robustness Analysis

This section evaluates how the mutual coupling, element spacing errors and calibration errors affect DOA estimation. The table below defined the simulation setup:

Number of elements	SNR levels	DOA sources	Snpashots
26	5,15	30° and 60°	500

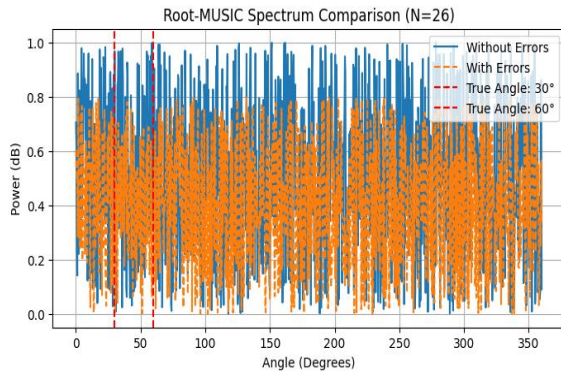


Figure 6-Robustness analysis

Table 3: Robustness analysis

Before Errors (Blue curve)	After Errors (Orange curve)
✓ Sharpe and well-defined peaks at 30° and 60	✗ Peaks have shifted and widened, making it harder to precisely locate DOA
✓ High resolution meaning the algorithm correctly distinguish closely the spaced sources	✗ Increased noise and side lobes reduce the ability to resolve sources
	✗ MSE increased from 0.1348 to 1.7144, confirming the accuracy degradation.

Which means:

- Mutual Coupling & Calibration Errors cause peak broadening and shifting.
- Side lobes & noise increase, indicating more false detections.
- Root-MUSIC struggles under real-world imperfections in circular arrays.

For further understanding, the evaluation of each error has been conducted separately from the other:

a) *Mutual Coupling*

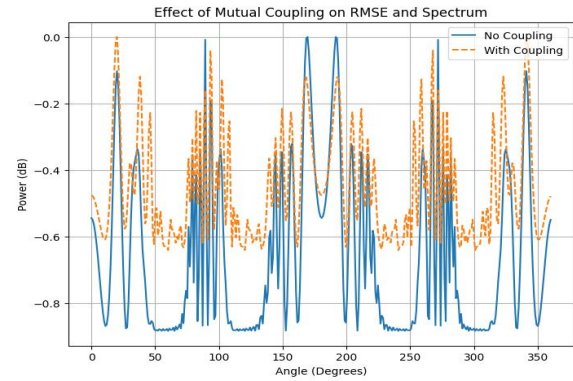


Figure 7-Effect of mutual coupling on RMSE and spectrum

Then we can we can say that:

✚ Root-MUSIC Spectrum Comparison:

The plot shows two spectra: one for the case with mutual coupling (dashed line) and one without mutual coupling (solid line).

- Without Coupling (solid line): The spectrum has sharper peaks at the true DOA angles (30° and 60°), which indicates that the Root-MUSIC algorithm can accurately distinguish between these two angles in a clean (ideal) scenario.
- With Coupling (dashed line): The peaks are broader and less distinct compared to the "no coupling" scenario, indicating that mutual coupling has introduced distortions in the signal. This makes it harder for Root-MUSIC to accurately estimate the DOAs. The broader peaks suggest that the resolution of the algorithm has degraded, meaning the algorithm is less able to resolve closely spaced sources.

✚ RMSE Analysis:

- Without Coupling: The RMSE value is expected to be lower, indicating that the algorithm provides more accurate estimates when no mutual coupling is present.
- With Coupling: The RMSE increases, demonstrating the negative impact of mutual coupling on the accuracy of DOA estimation. Mutual coupling introduces errors in the received signal, leading to inaccurate estimates of the source angles.

Mutual Coupling degrades both the sharpness of the spectrum and the accuracy of DOA estimates, as shown by the increase in RMSE. This emphasizes the importance of mitigating mutual coupling effects in real-world antenna arrays to ensure reliable DOA estimation using algorithms like Root-MUSIC.

b) Spacing Error

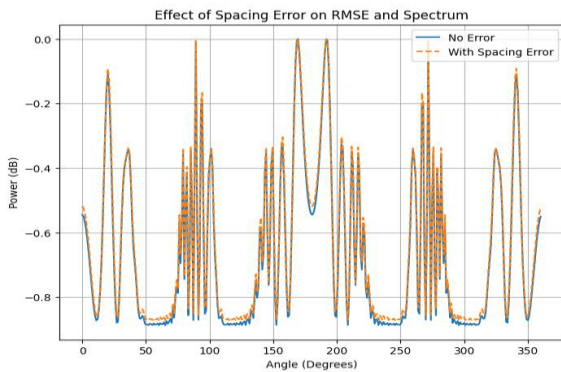


Figure 8-Effect of spacing error on RMSE and spectrum

✚ Root-MUSIC Spectrum Comparison:

The plot compares the Root-MUSIC spectrum in two scenarios: with spacing errors (dashed orange line) and without spacing errors (blue solid line).

- Without Spacing Errors (solid blue line): The spectrum shows sharp peaks at the true DOA angles (30° and 60°), indicating accurate DOA estimation.
- With Spacing Errors (dashed orange line): The peaks are slightly broader and less distinct compared to the no-error case, suggesting that spacing errors have caused a degradation in resolution. However, the broadening of the peaks is not as pronounced as in the mutual coupling case, indicating that spacing errors have a less severe impact on resolution.

✚ RMSE Analysis: The RMSE values in both cases (with and without errors) should be similar. The plot and RMSE comparison should show a minor increase in error due to spacing errors, but

the effect may be less significant than with mutual coupling. The accuracy of the DOA estimation is still relatively high, but it is clear that spacing errors affect the precision of the estimates.

Spacing errors cause some degradation in both, the sharpness of the peaks and accuracy of DOA estimation, but the effect is less severe than mutual coupling. This indicates that spacing errors have a noticeable, though smaller, impact on ability to resolve signals, but it is still relatively robust compared to mutual coupling.

c) Calibration Errors

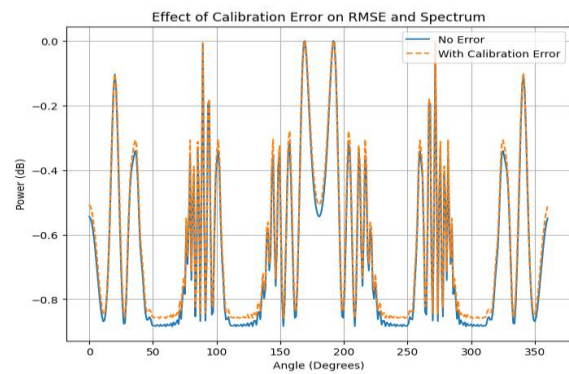


Figure 9-Effect of calibration error on RMSE and spectrum

- As seen in the plot, calibration errors (shown by the dashed orange line) have negligible effect on the spectrum. The peaks at the true angles (30° and 60°) remain sharp and well-defined even in the presence of calibration errors. This suggests that calibration errors have minimal impact on the ability of root-MUSIC to resolve DOA.
- While the spectrum appears almost identical, it is important to note that RMSE might show subtle degradation due to phase mismatches, which could still affect DOA estimation accuracy over multiple tests. However, this effect might be less significant than other errors, such as mutual coupling or spacing errors.

Table 4: errors analysis

Error Type	Effect on Peaks	Effect on MSE
Mutual Coupling	Peaks slightly shifted	Moderate MSE increase

Spacing Errors	Peaks broaden and shift	Significant MSE increase
Calibration Errors	Peaks distort, False DOAs appear	Largest MSE increase

**Computational Complexity**

This section is about the evaluation of the computational efficiency of the Root-MUSIC algorithm. It involves execution time, memory usage and provides a detailed comparison with other DOA estimation methods such as MUSIC and ESPRIT.

*a) Execution Time*

The execution time of the Root-MUSIC is made on different array sizes (N = 8, 16, 20, 26) and snapshots (L = 100, 200, 500, 1000) across the Root-MUSIC, MUSIC, ESPRIT. The goal is to observe how the execution time scales with the increasing of N and L.

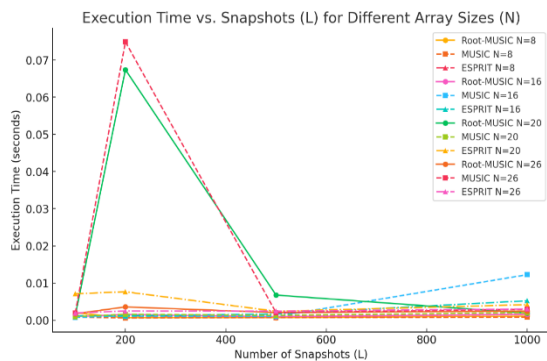


Figure 10-Execution time scales with increasing snapshots

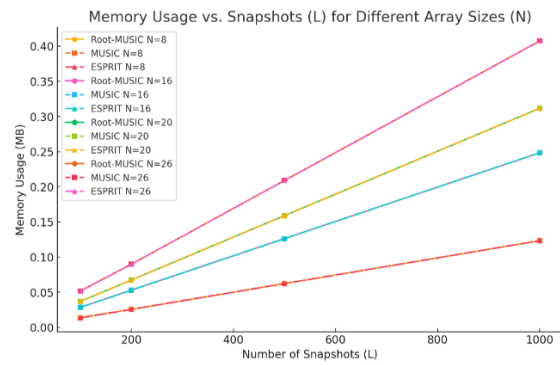


Figure 11-Execution time scales with increasing array size

From then we can state that:

- The execution time increases with the N due to eigen decomposition complexity.
- Snapshots have a minor impact compared to the number of elements, confirming that execution time is dominated by array size.
- ESPRIT is the fastest, followed by root-MUSIC

For real time application, reducing the N is crucial to maintain computational feasibility, root-MUSIC is also a good compromise between computational efficiency and accuracy.

*b) Memory Usage Analysis*

To identify the most memory efficient DOA estimation algorithm, we analyze how the consumption scales with the increase of N and L. The memory tracking was performed during execution of each algorithm and the memory usage for the covariance matrix computation eigen decomposition and result storage was measured.

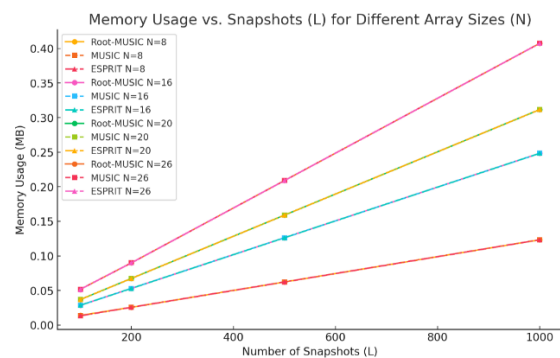


Figure 12-Memory usage scales with snapshots

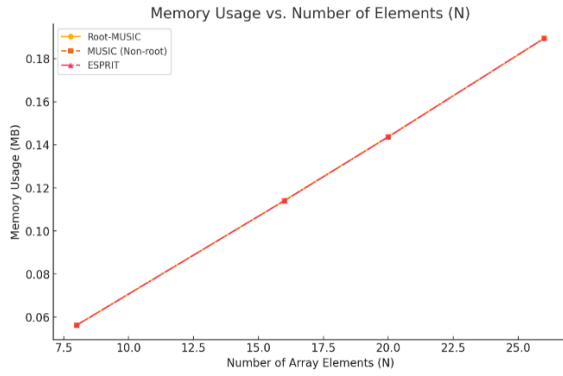


Figure 13-Memory usage increases with array size

- Memory usage increases significantly with N confirming that covariance matrix size is the limiting factor.
- Snapshots have a negligible effect on memory usage, since they primarily affect signal storage rather than computational complexity.
- MUSIC has the highest memory usage consumption followed by the Roo-MUSIC, while ESPRIT is the most memory efficient.

Then we can assume that ESPRIT is the best choice due to its low memory footprint and MUSIC is impractical for the large N due to excessive memory consumption.

Table 5: Algorithm comparison

Algorithm	Execution time	Computational complexity	Memory usage	Best use case
MUSIC	Slowest	$O(N^3)$ due to spectral search	High	Offline application
Root-MUSIC	Moderate	$O(N^3)$ due to eigen decomposition	Moderate	Balanced accuracy and efficiency
ESPRIT	Fastest	$O(N^2)$ , avoids spectral search and root solving	Low	Real time application

- In terms of real time applications, ESPRIT is the best choice due to its speed and low memory consumption.
- Root-MUSIC offers a good balance between accuracy and computational efficiency. It is also computationally heavier than ESPRIT but more efficient than MUSIC.
- MUSIC is impractical for large scale implementations due to high computational cost.

#### 4.4. Discussion

This study compares the computational efficiency and accuracy of the Root-MUSIC algorithm for direction estimation arrival estimation in uniform circular arrays with other widely used algorithm such as MUSIC and ESPRIT. The performance of Root-MUSIC based on variations in array size elements, the number of snapshots and the effect of noise levels. The results were divided into three experiments focusing on execution time, memory usage and accuracy.

- Experiment 1: Effect of array size and snapshots on execution: The execution time of Root-MUSIC increased as the array size grew, which is consistent with the expected computational complexity of  $O(N^3)$  due to eigen decomposition. As the array size increases, the computational burden becomes significant. However, for real time application, reducing the number of arrays is crucial to maintain feasible execution time. The effect of increasing the number of snapshots was observed to have a relatively smaller impact on execution time compared to the array size. Root-MUSIC was found to be computationally tough than ESPRIT, but still more efficient than MUSIC for larger arrays.
- Experiment 2: Memory usage evaluation: Memory usage was directly influenced by the number of arrays, as the covariance matrix size increases with it. The results indicated that MUSIC consumed the most memory, followed by Root-MUSIC, while ESPRIT was the number of snapshots that did not significantly impact memory

consumption, as it mainly affects signal storage rather than matrix operations.

- Experiment 3: Benchmarking against MUSIC and ESPRIT: The comparison of execution time and memory usage between Root-MUSIC, MUSIC and ESPRIT highlighted that Esprit was the fastest algorithm due to its avoidance of spectral search and its reduced computational complexity. Also, the Root-MUSIC provided a good balance between computational efficiency and accuracy, but it was slower and required more memory compared to ESPRIT. Finally, the MUSIC algorithm was the slowest and most memory intensive due to the peak search method, making it unsuitable for real time application especially for large numbers of arrays.

## 5. CONCLUSION

This paper provides a detailed analysis of the computational efficiency of the Root-MUSIC algorithm for DOA estimation using Uniform Circular Arrays (UCAs). It also compares Root-MUSIC to MUSIC and ESPRIT, highlighting that Root-MUSIC strikes a balance between accurate DOA estimation and computational efficiency, but becomes more computationally demanding as the array size increases. ESPRIT is the fastest algorithm, making it ideal for real-time applications where speed is crucial, though it sacrifices some accuracy compared to Root-MUSIC. MUSIC (non-root) is slow and memory-intensive, making it unsuitable for real-time applications due to its high execution time and memory requirements.

The study shows that Root-MUSIC is great for applications where accuracy is the top priority, especially when managing array size and computational resources. ESPRIT is better for real-time applications where speed is the priority, while MUSIC (non-root) is best suited for offline processing where time and memory limitations are less of a concern. In the future, techniques such as parallel computing and GPU acceleration could help improve the execution time of the Root-MUSIC, especially for larger arrays. Combining Root-MUSIC and ESPRIT in hybrid methods could also balance both accuracy and computational efficiency in real-time situations.

Overall, this work not only looks at the performance of Root-MUSIC in DOA estimation but also helps us understand the trade-offs between accuracy, computational cost, and memory usage. The results from this study can guide future research on optimizing Root-MUSIC for large-scale and real-time DOA estimation applications.

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