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# Identification Of Weak Transmission Lines For Effective Power Loss

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## Abstract

There are challenges in achieving this equitable loss allocation in power systems, which may lead to complicated methodological implications. This complexity inhibits the achievement of timely loss allocation hence overall system performance. The concept of topological-based solution for loss allocation is considered for intricate topological interconnections inherent in power systems. Having such a theoretical framework provides the basis that power does not circulate and losses are assigned throughout the network. The approach formulates the network losses from a topological viewpoint, thus evading the challenges associated with the selection of a slack bus in conventional load-flow analysis. This view not only brings greater precision in the calculations of power losses. The weakest and strongest transmission lines are then identified from the transmission line coupling strength of the network. Identification of these critical components enables illumination of the structural weaknesses and strengths of the power system. The study uses a 5-bus system of IEEE, 10-bus power system, standard IEEE 14-bus system and the Nigerian 28-bus power system to demonstrate the effectiveness of the method. This study deals with estimation and allocation of losses with respect to base case scenarios and contingency situations. The results obtained showed that bus 4 is the most vulnerable bus in the IEEE 5-bus system using the Degree centrality (DC), Eigenvector centrality (EC), Pagerank centrality (PRC), Closeness centrality (CC), Betweenness centrality (BC) and Node coupling (NC) methods. Also, in the IEEE 14-bus system, bus 8 is ranked number 1 by all the methods considered models. Also, in the Nigerian 28 bus system, Ajaokuta (bus 14) was seen as the most vulnerable bus

**Keywords**: Transmission Line, Power loss, IEEE-Bus

## 1. Introduction

In complex systems and infrastructures such as electrical power systems, good management of active and reactive power is needed to enhance stability and quality [1]. Since reactive power does not contribute to the performance of work but is necessary for functioning with inductive devices, its good management will reduce losses and make the grid more reliable [2,3]. However, one of the most important issues concerning this context is identification of the weak nodes and lines because it strongly affects the calculation of the reactive power loss and distribution of the resources supporting reactive power [4,5]. The weak nodes are the points in a power system that exhibit small magnitudes of voltage due to high demand or poor reactive support [6,7]. Their identification is important because once the weak elements are removed, performance improvement of a system comes along with the effective utilization of reactive power [8,9].

Integration of Flexible Alternating Current Transmission System (FACTS) devices into power systems has been of great importance in the improvement of power system network transient stability, especially during critical outage conditions. Traditionally, integration of FACTS devices has been the most effective approach, which has been widely deployed for transient stability enhancement. [10] explored the use of Particle Swarm Optimization (PSO)-based approach at tuning the Thyristor-Controlled Series

Compensator (TCSC) parameters to improve the transient stability of a power system. The main merit offered by this approach is that it is formulated as an optimization solution such that complex non-linear problems can easily be handled based on this method. Although, the approach has contributed significantly in resolving transient stability issues, it is only suitable for small-scale power systems, which makes its application to be limited when dealing with modern power systems consisting of thousands of buses and transmission lines. The approach is only suitable for small-scale power systems, which makes its application to be limited when dealing with modern power systems consisting of thousands of buses and transmission lines. The reinforcement of an existing power transmission network is a way of enhancing the performance of the power network and to cater for the ever-increasing demand of power. This could be seen as the expansion of the network through the addition of new transmission lines without actually introducing any additional new substations into the existing network. Consequently, the capital cost associated with the reinforced network is substantially minimized. The overall aim of this procedure is to improve the integrity of the existing network and it could be referred to as Transmission Expansion Planning (TEP) [11]. In recent years, Meta-heuristic-based optimization methods have been developed by various authors and have been documented in the open literature. For instance, a stochastic programming is proposed by the authors of reference [12], which considered random events in the formulation of the optimization problem. Markowitz theory is used to ingest a risk factor, which is then incorporated into the objective function. The major setback of this approach is that there is a need for considering several scenarios for proper modelling of the variables due to uncertainties [13].

Considering the ever-increasing environmental constraints, [14] proposed the TEP based on possible reconfiguration of the existing network, which is often the case in practical scenarios. It also considers likely repowering of circuits within the power network. The authors of [15] proposed a scheme based on the graph theory. This fuses the topological characteristics of a network with the shortest possible electrical distance between nodes. The algorithm proposed helps to obtain branch impedances that could be integrated into the existing network, such that the network becomes reinforced. [16] proposed a method based on a steady increment of load to determine the zone, where voltage collapse could (critical zone) be erupted. Although, this method has contributed significantly to the active stream of research, its practical application is limited because of the time complexity. [17] proposed prices using the Location Marginal Prices (LMP) technique because the consideration is majorly for very long transmission lines. However, this may not be technically effective as it does not put the power system stability into consideration. [18] developed a novel heuristic optimization technique which was named Brainstorm Optimization Algorithm (BSOA) as a follow up to his study on the PSO for location of FACTS devices in power systems. His results show that the BSOA leads to lower values of voltage deviations, overloads and losses than the other algorithms considered for optimal location of FACTS devices in power systems. [19] employed the use of Static Var Compensator (SVC) with the infusion of the Fuzzy logic controller to the existing PI controller. With the invention of different FACTS devices, the unified power flow controller (UPFC) is found to be most versatile and it has been used by different authors [20-22]. The UPFC consists of two AC/DC converters and it has a potential of supplying and absorbing real and reactive power on a transmission line.

Though these methods were useful in providing insights as to how vulnerable nodes in a power system could be detected, they fail to take into account the network topology as well as interconnectivity among the network generators, load, and transmission lines. The major advantage of the network topology, for the solution of this problem, is that it formulates the problem as a linear relationship between the nodal voltages and the current flowing through the links. The detail of various approaches for analyzing vulnerability has been reviewed and presented in this work. The vulnerability analysis of power systems can be considered as assessing the impact a local failure would have on the power network before such occurs, in order to devise necessary measures to prevent such failure.

## 2. Methodology

The electrical systems known for their power losses, mathematical formulations for the suggested coupling strength, which is capable of providing effective solutions to problems of weak/critical transmission lines and nodes identification, allocating real and reactive power losses to network players in just a computational time without the need for carrying out iterative processes is presented. This is swiftly followed by the presentation of the formulation based on the eigenvalue and eigenvector analysis which is suitable for determining the network strength.

### 2.2 Theoretical and Mathematical Formulations Based on System Topology

The power-flow equations are formulated from the basic circuit theory laws, which are linear in nature. Reformulation of these linear equations resulted in to complex non-linear power flow equations. In order to solve most power system problems, these non-linear equations need to be solved. The numerical solution can however be provided through iterative processes which are associated with various challenges. Consider an interconnected $N$-bus power system with $G$ number of generators while the remaining $N-G$ buses are load buses $(L)$. From the basic circuit-theory (KCL and KVL) standpoints:

$$\begin{bmatrix} V^L \\ I^G \end{bmatrix} = \begin{bmatrix} Z^{LL} & F^{LG} \\ K^{GL} & A^{GG} \end{bmatrix} \times \begin{bmatrix} I^L \\ V^G \end{bmatrix} \tag{1}$$

where

$$Z^{LL} = Y_{LL}^{-1} \tag{1a}$$

$$F^{LG} = -\left(Y^{LL}\right)^{-1} \times Y^{LG} \tag{1b}$$

$$K^{GL} = Y^{GL} \times \left(Y^{LL}\right)^{-1} \tag{1c}$$

$$A^{GG} = Y^{GG} - Y^{GL} \times \left(Y^{LL}\right)^{-1} \times Y^{LG} \tag{1d}$$

where $Y = \begin{bmatrix} Y^{GG} & Y^{GL} \\ Y^{LG} & Y^{LL} \end{bmatrix}$ (1e)

From the topological point of view, the network power loss is given by

$$\left[I^L\right]^*\left[V^L\right] = \left[I^L\right]^* \times \left[Z^{LL}\right] \times \left[I^L\right] + \left[I^L\right]^* \times \left[F^{LG}\right] \times \left[V^G\right] \tag{2}$$

Consequently, upon this, the network real and reactive power loss allocation can therefore be expressed using equation the matrix $F^{LG}$ in equation (1b), based on the network topology as:

$$S_{allocation} = diag\ \left(\left[S_{D1}, S_{D2}, \ldots, S_{Dn}\right]\right) \times \left[\left(F^{LG}\right)\right] \tag{3}$$

### 2.2 Vulnerable Transmission Line Identification

The structure of any network can easily be captured in the network bus admittance through the inter-relationship of the network elements and electrical parameters of the network. Therefore, the bus admittance of a network can easily be derived using a complex-weighted Laplacian method. Based on the foregoing, an electric power network can be modelled based on the perspective of the graph theory. Consider an electric power network represented by a weighted graph $G = (V, E, W)$ whose vertex set is defined by $V(G) = \{v_1, v_2, v_3, \ldots, v_n\}$ with the network edge set defined by such that for any two vertices and, the edge set is given by $E(G) = \{e_1, e_2, e_3, \ldots, e_n\}$ with each edge $k$ between any two vertices $i$ and $j$ defined by $e_k = (v_i, v_j)$. The weight attached to any edge $k$ of the network is $w_{ij}$. If no edge exists between vertices $i$ and $j$, then $w_{ij} = 0$. For any undirected weighted graph without any loop, $w_{ij} = w_{ji}$ and $w_{ii} = w_{jj} = 0$ and the weight matrix is symmetrical about its diagonal. The matrix for the weight matrix of the network graph can therefore be formulated as:

$$W(G)=\begin{pmatrix} 0 & w_{12} & w_{13} & . & . & . & w_{1n} \\ w_{21} & 0 & w_{23} & . & . & . & w_{2n} \\ w_{31} & w_{32} & 0 & . & . & . & w_{3n} \\ . & . & . & . & . & . & . \\ w_{n1} & w_{n2} & w_{n3} & . & . & . & 0 \end{pmatrix}$$

(4)

$$X(G)=\begin{pmatrix} x_1 & 0 & 0 & . & . & . & 0 \\ 0 & x_2 & 0 & . & . & . & 0 \\ 0 & 0 & x_3 & . & . & . & 0 \\ . & . & . & . & . & . & 0 \\ 0 & 0 & 0 & . & . & . & x_n \end{pmatrix}$$

(5)

where $x_i = \sum_{\substack{j=1 \\ j \neq i}}^{n} w_{ij}$ and the Laplacian for the network can be determined from:

$$L(G)=X(G)-W(G) \tag{6}$$

### 2.2.1 Degree Centrality (DC) Method

This centrality measure shows the degree of a vertex's connection to the remaining parts of the system. Generally, an individual vertex may be seen to be more connected to others within the system than others if it has links attached to it than any other vertex. Generally speaking, for any given node, the electrical degree centrality may be computed as:

$$DC(v) = \frac{\deg(v)}{n-1} = \frac{L(v,v)}{n-1}$$

(7)

where $n$ is the number of vertices.

### 2.2.2 Closeness Centrality (CC) Method

The closeness centrality of a vertex is determined by first determining the shortest path between such vertex and all other network vertices which are connected to it via a transmission line. Key node or vertex this centrality identifies requires a shorter electrical distance to communicate with all other connected vertices. It thus ranks the closest vertex higher than the farthest one. Mathematically speaking, the closeness centrality of a node $i$ in a network is defined by the expression given by:

$$CC(v) = \frac{n-1}{\sum_{i,j \in V} dy\,(i, j)}$$

(8)

where $dy\ (i,j)$ indicates the shortest electrical path length between the network vertices $i$, and $j$.

### 2.2.3 Eigenvector Centrality (EC) Method

The result of this approach is that the first eigenvector's weight will be assigned to each vertex as a value representing centrality. Normally, this is related to adjacency matrix A for determining the important of each vertex within a network. The electrical adjacency matrix of a network may be simply represented as:

$$A_t = -Y + Dg(Y)$$

(9)

where $Y$ represents the network admittance and $Dg(.)$ denotes the diagonal matrix, which is extracted from the original network matrix.The admittance matrix of the network $Y$ can easily be obtained using :

$$Y = A^T diag(y) A$$

(10)

where $y$ is the admittance vector of the network transmission links. The elements of the $n$-by-$n$ bus admittance matrix $Y$ are determined from:

$$\begin{cases} Y(i,j) = -y(i,j);\ \text{if}\ i \neq j \\ Y(i,j) = \sum_{\substack{i,j \in V \\ i \neq j}} y(i,j);\ \text{if}\ i = j \\ Y(i,j) = 0;\ \text{otherwise} \end{cases}$$

(11)

The eigenvector centrality of a vertex $i$ can therefore be determined from the entry $v$ of the eigenvector $\eta$ which corresponds to the largest eigenvalue $\mu_{max}$. Consequently, the weighted eigenvector centrality can be expressed as

$$EC(v) = \left\| \frac{1}{\mu_{\max}} \sum_{k=1}^{n} A(v,k)\eta_k \right\|$$

(12)

where $\mu\eta = A\eta$

(12a)

### 2.2.4 Betweenness Centrality (BC) Method

In this method of approach, the weight associated with the first eigenvector:

$$BC(v) = \frac{\sum_{i \neq v \neq t \in V} \sigma_{ij}(v) \big/ \sigma_{ij}}{n(n-1) \big/ 2}$$

(13)

where$\sigma_{ij}$ represents the shortest electrical path from vertex $i$ to vertex $j$ while $\sigma_{ij}$ $(v)$ represents the sum total of all shortest electrical paths from vertex $i$ to vertex $j$ which pass through $v$.

### 2.2.5 PageRank Centrality (PRC) Method

The PageRank Centrality approach is usually employed to rank webpages in order to evaluate the importance of the webpage through the structure of the hyperlink system. Application of PRC in identifying important nodes in a directed network has been reported in the open literature at large. From the graph-theoretical point of view, the webpage is usually modelled as a directed graph, where the vertices correspond to the web pages and the hyper-links between any two web pages correspond to the edges of the graph. Mathematically, for any given webpage $P_W$, the PRC can be expressed as:

$$PRC(P_w) = \sum_{P_k \in P(P_w)} \frac{PRC(P_k)}{N(P_k)}$$

(14)

where $P(P_W)$ denotes those pages that point towards $P_W$, $N(P_k)$denotes total out-link pages $P_k$.

### 2.3 Identification Based on Network Topology

In the study of physics, there is always a field of influence between two opposite charges separated by some distance. This can be likened to a field of influence between the generator and load connected by a transmission line. This field can easily be shown to be:

$$F_{12} = \frac{K(V_1\ V_2)}{(Z_{12})^2}$$

(15)

where $K$ is a constant, $V_1$is the complex-valued voltage at bus 1, $V_2$ is the voltage at bus 2, $Z_{12}$ is the equivalent electrical distance between the generator placed at bus 1and the load placed at bus 2

Maximum power transfer is given by

(16)
$$P_{\max} = \frac{(|V_1||V_2|)}{\left(\sqrt{(|Z_{12}|)}\right)^2}$$

The Network Coupling Matrix otherwise termed as Network Strength Matrix (NSM), in terms of the Relative Electrical Distance (RED), can be shown, by mathematical manipulations as :

Generator-to-Generator (G-G) region:

$$NSM_{GG} = K[V_i][V_j][RED]_{G\times G}^{-2}$$

(17)

Generator-to-Load (G-L) region:

$$NSM_{GL} = K[V_i][V_j][RED]_{G\times L}^{-2}$$

(18)

Load-to-Load (L-L) region:

$$NSM_{LL} = K[V_i][V_j][RED]_{L\times L}^{-2} \qquad (19)$$

By normalizing equations (17) to (19), we have the Coupling Index (CI) for each region as follows:

Generator-to-Generator (G-G) region:

$$CI_{GG} = \frac{NSM_{GG} - \min(NSM_{GG})}{\max(NSM_{GG}) - \min(NSM_{GG})}$$

(20)

Generator-to-Load (G-L) region:

$$CI_{GL} = \frac{NSM_{GL} - \min(NSM_{GL})}{\max(NSM_{GL}) - \min(NSM_{GL})}$$

(21)

Load-to-Load (L-L) region:

$$CI_{LL} = \frac{NSM_{LL} - \min(NSM_{LL})}{\max(NSM_{LL}) - \min(NSM_{LL})}$$

(22)

### 2.4 Description of the Case Study Network

The work uses three case studies, which include a simple 5-bus system of the standard IEEE network, 10-bus system and the Nigerian 28-bus system. The one-line diagram and the line data for the standard IEEE 5-bus network are shown and presented in Figure 1 and table 1. The simple 10-bus system is represented with the one-line diagram and system line data as presented in Figure 2 and Table 2 respectively. This simple test system consists of twelve transmission lines, three generators located at buses 1, 2 and 3 and seven load buses, which are located at buses 4, 5, 6, 7, 8, 9, and 10.

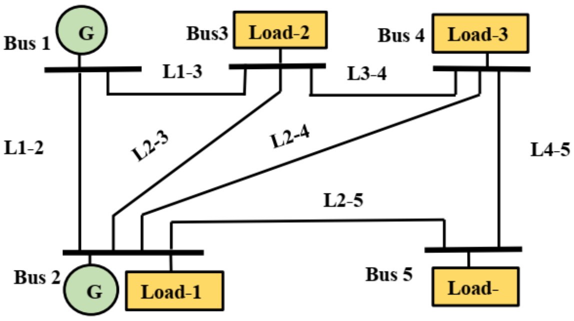

**Fig.1:** One-line diagram for the standard IEEE 5- bus system

**Table 1:** Network line data for the IEEE 5-bus system

| Line Number | From | To | R(pu) | X(pu) | B/2 (pu) |
|---|---|---|---|---|---|
| 1 | 1 | 2 | 0.04 | 0.07 | 0.08 |
| 2 | 1 | 3 | 0.08 | 0.32 | 0.06 |
| 3 | 2 | 3 | 0.07 | 0.16 | 0.05 |
| 4 | 2 | 4 | 0.07 | 0.16 | 0.05 |
| 5 | 2 | 5 | 0.05 | 0.14 | 0.03 |
| 6 | 3 | 4 | 0.02 | 0.04 | 0.04 |
| 7 | 4 | 5 | 0.09 | 0.32 | 0.06 |

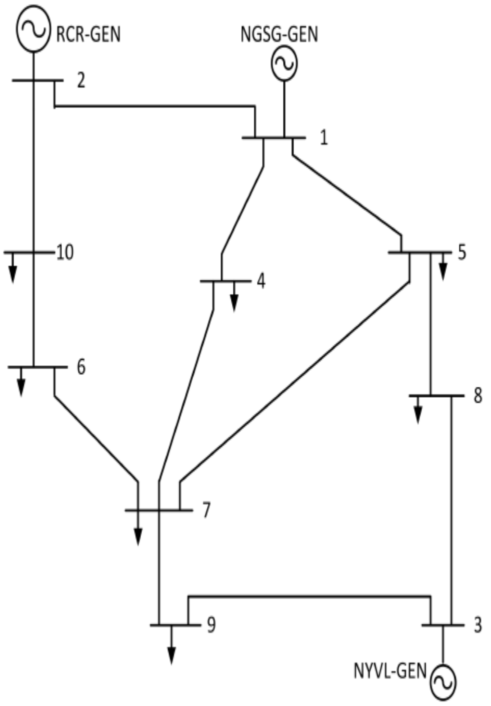

**Fig.2:** One-line of a 10-bus system

**Table 2:** Network line data for the 10-bus system

| Line Number | From | To | R(pu) | X(pu) | B/2 (pu) |
|---|---|---|---|---|---|
| 1 | 1 | 5 | 0.00272 | 0.02872 | 1.51829 |
| 2 | 1 | 4 | 0.00569 | 0.06008 | 0.79414 |
| 3 | 1 | 2 | 0.00477 | 0.05103 | 0.72673 |
| 4 | 2 | 10 | 0.00676 | 0.09429 | 0.75003 |

| | | | | | |
|---|---|---|---|---|---|
| 5 | 10 | 6 | 0.00546 | 0.06794 | 0.88836 |
| 6 | 7 | 9 | 0.00289 | 0.03603 | 0.46222 |
| 7 | 3 | 9 | 0.00145 | 0.01802 | 0.93968 |
| 8 | 7 | 4 | 0.00589 | 0.05995 | 0.78410 |
| 9 | 7 | 5 | 0.00430 | 0.04770 | 0.63700 |
| 10 | 5 | 8 | 0.00388 | 0.04834 | 0.65470 |
| 11 | 3 | 8 | 0.00297 | 0.03706 | 0.47543 |
| 12 | 6 | 7 | 0.00040 | 0.00400 | 0.15000 |
| 13 | 1 | 5 | 0.00272 | 0.02872 | 1.51829 |

The Nigerian 28-bus 330kV network has 10 generator buses (buses 1 to 10), 18 load buses (buses 11 to 28) with 31 transmission lines. The bus numbers and their associated bus names in the Nigerian system are presented in Table 3. The line data and the one-line diagram for this system are presented in Figure 3 and Table 3 respectively.

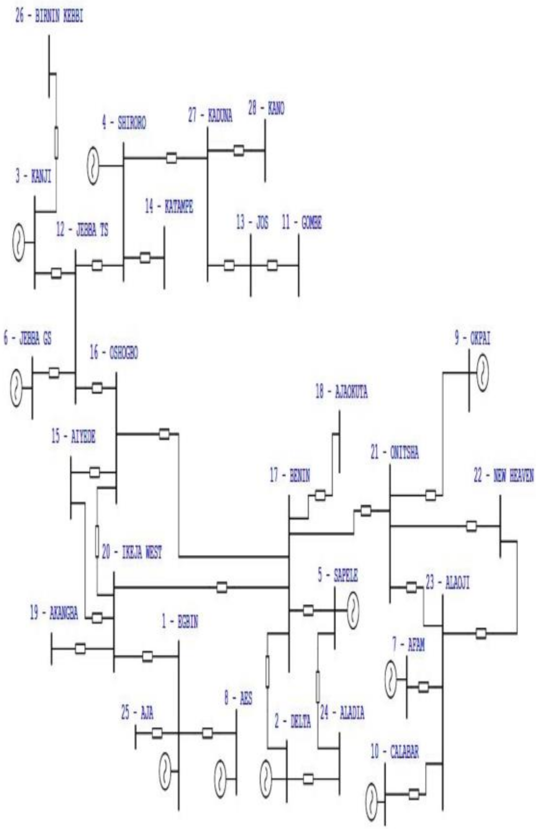

**Fig.3:** Single-line diagram of the practical Nigerian 28-bus system

**Table 3:** Line Data for the Nigerian 28 - Bus System

| From | To | R (in p.u) | X(in p.u) | B/2 (in p.u) | tap setting | Line No. |
|---|---|---|---|---|---|---|
| 1 | 8 | 0.0001 | 0.0004 | 0.0498 | 1 | L1 |
| 1 | 20 | 0.0004 | 0.0029 | 0.0386 | 1 | L2 |
| 1 | 25 | 0.0007 | 0.0057 | 0.0386 | 1 | L3 |
| 3 | 17 | 0.0008 | 0.0063 | 0.1793 | 1 | L4 |
| 2 | 24 | 0.0008 | 0.0063 | 0.1793 | 1 | L5 |
| 3 | 26 | 0.0041 | 0.0304 | 0.9068 | 1 | L6 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 3 | 12 | 0.001 | 0.0082 | 0.462 | 1 | L7 |
| 4 | 27 | 0.0011 | 0.0097 | 0.273 | 1 | L8 |
| 4 | 12 | 0.0022 | 0.0234 | 0.6953 | 1 | L9 |
| 4 | 14 | 0.009 | 0.0067 | 0.8967 | 1 | L10 |
| 5 | 17 | 0.002 | 0.0015 | 0.468 | 1 | L11 |
| 5 | 24 | 0.0008 | 0.0063 | 0.1793 | 1 | L12 |
| 6 | 12 | 0.0001 | 0.0004 | 0.0498 | 1 | L13 |
| 7 | 23 | 0.0015 | 0.0012 | 0.156 | 1 | L14 |
| 9 | 21 | 0.0008 | 0.0063 | 0.1793 | 1 | L15 |
| 10 | 23 | 0.0163 | 0.014 | 0.393 | 1 | L16 |
| 11 | 13 | 0.0032 | 0.0027 | 0.7575 | 1 | L17 |
| 12 | 16 | 0.0019 | 0.0159 | 0.4478 | 1 | L18 |
| 13 | 27 | 0.0027 | 0.0202 | 0.6057 | 1 | L19 |
| 15 | 16 | 0.0013 | 0.01 | 0.2999 | 1 | L20 |
| 15 | 20 | 0.0016 | 0.0134 | 0.4029 | 1 | L21 |
| 16 | 17 | 0.003 | 0.0254 | 0.7155 | 1 | L22 |
| 16 | 20 | 0.0033 | 0.0227 | 0.741 | 1 | L23 |
| 17 | 18 | 0.0023 | 0.0198 | 0.5559 | 1 | L24 |
| 17 | 20 | 0.0034 | 0.0016 | 0.8508 | 1 | L25 |
| 17 | 21 | 0.0016 | 0.00139 | 0.3905 | 1 | L26 |
| 19 | 20 | 0.0007 | 0.0057 | 0.1928 | 1 | L27 |
| 21 | 22 | 0.0011 | 0.0097 | 0.2738 | 1 | L28 |
| 21 | 23 | 0.0163 | 0.014 | 0.393 | 1 | L29 |
| 22 | 23 | 0.0023 | 0.0171 | 0.6953 | 1 | L30 |
| 27 | 28 | 0.0027 | 0.0202 | 0.6057 | 1 | L31 |

## 3. Results and Discussion

This section presents the numerical results obtained in order to test the effectiveness of the method suggested in this work. Using the standard IEEE 5-bus system as a case study, the results obtained using five different Complex Network Theory (CNT)-based methods (Degree Centrality (DC), Eigenvector Centrality (EC), Pagerank Centrality (PRC), Closeness Centrality (CC) and Betweenness Centrality (BC)) as well as NSCT-based method (Node Coupling (NC) otherwise referred to as Bus Coupling (BC)) are presented in Table 4 based on results obtained in Figure 4. Based on these results, the load nodes are then ranked, using each method, in the order of magnitudes. For example, as presented in Table 4, it is obvious that the load bus 4 has the maximum values of 9.4743, 8.3566, 3.5434, 8.6684, 1.5376 and 13.5315 using DC method, EC method, PRC method, CC method, BC method and NC method respectively.

This implies that load node 4 will be ranked number 1 for all the methods. This is repeated for all the load nodes in the network. The ranking results are presented as shown in Table 5. Also power allocation using ) using N-1 criterion is presented in figure 5.

**Table 4:** Results for different approaches using a 5-bus system

| Load node | DC | EC | PRC | CC | BC | NC |
|---|---|---|---|---|---|---|

| | | | | | | |
|---|---|---|---|---|---|---|
| 3 | 7.1479 | 4.352 | 2.5453 | 7.5675 | 1.3531 | 12.1211 |
| 4 | 9.4743 | 8.3566 | 3.5434 | 8.6684 | 1.5376 | 13.5315 |
| 5 | 1.3546 | 6.5474 | 1.6356 | 4.5875 | 0.4759 | 3.8637 |

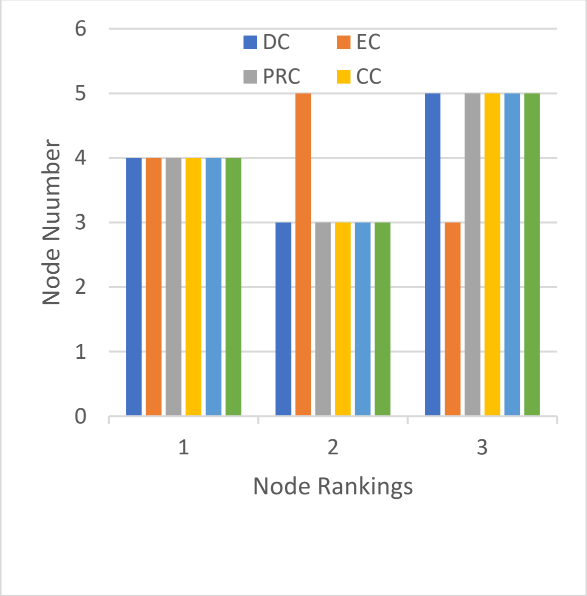

**Fig.4:** Vulnerable Node Identification in the standard IEEE 5-bus system

**Table 5:** Vulnerable Node Identification in the standard IEEE 5-bus system

| Node Ranking | DC | EC | PRC | CC | BC | NC |
|---|---|---|---|---|---|---|
| 1 | **4** | **4** | **4** | **4** | **4** | **4** |
| **2** | 3 | 5 | 3 | 3 | 3 | 3 |
| **3** | 5 | 3 | 5 | 5 | 5 | 5 |

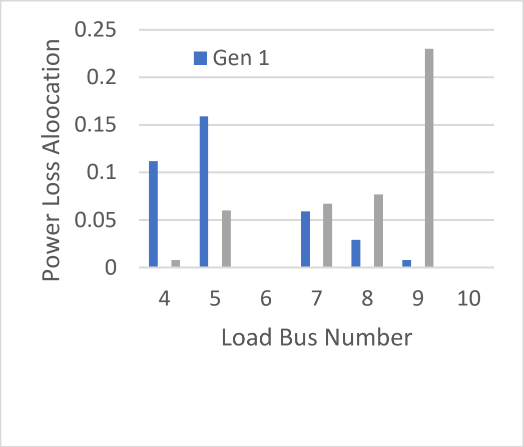

**Fig.5:** Power loss allocation during the outage of line 5

(Weakest line) using N-1 criterion

The results obtained for vulnerable node identification within the IEEE 14-bus system is plotted and shown in Figure 6. the results obtained for all the methods are ranked as presented in Table 6 to show the relative importance of each node within the system under consideration.

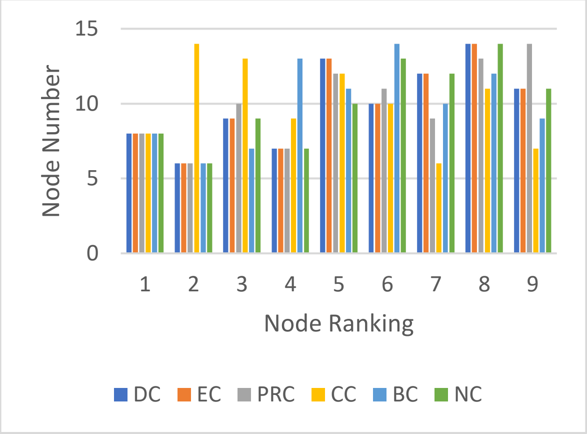

**Fig.6:** Vulnerable nodes identification in the modified

IEEE 14-bus

**Table 6:** Vulnerable nodes identification in the modified IEEE 14-bus

| Node Ranking | DC | EC | PRC | CC | BC | NC |
|---|---|---|---|---|---|---|
| 1 | **8** | **8** | **8** | **8** | **8** | **8** |
| **2** | **6** | **6** | **6** | **14** | **6** | **6** |

| | | | | | | |
|---|---|---|---|---|---|---|
| **3** | 9 | 9 | 10 | 13 | 7 | 9 |
| **4** | 7 | 7 | 7 | 9 | 13 | 7 |
| **5** | 13 | 13 | 12 | 12 | 11 | 10 |
| **6** | 10 | 10 | 11 | 10 | 14 | 13 |
| **7** | 12 | 12 | 9 | 6 | 10 | 12 |
| **8** | 14 | 14 | 13 | 11 | 12 | 14 |
| **9** | 11 | 11 | 14 | 7 | 9 | 11 |

The structural topology of the Nigerian 28-bus system, with all nodes intact, is as shown in Figure 7. The results of analysis obtained for a set of vulnerable nodes identified by using the CNT-based methods and the method of NC suggested in this study, are presented in Table 7. These results are ranked based on the order of magnitudes and the top 10 vulnerable nodes identified are presented in Table 7.

## 4. Conclusion

A quick and more efficient methodology based on the topological interconnections of power systems for identifying weakest/strongest bus and transmission line, estimating the power loss allocation as well as allocating the estimated loss. The mathematical formulations for the approach are presented from the circuit theory and structural interconnection points of view. The study considered power loss estimation and its allocation during the normal operation, and contingency outage conditions using N-1 criterion. The approach is a non-iterative in nature thereby eliminates the issues arising from slack bus selection, divergence of solution, computational complexities in terms of time and computer memory, etc. The results showed that critical outage lines have a significant influence on the loss allocation while the influence of the weakest line outage is highly insignificant.

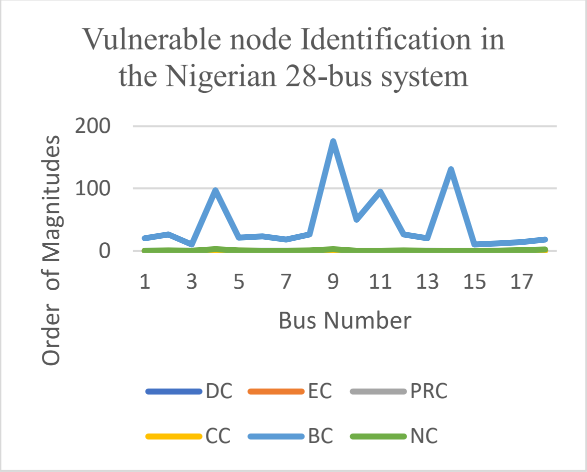

**Fig.7:** Vulnerable node Identification in the Nigerian 28-bus system

**Table 7:** Vulnerable nodes identification in the modified IEEE 14-bus

| Bus Name | Bus number | DC | EC | PRC | CC | BC | NC |
|---|---|---|---|---|---|---|---|
| Ajah | 11 | 0.037 | 0.0071 | 0.0213 | 0.0372 | 20 | 0.2119 |
| Akangba | 12 | 0.0741 | 0.0228 | 0.0375 | 0.0438 | 26 | 0.3491 |
| Ikeja-West | 13 | 0.037 | 0.0206 | 0.0194 | 0.043 | 10 | 0.1464 |
| Ajaokuta | 14 | 0.1481 | 0.0661 | 0.0661 | 0.0466 | 97 | 2.3109 |
| Aladja | 15 | 0.0741 | 0.0726 | 0.0295 | 0.0395 | 21 | 0.2143 |
| Benin | 16 | 0.037 | 0.004 | 0.0209 | 0.035 | 23 | 0.037 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Jebba | 17 | 0.037 | 0.0159 | 0.0198 | 0.0278 | 18 | 0.0188 |
| Ayede | 18 | 0.0741 | 0.0007 | 0.0406 | 0.0165 | 26 | 0.4743 |
| Jos | 19 | 0.1111 | 0.0377 | 0.0476 | 0.0473 | 176 | 2.4478 |
| Kaduna | 20 | 0.0741 | 0.002 | 0.0376 | 0.0209 | 50 | 0.108 |
| Osogbo | 21 | 0.1111 | 0.0059 | 0.053 | 0.022 | 95 | 0.1484 |
| Kano | 22 | 0.0741 | 0.013 | 0.0366 | 0.0364 | 26 | 0.4417 |
| Alaoji | 23 | 0.037 | 0.0018 | 0.0204 | 0.0182 | 20 | 0.0169 |
| New Haven | 24 | 0.1111 | 0.015 | 0.0506 | 0.0365 | 131 | 0.0592 |
| Onitsha | 25 | 0.0741 | 0.0573 | 0.0305 | 0.0366 | 10 | 0.1715 |
| Katampe | 26 | 0.037 | 0.0047 | 0.0197 | 0.0325 | 12 | 0.0033 |
| Birnin Kebbi | 27 | 0.037 | 0.0206 | 0.0194 | 0.034 | 14 | 1.1789 |
| Gombe | 28 | 0.037 | 0.0002 | 0.0226 | 0.0142 | 18 | 2.1951 |

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