

Impact of Variable Holding Cost on Two Level Trade Credit Policy for an Optimal Supply Chain Management

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Abstract

The supply chain system purpose is to integrate environment thinking with adroit energy consumption into supply chain management (SCM). It involves flexible manufacturing process of the items, more items delivery, optimal energy consumption, and deduced the waste. The manufacturing process may be made much flexible through volume agility. In this case, production of the items can't be certain, and with the concept of volume agility, production is taken as decision variable under the impact of optimum energy consumption. We consider supplier and producer with two level trade credit policies with optimum consumption. To reduce the total cost, we believe that demand function on selling price. Energy and carbon emission costs introduced in the holding cost, setup cost, ordering cost and items cost for supplier and producer. The supply chain model was established analytically and obtained the optimal decision variable in closed form solution. The optimality of the objective function is being obtained analytically as well as graphically. Sensitivity analysis solved to show the variation in the decision parameters. Finally, the numerical examples are employed to illustrate the present study with marginal insights.

Keywords - Supply chain, management, variable, holding cost, credit, production.

1. Introduction

The efficient use of energy may make supply chain more efficient. (SCM) supply chain management characterized by complementing the traditional economic (cost) focus on environmental (emission) considering only without concept of carbon emission. In this study research gap for the impact of carbon emission and energy can be conducting. It is a great challenging to reduce the carbon emission and energy consumption in the traditional production, but energy consumption may be reduced by controlling the production rate of items. Due to a lack of coordination in the supply chain the impact of deterioration from one stage to other increases. Impacts of carbon emission in the global sustainable supply chain play a vital role. Carbon emission and energy cost can be occurred in various different production stages while holding stock, in setup, and other cost.

Consequently, due to deterioration of the items, waste is generated in every supply chain stage. It is pretence a severe risk to nature and human health. It introduced global warming to the environment. Global warming increases due to carbon di-oxide, sulphur di-oxide gases. So in this context many prominent researchers and authors worked such as- Leopoldo et

al.[1] developed an economic production quantity model (EPQ) under the multi-shipment policy. They examined the combined impacts of fluctuating carbon emission costs and various payment schemes. Sarkar and team[2] developed the three-echelon supply chain model for deteriorating items. And they managed credit policy for imperfect quality items. In the same manner three-echelon supply chain was developed by Khanna et al. [3]. Huang and Hsu[4] extended Huang's [5] model by incorporating a two-level trade credit policy (TLTCP). In this policy, the retailer received the complete trade-credit from the supplier. And demand was dependent on selling price. Jaggi & Verma [6] investigated partial trade credit financing for two level of supply chain system. The main aim of this work was to provide the best ordering inventory policies. Sarkar [7] developed the multi stage production model for optimum production with imperfect production. Dam and Singh [8] investigated a multi-item ordering process with green technology. They worked on the agile production. Yadav et al.[9] Established an inventory model in the fuzzy environment with trade credit and selling price demand dependent. Ouyang et al.[10] examined the optimal inventory policy under trade credit. Jaggi et al.[11] Determined the optimum credit policy and replenishment policy for the retailers.

Gautam et al. [12]evaluated the impacts of preservation technique, volume agility, and fee-sensitive demand. Chung's [13] established production inventory control model, which was based on TLTCP. Teng et al.[14] Established a model for favourable price of the items and lot size strategy. They included delay in payments and discrepancy between the selling price and the buying cost. Aljazzar et al.[15] Developed a model for deteriorating items with delay in payment. They reduced the carbon emission from industries and transportation. Tiwari et al.[16]developed a sustainable green production model with the impact of partial backordering and delay in payment. Shah et al.[17] Presented the economic order quantity model for effective regulating of non-instantaneous deteriorating products. And demand was dependent on selling price and advertisement. Bhunia et al.[18] Introduced an inventory model considering the selling

price, frequency of advertisement, and time dependent demand. Namdeo et al.[19] Presented inventory model with constant deterioration of the items. They applied the stock and price dependent demand function. And preservation technology applied to control the deterioration of the items. Shaikh et al.[20] Developed an Economic Order Quantity (EOQ) model incorporating time dependent and ramp type demand function. Handa et al.[21] Investigated the impact of inflation on their inventory strategy. Preservation technology applied to control the deterioration of the items.

This research articles helps to find the gap regarding the relationship between energy use and carbon emissions by using agile manufacturing and holding cost variable showing two level trade credit policy

2. Statement of problem, symbols and assumptions

2.1 Statement of problem

In this order to reduce the energy cost and carbon emissions with agile manufacturing the supply chain system is studied. Production rate is considered as a decision variable in agile production. Where, per-unit production costs differ from raw material costs. In this example credit period and the purchase cost have impact on the demand rate. During the setup, and holding costs, carbon emissions and energy costs are increases. This study main aim is to reduce total cost and control the carbon emissions cost. And to maintain the best approach to integrated model then TLTCP is the important factor. This research work studied to increase and boosting demand.

2.2 Symbols

Table-1

Notations	Description
a	Decision-making Factors
T	length of cycle of the producer
N	Supplier-provided credit terms
P	The producer's production rate

	Parameters for producers
$E_{1p} = h_1 + h_2 t$	Producer inventory holding price is variable
E_p	Cost of a producer's item
E_{2p}	Producer setup cost
E'_{1p}	Costs associated with holding objects include energy and carbon emissions
E'_p	Costs resulting from energy use and carbon emissions from deterioration
E'_{2p}	Costs included for setup-related carbon emissions and energy use
$\phi(p)$	Cost of production
c	Parameters for Supplier
M	Producer-provided credit period
I_e	Interest rate received by the supplier
I_p	interest paid by the supplier
E_{1s}	Supplier holding costs

E_s	Supplier Item cost
E_{2s}	Supplier ordering cost
E'_{1s}	Costs associated with holding objects include energy and carbon emissions
E'_{2s}	Costs associated with ordering products, including energy and carbon emissions
c	Purchasing cost
E'_s	Costs resulting from energy use and carbon emissions from deterioration
n	Number of cycle
T_2	Per delivery cycle length $T_2 = \frac{T}{n}$

2.3 Assumptions

1. Regarding the requirements of the situation, this model made an assumption regarding the cost of energy and carbon emissions in a cost of holds item, and cost of setup/ordering for producers and suppliers.[22]
2. Because of agile production, an adjustable manufacturer rate regulates any excess holding of items or shortage throughout the entire cycle. Because the manufacturing rate is a decision variable, there are no shortages.[23]
3. Here, the rate of demand fluctuates according to the credit time and selling price of the supplier. Therefore, equal to $D = V - (V - b)(1 - \gamma)N + \alpha p - \delta$, where V and b are the Highest and primary demands for the products, respectively, and z is the demand saturation rate. α and δ are the scaling parameter and price elasticity parameter of the selling price p , respectively.[11]
4. There is a very low rate of deterioration ($0 \leq \theta \leq 1$)
5. TLTCP is utilised in this study. In accordance with this procedure, the manufacturer assigns a constant time M to the seller, The seller also suggests to their

customers a duration N for account settlement.[11]

6. There is not considering the effect of the inflation.
 7. Production cannot remain steady due to agile manufacturing. Because of this, the production rate is provided below [23]
- $$\phi(P) = \sigma + \frac{G}{P} + SP + \beta(P - P_{cs}).g(P - P_{cs}), \text{ where } g(P - P_{cs}) = \begin{cases} 1, & P > P_{cs} \\ 0, & P \leq P_{cs} \end{cases}$$

- σ is the article price.
- Energy costs and employment are related to the term $(\frac{G}{P})$. This cost declines as productivity (P) grows.
- The symbol (SP) stands for tool or dies price.
- The final term reflects a crucial level for the machine's output rate (P_{cs}). For an extremely high manufacturing rate, the produced stock is supposed to be flawed ($P > P_{cs}$).

3. Materials and methods

In this segment, we introduced an integrated two-echelon supply chain model for a supplier and the producer. The items are produced here by the producer at a certain rate (P). These items are delivered in a set number of shipments to the supplier. The selling price and credit period both factors affected with the demand. The producer gives time to the supplier during they must return a certain amount of the product. The retailer offers its customers a credit period as well

3.1. Model of Producer's

The mathematical model developed for the producer using agile manufacture items. The manufacturer production begins at time $t = 0$ and continuously produced at time $t = T_1$, when the stock level reaches its maximum level. Then inventory level decrease due to demand and deterioration. Deterioration starts at the time T_1 , after that inventory level reach at maximum level then manufacturing process stops and inventory level declines. At time $t = T$, the stock level reaches

at zero level. Figure 1 shows the behaviour of the stock system during the interval

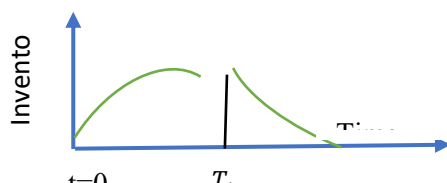


Fig.1 Representation of the levels for a producer with time

Stock levels are demonstrated to rise with the pace of production and fall with the rate of demand and deterioration. As seen below, the situation of inventory can be described.

$$\frac{dI_{p1}}{dt} = -\theta I_{p1}(t) - D + P, \quad 0 \leq t \leq T_1 \quad (1)$$

At the time $T_1 \leq t \leq T$ we see that the demand and deterioration rates cause the stock level rate to fall. The following differential equation is a representation of the inventory system:

$$\frac{dI_{p2}}{dt} = -\theta I_{p2}(t) - D, \quad T_1 \leq t \leq T \quad (2)$$

Solving equations (1), (2) by using boundary conditions $I_{p1}(0) = 0$ and $I_{p2}(T) = 0$ we have.

$$I_{p1}(t) = \frac{P - D}{\theta} (1 - e^{\theta t}) \quad (3)$$

The inventory level at any time when there is no manufacturing.

$$I_{p2}(t) = \frac{D}{\theta} (e^{\theta(T-t)} - 1) \quad (4)$$

Using condition $I_{p1}(t) = I_m$, the maximum inventory level, we have.

$$I_m = \frac{P - D}{\theta} (1 - e^{\theta T_1}) \quad (5)$$

Applying the continuity condition of inventory at T_1 i.e., $I_{p1}(T_1) = I_{p2}(T_1)$, We have.

$$\frac{P - D}{\theta} (1 - e^{\theta T_1}) = \frac{D}{\theta} (e^{\theta(T-T_1)} - 1)$$

On solving this we find production time as

$$T_1 = \frac{1}{\theta} \log \frac{De^{\theta T} + P - D}{P}$$

(a) Holding Cost for item

The industries have must covered the extra expense and environmental cost associated with the EURO-14. There are two types of the holding cost. One is related to the holding of the products as E_{1p} , where $E_{1p} = h_1 + h_2 t$, and other is E'_{1p} carbon emissions.

$$H_p = \frac{(E_{1p} + E'_{1p})}{T} \int_0^{T_1} I_{p1}(t) dt + \int_{T_1}^T I_{p2}(t) dt$$

Here $E_{1p} = h_1 + h_2 t$

$$= \frac{h_1 + c}{T} \left[\frac{P - D}{\theta} \left\{ T_1 + \frac{(e^{-\theta T_1} - 1)}{\theta} \right\} + \frac{D}{\theta} \left\{ \frac{1}{\theta} (e^{-\theta(T-T_1)} - 1) + (T_1 - T) \right\} + \frac{h_2}{T} \left[\frac{P - D}{\theta} \left\{ \frac{T_1^2}{2} + \frac{T_1 e^{-\theta T_1}}{\theta} + \frac{e^{-\theta T_1}}{\theta^2} - \frac{1}{\theta^2} \right\} + \frac{D}{\theta} \left\{ -\frac{T}{\theta} - \frac{1}{\theta^2} + \frac{T_1 e^{-\theta(T-T_1)}}{\theta} + \frac{e^{-\theta(T-T_1)}}{\theta^2} + \frac{T_1^2 - T^2}{2} \right\} \right]$$

(b) Setup Cost

The setup cost of the manufacturer is divided in two parts: E_{2p} , which is related to the setup of the products, and E'_{2p} , which is related to energy use and carbon emissions. Finally, the total setup cost taking in the environment account.

$$ST_p = \frac{(E_{2p} + E'_{2p})}{T}$$

(c) Item Cost

The item cost from the producer has two parts. One is connected with deterioration of the products as E_p and other part is the energy use and carbon emissions as E'_p . So the total item cost is given below

$$IT_p = \frac{(E_p + E'_p)}{T} \int_0^{T_1} P dt$$

$$IT_p = \frac{(E_p + E'_p)}{T} (P T_1)$$

(d) Cost of Production

The production cost for the producer consisting the labour costs, material costs, and tool, die costs, and possibly additional labour and energy costs.

$$PC = \varphi(P) \int_0^{T_1} P dt = \left[\sigma + \frac{K}{P} + SP + \beta(P - P_{cs}) * g(P - P_{cs}) \right] P T_1$$

Total cost of producer.

$$\begin{aligned} TCP = PC + ST_p + IT_p + H_p \\ = \left[\sigma + \frac{K}{P} + SP + \beta(P - P_{cs}) * g(P - P_{cs}) \right] P T_1 + \\ \frac{(E_{2p} + E'_{2p})}{T} + \frac{(E_p + E'_p)}{T} (P T_1) + \frac{h_1 + c}{T} \left[\frac{P-D}{\theta} \{ T_1 + \frac{(e^{-\theta} T_1 - 1)}{\theta} \} + \frac{D}{\theta} \left\{ \frac{1}{\theta} (e^{-\theta(T-T_1)} - 1) + (T_1 - T) \right\} \right] + \\ \frac{h_2}{T} \left[\frac{P-D}{\theta} \left\{ \frac{T_1^2}{2} + \frac{T_1 e^{-\theta T_1}}{\theta} + \frac{e^{-\theta} T_1}{\theta^2} - \frac{1}{\theta^2} \right\} + \frac{D}{\theta} \left\{ \frac{-T}{\theta} - \frac{1}{\theta^2} + \frac{T_1 e^{-\theta(T-T_1)}}{\theta} + \frac{e^{-\theta(T-T_1)}}{\theta^2} + \frac{T_1^2 - T^2}{2} \right\} \right] \end{aligned}$$

3.2 Supplier's model

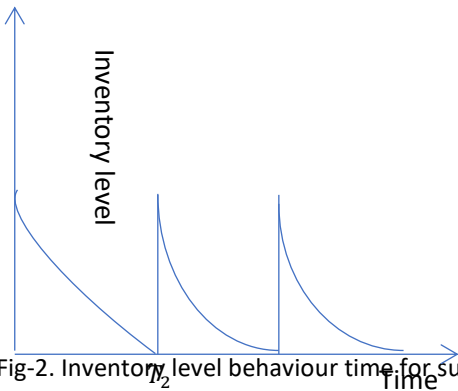


Fig-2. Inventory level behaviour time for supplier

The seller storage the items and demand pattern is shown in Figure 2. $I(t)$ is the stock that has been received from the producer at the initial time t . It is expected that the manufacturer will give a credit term to the supplier. And the supplier provides a period to his customers. Due to deterioration and demand inventory level continually decreases. And stock level reaches zero at $t = T_2$. Here, the seller fulfils the demand of the customers in the orders.

Due to demand and the rate of deterioration the inventory level falls during the interval $0 \leq t \leq T_2$. So the governing differential equation is.

$$\frac{dI_s}{dt} = -\theta I_s(t) - D, \quad 0 \leq t \leq T_2$$

With the inventory condition $I_{s1}(T_2) = 0$ stock level varies in interval $0 \leq t \leq T_2$ at every time t .

$$I_s(t) = \frac{D}{\theta} (e^{\theta(T_1-t)} - 1)$$

(a) Holding cost of items

Here, the holding cost has two parts. One part is connected to the holding cost of items as E_{1s} , and other is energy use and carbon emissions as E'_{1s} . Hence, the total holding calculated for the environment's impact is-

$$\begin{aligned} H_s &= \frac{n(E_{1s} + E'_{1s})}{T} \int_0^{T_2} I_s(t) dt \\ &= \frac{nD(E_{1s} + E'_{1s})}{T} \left[\frac{1}{\theta} (e^{\theta T_2} - 1) - T_2 \right] \end{aligned}$$

(b) Ordering Cost

There are two parts of the supplier's ordering cost. The first part of ordering cost is E_{2s} , and the energy use and carbon emissions as a E'_{2s} second component. The finally, we have calculated the ordering cost as-

$$ST_s = \frac{n(E_{2s} + E'_{2s})}{T}$$

(c) Items cost

There are two parts to the supplier's item cost. The first is related to deteriorate item being sold as a C , and the second is related to energy use and carbon emissions and is also a C . Finally, the total item cost is

$$IT_s = \frac{n(E_s + E'_s)}{T} I_s(0) = \frac{n(E_s + E'_s)}{T} \frac{D}{\theta} (e^{\theta T_2} - 1)$$

Supplier total cost

The total cost for supplier is given by

$$TCS = ST_s + IT_s + H_s$$

$$\begin{aligned} TCS &= \frac{nD(E_{1s} + E'_{1s})}{T} \left[\frac{1}{\theta} (e^{\theta T_2} - 1) - T_2 \right] + \\ &\frac{n(E_s + E'_s)}{T} \frac{D}{\theta} (e^{\theta T_2} - 1) + \frac{n(E_{2s} + E'_{2s})}{T} \end{aligned}$$

Total cost to the supplier across two different credit periods.

Case-I When $N \leq M \leq T_2 + N$

In this case, the TCP that is supplier offers to his customers. The supplier paid interest based on the period's average sales revenue $(M-N)$. The financial institutes are pay to the producer at time M .

The earn interest calculated as

$$IE_1 = \frac{n i_e p}{T} \int_N^M D dt$$

$$= \frac{n i_e p D(M-N)}{T}$$

The paid interest calculated as

$$IP_1 = \frac{n i_p c}{T} \int_M^{N+T} I_s(t) dt$$

$$= \frac{n i_p c D}{T} \frac{1}{\theta} \{ (e^{\theta(T_2-M)} - e^{\theta(T_2-N-T)}) + M - N - T \}$$

The suppliers total cost TCS_1 per unit of time is given below.

$$TCS_1 = TCS + IP_1 - IE_1$$

$$= \frac{nD(E_{1s}+E'_{1s})}{T} [\frac{1}{\theta}(e^{\theta T_2} - 1) \cdot T_2] + \frac{n(E_s+E'_s)}{T} \frac{D}{\theta} (e^{\theta T_2} - 1) + \frac{n(E_{2s}+E'_{2s})}{T} + \frac{n i_p c D}{T} \frac{1}{\theta} \{ (e^{\theta(T_2-M)} - e^{\theta(T_2-N-T)}) + M - N - T \} + \frac{n i_e p D(M-N)}{T}$$

Case-II when $N \leq T_2 + N \leq M$

The earned interest calculated as

$$IE_2 = \frac{n i_e p}{T} \int_N^{N+T_2} D dt + \int_{N+T_2}^M D dt$$

$$= \frac{n i_e p}{T} [D T_2(1 + M - T_2 - N)]$$

Supplier total cost TCS_2 of stock per cycle is given below

$$TCS_2 = TCS - IE_2$$

$$= \frac{nD(E_{1s}+E'_{1s})}{T} [\frac{1}{\theta}(e^{\theta T_2} - 1) \cdot T_2] + \frac{n(E_s+E'_s)}{T} \frac{D}{\theta} (e^{\theta T_2} - 1) + \frac{n(E_{2s}+E'_{2s})}{T} - \frac{n i_e p}{T} [D T_2(1 + M - T_2 - N)]$$

Case-III When $M \leq N \leq T_2 + N$

Paid interest calculated as

$$IP_2 = \frac{n i_p c}{T} [D T_2(N - M) + \int_N^{N+T_2} D dt]$$

$$= \frac{n i_p c D}{T} [T_2(N - M + 1)]$$

Supplier total cost TCS_3 of stock per cycle is given below

$$TCS_3 = H_s + ST_s + IT_s + IP_2$$

$$= \frac{nD(E_{1s}+E'_{1s})}{T} [\frac{1}{\theta}(e^{\theta T_2} - 1) \cdot T_2] + \frac{n(E_{2s}+E'_{2s})}{T} + \frac{n(E_s+E'_s)}{T} \frac{D}{\theta} (e^{\theta T_2} - 1) + \frac{n i_p c D}{T} [T_2(N - M + 1)]$$

The total cost is

$$TCS = \begin{cases} TCS_1 & \text{if } N \leq M \leq T_2 + N \\ TCS_2 & \text{if } N \leq T_2 + N \leq M \\ TCS_3 & \text{if } M \leq N \leq T_2 + N \end{cases}$$

There are three scenarios occur during established the model.

$$TCS_A(P, N, T) = TCP + TCS_1, \quad N \leq M \leq T_2 + N$$

$$TCS_B(P, N, T) = TCP + TCS_2, \quad N \leq T_2 + N \leq M$$

$$TCS_C(P, N, T) = TCP + TCS_3, \quad M \leq N \leq T_2 + N$$

In Case-I Integrated total cost is

$$TCS_A(P, N, T) = [\sigma + \frac{K}{P} + S P + \beta(P - P_{cs}) \cdot g(P - P_{cs})] P T_1 + \frac{(E_{2p} + E'_{2p})}{T} + \frac{(E_p + E'_p)}{T} (P T_1) + \frac{h_1 + c}{T} [\frac{P-D}{\theta} \{ T_1 + \frac{(e^{-\theta T_1} - 1)}{\theta} \} + \frac{D}{\theta} \{ \frac{1}{\theta} (e^{-\theta(T-T_1)} - 1) + (T_1 - T) \} + \frac{h_2}{T} [\frac{P-D}{\theta} \{ \frac{T_1^2}{2} + \frac{T_1 e^{-\theta T_1}}{\theta} + \frac{e^{-\theta T_1}}{\theta^2} - \frac{1}{\theta^2} \} + \frac{D}{\theta} \{ \frac{-T}{\theta} - \frac{1}{\theta^2} + \frac{T_1 e^{-\theta(T-T_1)}}{\theta} + \frac{e^{-\theta(T-T_1)}}{\theta^2} + \frac{T_1^2 - T^2}{2} \} + \frac{nD(E_{1s}+E'_{1s})}{T} [\frac{1}{\theta}(e^{\theta T_2} - 1) \cdot T_2] + \frac{n(E_s+E'_s)}{T} \frac{D}{\theta} (e^{\theta T_2} - 1) + \frac{n(E_{2s}+E'_{2s})}{T} + \frac{n i_p c D}{T} \frac{1}{\theta} \{ (e^{\theta(T_2-M)} - e^{\theta(T_2-N-T)}) + M - N - T \} + \frac{n i_e p D(M-N)}{T}$$

In Case-II Integrated total cost is

$$TCS_B(P, N, T) = [\sigma + \frac{K}{P} + S P + \beta(P - P_{cs}) \cdot g(P - P_{cs})] P T_1 + \frac{(E_{2p} + E'_{2p})}{T} + \frac{(E_p + E'_p)}{T} (P T_1) + \frac{h_1 + c}{T} [\frac{P-D}{\theta} \{ T_1 + \frac{(e^{-\theta T_1} - 1)}{\theta} \} + \frac{D}{\theta} \{ \frac{1}{\theta} (e^{-\theta(T-T_1)} - 1) + (T_1 - T) \} + \frac{h_2}{T} [\frac{P-D}{\theta} \{ \frac{T_1^2}{2} + \frac{T_1 e^{-\theta T_1}}{\theta} + \frac{e^{-\theta T_1}}{\theta^2} - \frac{1}{\theta^2} \} + \frac{D}{\theta} \{ \frac{-T}{\theta} - \frac{1}{\theta^2} + \frac{T_1 e^{-\theta(T-T_1)}}{\theta} + \frac{e^{-\theta(T-T_1)}}{\theta^2} + \frac{T_1^2 - T^2}{2} \} + \frac{nD(E_{1s}+E'_{1s})}{T} [\frac{1}{\theta}(e^{\theta T_2} - 1) \cdot T_2] + \frac{n(E_s+E'_s)}{T} \frac{D}{\theta} (e^{\theta T_2} - 1) + \frac{n(E_{2s}+E'_{2s})}{T} - \frac{n i_e p}{T} [D T_2(1 + M - T_2 - N)]$$

Case-III the integrated total cost is

$$TCS_C(P, N, T) = [\sigma + \frac{K}{P} + S P + \beta(P - P_{cs}) \cdot g(P - P_{cs})] P T_1 + \frac{(E_{2p} + E'_{2p})}{T} + \frac{(E_p + E'_p)}{T} (P T_1) + \frac{h_1 + c}{T} [\frac{P-D}{\theta} \{ T_1 + \frac{(e^{-\theta T_1} - 1)}{\theta} \} + \frac{D}{\theta} \{ \frac{1}{\theta} (e^{-\theta(T-T_1)} - 1) + (T_1 - T) \} + \frac{h_2}{T} [\frac{P-D}{\theta} \{ \frac{T_1^2}{2} + \frac{T_1 e^{-\theta T_1}}{\theta} + \frac{e^{-\theta T_1}}{\theta^2} - \frac{1}{\theta^2} \} + \frac{D}{\theta} \{ \frac{-T}{\theta} - \frac{1}{\theta^2} + \frac{T_1 e^{-\theta(T-T_1)}}{\theta} + \frac{e^{-\theta(T-T_1)}}{\theta^2} + \frac{T_1^2 - T^2}{2} \} + \frac{nD(E_{1s}+E'_{1s})}{T} [\frac{1}{\theta}(e^{\theta T_2} - 1) \cdot T_2] + \frac{n(E_{2s}+E'_{2s})}{T} + \frac{n(E_s+E'_s)}{T} \frac{D}{\theta} (e^{\theta T_2} - 1) + \frac{n i_p c D}{T} [T_2(N - M + 1)]$$

4. Solution procedure

To get the optimum value we differentiate total cost $C = TCS_A(P, N, T)$ with respect to P, N, and T we get

$$\frac{\partial TCS_A(P,N,T)}{\partial P} = 0, \frac{\partial TCS_A(P,N,T)}{\partial N} = 0, \frac{\partial TCS_A(P,N,T)}{\partial T} = 0$$

The determinants of the principal minor of hessian matrix are positive definite, i.e. $\det(Z_1) > 0, \det(Z_2) > 0, \det(Z_3) > 0$ where Z_1, Z_2, Z_3 is the principal minor of the Hessian-matrix. Hessian Matrix of the total cost function is as follows:

$$\begin{bmatrix} \frac{\partial^2 TC}{\partial P^2} & \frac{\partial^2 TC}{\partial P \partial N} & \frac{\partial^2 TC}{\partial P \partial T} \\ \frac{\partial^2 TC}{\partial N \partial P} & \frac{\partial^2 TC}{\partial N^2} & \frac{\partial^2 TC}{\partial N \partial T} \\ \frac{\partial^2 TC}{\partial T \partial P} & \frac{\partial^2 TC}{\partial T \partial N} & \frac{\partial^2 TC}{\partial T^2} \end{bmatrix}$$

5. Numerical example

To find the optimum values of the decision parameters we have taken the appropriate values of the parameters.

Numerical example-1

To calculate the values of the decision parameters we have taken the appropriate values of the parameters and optimum values are given below as-

Let $E_p = 5$ unit per week, $E_{2p} = 992/\text{setup}$, $E'_{1p} = 0.9$ unit per week, $E'_p = 2$, $E'_{2p} = 10/\text{set up}$, $c = 11$ unit per week, $\theta = 0.02, V = 100 \text{units/week}$, $b = 20 \text{units/week}$, $\sigma = 90/\text{week}$, $K = 3500/\text{week}$, $\beta = 0.03/\text{week}$, $z = 0.2, S = 0.02/\text{week}$, $E_s = 4.5, E_{1s} = 2, E_{2s} = 290, E'_{1s} = 1, E'_s = 0.4, E'_{2s} = 2/\text{order}$, $n = 2, I_e = 0.15, I_p = 0.10, \alpha = 0.11, h_1 = 0.4, h_2 = 0.1, P_{cs} = 220 \text{units}, \tau = 0.03, p = 5/\text{unit}$

The optimum value of $TCS_1 = 36570.3$, $P = 14.0388$ units, $T = 1.21049$ weeks, $N = 3.31321$ weeks. T_1 and T_2 can be found out from of the equation and $T_2 = \frac{T}{n}$. The Mathematica software is used to analyse the numerical values.

6. (A). (a). Effects of different parameters on the cost factors and decision variables

Table-2 Showing the effect of number of shipments. Table-3 & 4 showing the integrated total cost function.

TABLE 2. THE EFFECT OF THE NUMBER OF SHIPMENTS

Parameter Change	P	T	N	TCS
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n	2	14.0388	1.21049	3.31321	36570.3
	3	18.5442	1.92391	3.28698	48427
	4	21.773	2.66424	3.19709	59324.7
	5	24.0031	3.42906	3.07625	69186.4
	6	25.4062	4.22033	2.93334	77945.2

TABLE 3- THE EFFECT OF CARBON EMISSION AND ENERGY COST AS E'_{1p}

Parameter Change	P	T	N	TCS	
E'_{1p}	0.6	13.6152	1.20025	3.29839	36862.1
	0.7	13.7565	1.20371	3.30341	36762.9
	0.8	13.8977	1.20712	3.30341	36665.3
	0.9	14.0388	1.21049	3.31321	36570.3
	1	14.1797	1.21381	3.318	36475

TABLE 4- THE EFFECT OF CARBON EMISSION AND ENERGY COST AS E'_p

Parameter Change	P	T	N	TCS	
E'_p	1	14.0109	1.20998	3.30824	36854.4
	2	14.0388	1.21049	3.31321	36570.3
	3	14.0665	1.21099	3.31815	36577.8
	4	14.094	1.21149	3.32305	36442.5
	5	14.1214	1.21198	3.32792	36701.4
	6	14.1486	1.21247	3.33275	36763.3

Table-5 ANALYSIS OF TABLE 2, 3, 4

No.	Parameters	Changes	P	N	T	TCS
1	n	↑	↓	↑	↓	↑
		↓	↑	↓	↑	↓
2	E'_{1p}	↓	↑	↓	↑	↓
		↑	↓	↑	↓	↑
3	E'_p	↓	↑	↓	↑	↓
		↑	↓	↑	↓	↑

Here upper arrow [↑] shows increment and down arrow [↓] shows decrement.

Graphical presentation

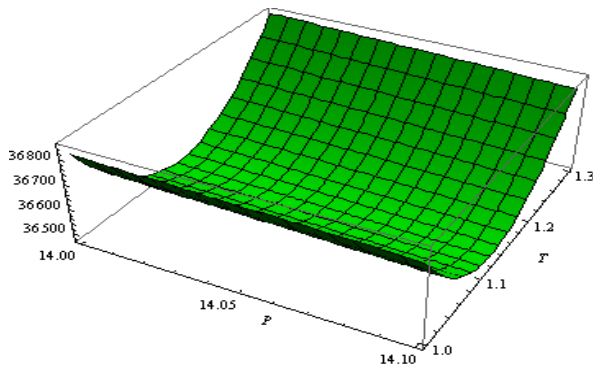


Fig. 3 Graphical representation between T, P with TC

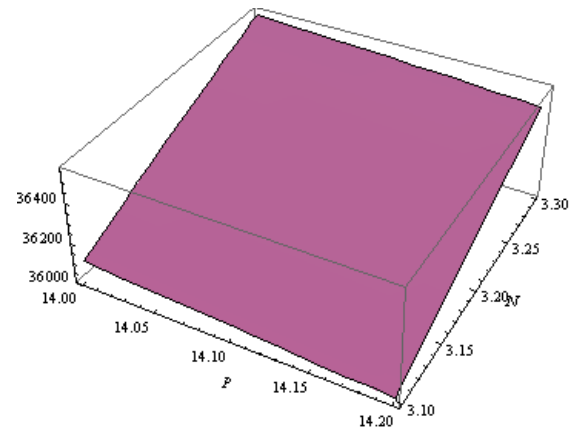


Fig-5 Optimum nature between P, N with TC

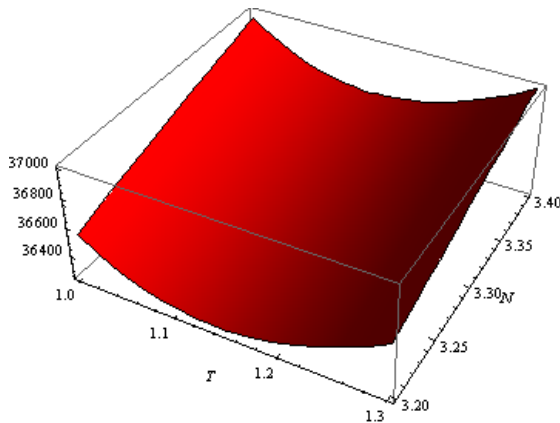


Fig-4 Optimality between T, N with TC

changing according to the production rate of items and cycle length. Our computational results supply chain managerial insights to the production and marketing managers to help plan the supply chain's successful coordination. The problem addressed in this research is applied to a wide array of production practices and motivated by real business concerns. Numerical examples are solved to find the optimality of the model. And sensitivity analysis carried out to show the variation in the parameters. Graphical part shows the optimality of the present work. In the future this model can be extended with different demand like- time dependent demand, stock dependent demand, and seasonal demand function. Further model may be extended in the fuzzy environment and with fuzzy demand function. Also model expend including different inventory parameters.

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6. Conclusion

In the present research work we developed a two-warehouse inventory control model for supply chain system. And especially we focused to maintain the inventory level for the manufacturer, retailer, supplier and distributors. The special aspect of the current study is the utilization of the variable holding cost. We concluded that the production, cycle length and total cost are highly affected by carbon emission and number of cycle length. And the production, and total cost affect in the number of shipments. Carbon emission cost

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