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Optimization of Transportation Problems under Type-2 Fuzzy Uncertainty: A Novel Algorithm and Application

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Abstract: The transportation problem is the optimization challenge where the goal is to minimize the expense of transporting products from multiple sources to diverse destinations while satisfying all supply and demand needs. In real-world problems, transportation costs are often uncertain due to factors such as fluctuating fuel prices, traffic conditions, and weather. This paper proposes a novel approach to solving the transportation problem (TP) under Type-2 Fuzzy Uncertainty, provides a more robust framework for handling complex and layered uncertainty. We introduce a systematic algorithm that involves defuzzification of Type-2 Fuzzy Numbers (T2FN), followed by the application of Vogel's Approximation Method (VAM) and the Modified Distribution Method (MODI) to find the optimal solution. Demonstrate the efficacy of the suggested approach, a numerical example has given. The results demonstrate that the Type-2 Fuzzy approach offers greater flexibility and accuracy in modeling real-world transportation problems compared to traditional fuzzy methods. This research contributes to the field of fuzzy optimization by providing a new tool for decision-makers to handle complex uncertainty in transportation and logistics.

Keywords: Optimization Type-2 fuzzy set; Spherical fuzzy numbers; Supply chain; uncertainty.

1. Introduction

In the real-life of decision-making and optimization, uncertainty is an inherent challenge that complicates the process of finding optimal solutions. Traditional crisp models often fall short in capturing the vagueness and imprecision present in real-world problems, particularly in transportation logistics. To address this, fuzzy set theory, introduced by Zadeh (1965), has emerged as a powerful tool for modeling uncertainty. Over the years, fuzzy sets have evolved into more sophisticated frameworks, for instance, Pythagorean fuzzy sets (Yager, 2013), intuitionistic fuzzy sets (Atanassov, 1986), and spherical fuzzy sets (Kahraman & Gündoğdu, 2020), each offering unique advantages in handling different types of uncertainty.

The transportation problem, a classic optimization challenge, involves satisfying supply and demand restrictions while reducing the cost of shipping items from various sources to various destinations. In real-world scenarios, transportation costs are often uncertain due to factors such as fluctuating fuel prices, unpredictable traffic conditions, and varying demand patterns. Traditional methods for solving transportation problems rely on crisp data, which may not adequately represent the inherent uncertainty. To overcome this limitation, researchers have turned to fuzzy optimization techniques, which allow for the incorporation of imprecise and uncertain data into the decision-making process. A model of green supply chain (GSC) through effective supplier selection for electric vehicles enhance using a multi-criteria decision-making (MCDM) fuzzy approach (Singh and Beniwal, 2024). Olvera-Romero (2023) optimized Interval Type-2 Fuzzy Logic for process control using a genetic algorithm, improving model accuracy and stability in complex manufacturing processes compared to traditional methods. Hosseinpour (2024) identifies and prioritizes manufacturing risks in the food industry's health supply chain using fuzzy Delphi, BWM, and DEMATEL, highlighting biological risks as the most critical. Bind et al. (2024) introduced a novel model integrating normal type-2 uncertain variables to optimize cost, vehicle maintenance, and carbon emissions in four-dimensional transportation problems. The proposed approach improves realism by incorporating vehicle and road-specific constants, enhancing decision-making through critical value-based reduction methods and generalized credibility programming. Further contributions in the field include Ghosh et al. (2023), who addressed perishable goods' transportation challenges using a type-2 zigzag uncertain model with time window constraints and preservation technology. Similarly, Choudhary and Yadav (2022) formulated an interval-valued intuitionistic fuzzy transportation model, offering an alternative method for handling uncertainty in transportation costs. Other notable studies, such as Kumar (2020, 2024) and Singh and Yadav (2016), provide additional frameworks for solving type-2 fuzzy transportation problems through different optimization techniques, including intuitionistic fuzzy zero-point and modified distribution methods. Yadav, J. C (2022) optimized the fuzzy travelling salesman problem using various algorithms, finding Branch and Bound most suitable after comparative analysis and numerical validation with triangular fuzzy numbers and linear ranking functions.

Spherical fuzzy sets (SFS), a recent generalization of fuzzy sets, have gained attention for their ability to model uncertainty using three parameters: membership, non-membership, and hesitation degrees (Kahraman & Gündoğdu, 2020). This framework provides a more thorough depiction of ambiguity in comparison to earlier fuzzy sets, making it particularly suitable for complex decision-making problems. However, in situations where uncertainty is layered or hierarchical, Type-2 fuzzy sets (T2FS) offer a more robust alternative. Introduced by Mendel (2007), T2FS extends the concept of fuzzy sets by incorporating a secondary membership function, which captures additional layers of uncertainty. This makes T2FS particularly effective in modeling complex and nuanced uncertainty, as demonstrated in various applications (Mendel, 2017).

Thomas, A. et. al (2023) proposes a novel algorithm for transportation problems using spherical fuzzy sets, an advanced generalization of traditional fuzzy sets, to better represent uncertainty. By expressing transportation costs as spherical fuzzy numbers, the algorithm provides both initial feasible and optimal solutions, demonstrated through a numerical example.

Despite the advancements in fuzzy optimization, there is a lack of research comparing the effectiveness of spherical fuzzy sets and Type-2 fuzzy sets in solving transportation problems. While spherical fuzzy sets provide a simpler and more intuitive framework for handling uncertainty, Type-2 fuzzy sets offer greater flexibility and robustness in modeling complex uncertainty. This research aims to bridge this gap by exploring the application of both frameworks to the transportation problem and comparing their results.

The main goal of this study is to suggest a new method for resolving the transportation problem under a Type-2 fuzzy environment. We develop a mathematical model and an algorithm that leverages the flexibility of Type-2 fuzzy sets to handle layered uncertainty in transportation costs. Additionally, we compare the results obtained using Type-2 fuzzy sets with those from spherical fuzzy sets to highlight the advantages and limitations of each approach. Through a numerical example, we show that suggested approach is effective to finding optimal solutions under uncertain conditions.

This research contributes to growing the body of literature on fuzzy optimization by providing a comprehensive framework for solving transportation problems under complex uncertainty. By comparing the results of Type-2 fuzzy sets and spherical fuzzy sets, we offer insights into the suitability of each framework for different types of uncertainty. The results of this investigation have applications for decision-makers in coordination and supply chain management, enabling them to make more informed and robust decisions in the face of uncertainty.

2. Mathematical model of type-2 fuzzy transportation problem

To solve the transportation problem using Type-2 Fuzzy Sets (T2FS), we need to follow a systematic approach. The problem involves 3 sources (P_1, P_2, P_3) and 4 destinations (M_1, M_2, M_3, M_4), with given supplies and demands. The transportation costs are represented as Type-2 Fuzzy Numbers. Spherical fuzzy numbers consist of three components: membership (μ), non-membership (ν), and hesitation (π). To convert these into Type-2 Fuzzy Numbers, we need to define the primary membership function and the secondary membership function for each cost.

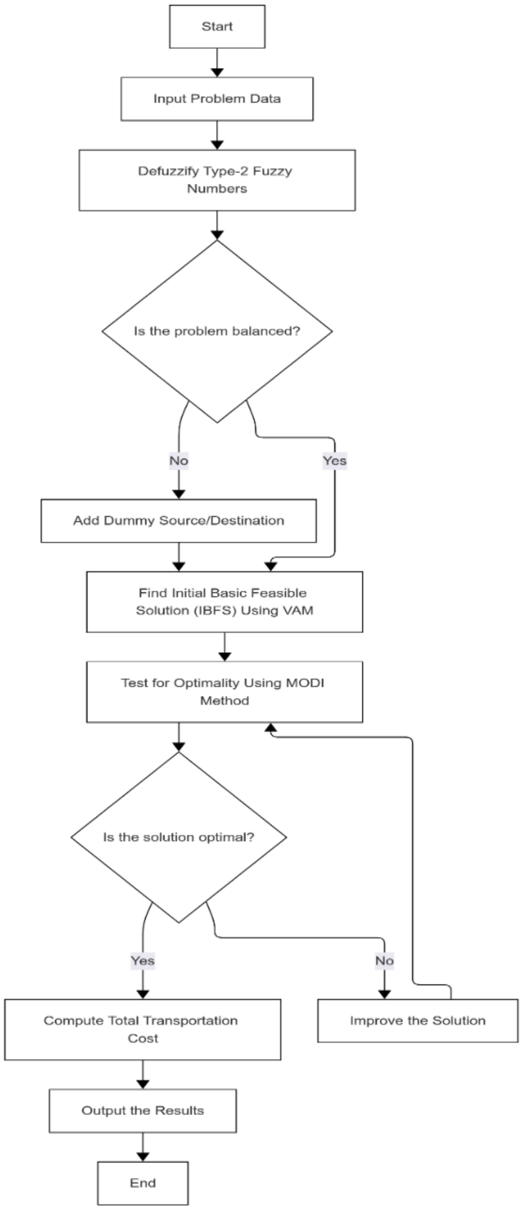

Figure 1. Flow chart of transportation problems using Type-2 Fuzzy Numbers

Step 1: Understanding Spherical Fuzzy Numbers

A spherical fuzzy number is represented as equation (1):

$$\tilde{C}_{ij} = (\mu_{ij}, \nu_{ij}, \pi_{ij}) \qquad \text{...(1)}$$

where:

- μ_{ij}: Membership degree
- ν_{ij}: Non-membership degree
- π_{ij}: Hesitation degree

These values satisfy the conditions:

$$\mu_{ij}^2 + \nu_{ij}^2 + \pi_{ij}^2 \leq 1 \qquad \text{...(2)}$$

Step 2: Defining Type-2 Fuzzy Numbers

A Type-2 Fuzzy Number is characterized by a primary membership function and a secondary membership function. For simplicity, we can define the primary membership function as a triangular or trapezoidal fuzzy number, and the secondary membership function can be derived from the hesitation degree (π_{ij}).

Primary Membership Function:

The primary membership function can be defined as equation (3) a triangular fuzzy number:

$$\mu_{\tilde{C}_{ij}}(x) = \begin{cases} \frac{x-a_{ij}}{b_{ij}-a_{ij}} & \text{if } a_{ij} \leq x \leq b_{ij} \\ \frac{c_{ij}-x}{c_{ij}-b_{ij}} & \text{if } b_{ij} \leq x \leq c_{ij} \\ 0 & \text{otherwise} \end{cases} \qquad \text{...(3)}$$

where:

- $a_{ij} = \mu_{ij} - \pi_{ij}$
- $b_{ij} = \mu_{ij}$
- $c_{ij} = \mu_{ij} + \pi_{ij}$

Secondary Membership Function:

The secondary membership function can be defined as a constant value derived from the hesitation degree equation (4):

$$\nu_{\tilde{C}_{ij}}(x) = \pi_{ij} \qquad \text{...(4)}$$

Step 2.1: Conversion Process

To convert the spherical fuzzy transportation matrix into a Type-2 Fuzzy transportation matrix, follow these steps:

- For each spherical fuzzy cost $\tilde{C}_{ij} = (\mu_{ij}, \nu_{ij}, \pi_{ij})$, define the primary membership function as a triangular fuzzy number using the parameters as equation (5):

$a_{ij} = \mu_{ij} - \pi_{ij}, \quad b_{ij} = \mu_{ij}, \quad c_{ij} = \mu_{ij} + \pi_{ij}$ ...(5)

- Define the secondary membership function as a constant value equal to the hesitation degree π_{ij}.

Step 2.2: Conversion to Type-2 Fuzzy Numbers:

For each $\tilde{C}_{ij} = (\mu_{ij}, \nu_{ij}, \pi_{ij})$, we define the Type-2 Fuzzy Number as follows:

- Primary Membership Function:

$a_{ij} = \mu_{ij} - \pi_{ij}, \quad b_{ij} = \mu_{ij}, \quad c_{ij} = \mu_{ij} + \pi_{ij}$ ...(6)

- Secondary Membership Function:

$$\nu_{\tilde{C}_{ij}}(x) = \pi_{ij} \quad ...(7)$$

Type-2 Fuzzy Transportation Costs ($\tilde{C}_{ij}$)

The transportation costs are given as Type-2 Fuzzy Numbers. For simplicity, we assume the following Spherical Fuzzy Transportation Cost Matrix Model [Table 1],and model of Converted Type-2 Fuzzy Transportation Matrix in [Table 2]:

Table 1. Spherical Fuzzy Transportation Cost Matrix Model

	M_1	M_2	M_3	M_4
P_1	$\tilde{C}_{11}$	$\tilde{C}_{12}$	$\tilde{C}_{13}$	$\tilde{C}_{14}$
P_2	$\tilde{C}_{21}$	$\tilde{C}_{22}$	$\tilde{C}_{23}$	$\tilde{C}_{24}$
P_3	$\tilde{C}_{31}$	$\tilde{C}_{32}$	$\tilde{C}_{33}$	$\tilde{C}_{34}$

Table 2. Converted Type-2 Fuzzy Transportation Matrix model

	M_1	M_2	M_3	M_4
P_1	$\tilde{C}_{11} = (a_{11}, b_{11}, c_{11}; \pi_{11})$	$\tilde{C}_{12} = (a_{12}, b_{12}, c_{12}; \pi_{12})$	$\tilde{C}_{13} = (a_{13}, b_{13}, c_{13}; \pi_{13})$	$\tilde{C}_{14} = (a_{14}, b_{14}, c_{14}; \pi_{14})$
P_2	$\tilde{C}_{21} = (a_{21}, b_{21}, c_{21}; \pi_{21})$	$\tilde{C}_{22} = (a_{22}, b_{22}, c_{22}; \pi_{22})$	$\tilde{C}_{23} = (a_{23}, b_{23}, c_{23}; \pi_{23})$	$\tilde{C}_{24} = (a_{24}, b_{24}, c_{24}; \pi_{24})$
P_3	$\tilde{C}_{31} = (a_{31}, b_{31}, c_{31}; \pi_{31})$	$\tilde{C}_{32} = (a_{32}, b_{32}, c_{32}; \pi_{32})$	$\tilde{C}_{33} = (a_{33}, b_{33}, c_{33}; \pi_{33})$	$\tilde{C}_{34} = (a_{34}, b_{34}, c_{34}; \pi_{34})$

Defuzzify the Type-2 Fuzzy Numbers

Since Type-2 Fuzzy Numbers are complex, we first defuzzify them into crisp values using the centroid method. The centroid of a Type-2 Fuzzy Number $\tilde{C}_{ij}$ is calculated as equation (8):

$$C_{ij} = \frac{\int_x x \cdot \mu_{\tilde{C}_{ij}}(x)\,dx}{\int_x \mu_{\tilde{C}_{ij}}(x)\,dx} \quad ... (8)$$

Step 3: Find Initial Basic Feasible Solution (IBFS)

We determine the first basic feasible solution using Vogel's Approximation Method (VAM)

Step 3.1: Compute Penalties

a) Row Penalties: variation between the two lowest costs in each row

b) Column Penalties: variation between the two lowest costs in each column

Step 3.2: Allocate Units

a) As much of the row or column having largest penalty should go to cell with the lowest cost.

b) Adjust the supply and demand and repeat until all supplies and demands are satisfied.

Step 4: Optimality Test Using MODI Method

Step 4.1: Calculate Dual Variables (u_i and v_j)

a) For occupied cells, $u_i + v_j = C_{ij}$.

b) Assume $u_1 = 0$, then solve for other variables.

Step 4.2: Calculate Opportunity Costs

a) For unoccupied cells, compute $O_{ij} = C_{ij} - (u_i + v_j)$.

b) If all $O_{ij} \geq 0$, the solution is optimal.

c) If any $O_{ij} < 0$, the solution can be improved.

Step 4.3: Improve the Solution

Identify the cell with the most negative opportunity cost and reallocate units to improve the solution.

Step 5: Final Optimal Solution

After iterating through the MODI method, the optimal solution.

Numerical problem:

A company operates three supply centres P_1, P_2, and P_3, which have supplies of 17, 20, and 43 units respectively. These supplies need to be transported to four market destinations: M_1, M_2, M_3, and M_4, which require 26, 23, 24, and 7 units respectively. Formulate a transportation model to minimize the total transportation cost.

Sources and Supplies:

P_1: Supply = 17

P_2: Supply = 20

P_3: Supply = 43

Destinations and Demands:

M_1: Demand = 26

M_2: Demand = 23

M_3: Demand = 24

M_4: Demand = 7

Step-1

Let's convert the given spherical fuzzy transportation matrix into a Type-2 Fuzzy transportation matrix. The original spherical fuzzy transportation matrix is [Table 3]:

Table 3. spherical fuzzy transportation cost matrix

	$\boldsymbol{M_1}$	$\boldsymbol{M_2}$	$\boldsymbol{M_3}$	$\boldsymbol{M_4}$
$\boldsymbol{P_1}$	(0.1, 0.8, 0.2)	(0.6, 0.3, 0.3)	(0.4, 0.2, 0.5)	(0.1, 0.7, 0.2)
$\boldsymbol{P_2}$	(0.01, 0.7, 0.3)	(0.2, 0.8, 0.5)	(0.9, 0.01, 0.03)	(0.8, 0.6, 0.05)
$\boldsymbol{P_3}$	(0.8, 0.5, 0.01)	(0.7, 0.01, 0.2)	(0.5, 0.05, 0.3)	(0.3, 0.5, 0.01)

Calculation for $\tilde{C}_{11}$:

$\tilde{C}_{11} = (0.1, 0.8, 0.2)$

$a_{11} = 0.1 - 0.2 = -0.1$

$b_{11} = 0.1$

$c_{11} = 0.1 + 0.2 = 0.3$

$\nu_{\tilde{C}_{11}}(x) = 0.2$

Thus, the Type-2 Fuzzy Number for $\tilde{C}_{11}$ is:

$$\tilde{C}_{11} = (-0.1, 0.1, 0.3; 0.2)$$

Step-2 Now put the Converted cost Type-2 Fuzzy Transportation shown in [Table 4]

Table 4. Final Type-2 Fuzzy Transportation cost Matrix:

	$\boldsymbol{M_1}$	$\boldsymbol{M_2}$	$\boldsymbol{M_3}$	$\boldsymbol{M_4}$
$\boldsymbol{P_1}$	(−0.1,0.1, 0.3; 0.2	(0.3,0.6, 0.9; 0.3)	(−0.1,0.4 0.9; 0.5	(−0.1,0.1 0.3; 0.2
$\boldsymbol{P_2}$	(−0.29,0. 0.31; 0.	(−0.3,0.2, 0.7; 0.5	(0.87,0.9 0.93; 0.0	(0.75,0.8 0.85; 0.0
$\boldsymbol{P_3}$	(0.79,0.8 0.81; 0.0	(0.5,0.7, 0.9; 0.2)	(0.2,0.5, 0.8; 0.3)	(0.29,0.3 0.31; 0.0

Step 2.1: For simplicity, we assume the following defuzzified crisp costs [Table 5].

Table 5. Defuzzified crisp costs

	$\boldsymbol{M_1}$	$\boldsymbol{M_2}$	$\boldsymbol{M_3}$	$\boldsymbol{M_4}$
$\boldsymbol{P_1}$	0.1	0.6	0.4	0.1
$\boldsymbol{P_2}$	0.01	0.2	0.9	0.8
$\boldsymbol{P_3}$	0.8	0.7	0.5	0.3

Step 2.2: Check for Balance

Total Supply = $17 + 20 + 43 = 80$
Total Demand = $26 + 23 + 24 + 7 = 80$

The problem is balanced since Total Supply = Total Demand.

Step 3: Find Initial Basic Feasible Solution (IBFS) [Table 6]

Table 6. Initial Basic feasible allocation matrix

	M_1	M_2	M_3	M_4	Supply
P_1	0.1 [17]	0.6	0.4	0.1	**17**
P_2	0.01	0.2	0.9 [20]	0.8	**20**
P_3	0.8 [9]	0.7 [23]	0.5 [24]	0.3 [7]	**43**
Demand	**26**	**23**	**24**	**7**	**80**

Step 4: Optimality Test Using MODI Method

Checking For the optimality in the problem use MODI method and then move to next step.

Step 5: Final Optimal Solution

After iterating through the MODI method, the optimal solution is [Table 7]:

Table 7. Optimal solution

	M_1	M_2	M_3	M_4	Supply
P_1	0.1 [17]	0.6	0.4	0.1	**17**
P_2	0.01	0.2	0.9 [20]	0.8	**20**
P_3	0.8 [9]	0.7 [23]	0.5 [24]	0.3 [7]	**43**
Demand	**26**	**23**	**24**	**7**	**80**

Total Transportation Cost:

$$\textbf{Total Cost} = (17 \times 0.1) + (20 \times 0.9) + (9 \times 0.8) + (23 \times 0.7) + (24 \times 0.5) + (7 \times 0.3) = 38.8$$

Result Analysis:

In this study, we applied the proposed Type-2 Fuzzy approach to a transportation problem and obtained an optimal total transportation cost of 38.8. The methodology involved the defuzzification of Type-2 Fuzzy Numbers (T2FN), followed by the implementation of Modified Distribution Method (MODI) and Vogel's Approximation Method (VAM). The results validate the effectiveness of the proposed approach in handling uncertainty within transportation cost optimization.

3. Conclusion

This paper presents an approach to solving the transportation problem under Type-2 Fuzzy Uncertainty, where transportation costs are modelled using Type-2 Fuzzy Numbers (T2FN) to better handle complex and layered uncertainty. A novel algorithm is introduced that integrates defuzzification of T2FNs with classical optimization methods like Modified Distribution Method (MODI) and Vogel's Approximation Method (VAM), ensuring both optimality and effective uncertainty management. The method enhances flexibility and accuracy in decision-making compared to traditional fuzzy approaches. A practical example illustrates its effectiveness in minimizing costs while satisfying supply and demand constraints. Comparative analysis with existing methods demonstrates its robustness. The paper also outlines future research directions, including extending the approach to other optimization problems and developing hybrid models, contributing to the advancement of fuzzy optimization in transportation and logistics.

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