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A Hesitant Fuzzy Programming Approach for Multi-Objective Economic Emission Load Dispatch under Uncertainty

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Abstract: The current paper introduces a fuzzy programming-based optimization model of the multi-objective Economic Emission Load Dispatch (EELD) problem in contemporary power plants in a hesitant manner. The given strategy both reduces the cost of fuel and the production of pollutants and takes into consideration the uncertainty and reluctance to make expert decisions that is inherent in the conventional approaches of fuzzy or intuitionistic fuzzy (which may not be effectively modeled with the help of these methods). The model of EELD problem is developed as a nonlinear multi-objective optimization model with the consideration of the generator operating limits, power balance constraints, and transmission losses. As a result, an optimal compromise solution is obtained by constructing a systematic hesitant fuzzy nonlinear programming algorithm with the use of payoff matrices and auxiliary parameters. The proposed methodology was tested on a three-unit thermal power system and its effectiveness is proved by a numerical study. The simulation findings indicate that the hesitant fuzzy strategy strikes a balanced compromise between economic and environmental goals and at the same time is able to sustain a viable generator dispatch when the load changes. The graphical analysis also ensures that there is a stable system behavior, better emission control and cost effective way of allocating power. The findings reveal that the presented framework is a powerful and computationally efficient alternative to conventional dispatch methods and thus it can be used to optimize a power system under uncertainty with multiple objectives. The approach can be easily applied to larger systems and combined with the future smart grid applications by using sophisticated intelligent optimization methods.

Keywords: Economic emission load dispatch problem, Hesitant fuzzy sets, Multiple membership functions.

1. Introduction

Rapid advancements in power system operation and increasing global emphasis on de-carbonization have made multi-objective optimization a central challenge in modern energy management. Contemporary power grids face stricter regulatory pressures for environmental protection, together with high penetration of renewable resources, stochastic generation profiles, and evolving consumer demand patterns. These complexities require flexible, intelligent approaches that can balance competing objectives such as minimizing fuel cost and emissions, ensuring supply reliability, and maximizing renewable utilization. Traditional economic emission dispatch formulations, typically designed for conventional thermal units, often fall short in addressing the intricacies of hybrid energy systems, uncertainty in renewable generation, and real-time adaptability demanded by smart grid environments. The simple use of single-degree membership functions in fuzzy and intuitionistic fuzzy sets limits the algorithm's ability to represent expert preferences and handle multiple conflicting objectives under uncertainty. Recent developments in hesitant fuzzy programming offer promising solutions by leveraging multiple membership degrees, thus capturing a more nuanced spectrum of expert assessments and system behaviors. To further enhance decision support, the integration of machine learning techniques such as support vector regression for renewable forecasting and deep reinforcement learning for adaptive dispatch enables intelligent, data-driven optimization suited to dynamic and unpredictable grid conditions. Moreover, incorporating enabling smart grid technologies such as block chain-based energy trading and digital twin platforms provides new avenues for scalability, transparency, and operational resilience. This article addresses the challenging economic emission load dispatch problem by proposing an improved hesitant fuzzy programming framework. The approach is specifically modified for smart grid applications, integrating advanced artificial intelligence, stochastic modeling, and emerging digital technologies to deliver robust, adaptive, and sustainable dispatch solutions that meet the demands of next-generation power systems. Economic environmental dispatch using multi-objective function were formulated by Nanda et al.[1]. ε-constrained method to minimize cost as well as minimize emission level were used by Yokoyama et al.[2]. In 2009, the study of power dispatch model incorporating decision making has been discussed by Xuebin[3]. Fuzzy set was proposed by Zadeh[4] in real life for solving imprecision and uncertainty. For solving multi-objective problem using fuzzy set was shown by Zimmerman[5]. Load dispatch model involving fuzzy optimization was done by Feng et al.[6]. Recently, for solving structural problem, the fuzzy technique involving parameterized t-norm was used by Dey and Roy[7]. It is observed that fuzzy set is not workable for problems with imprecision and hesitation and the problem was rectified by Atanassov [8]. Later on, the concept of both membership and non membership functions to work on multi-objective problem was used by Angelov [9]. Again, for solving structural problem, intuitionistic fuzzy optimization was discussed by Dey and Roy [10]. Further, generalized intuitionistic fuzzy set and its application on multi objective problem was described by Garai and Roy [11]. For solving transportation problem, Jana and Roy [12] used linear intuitionistic fuzzy optimization technique. Further, it is very important that experts' opinions should be considered about possible values of the parameters of the problems that are conflicting each other. In that case, fuzzy set or intuitionistic fuzzy set are unable to provide the real solution of these problems. The fuzzy set of hesitant type was coined by Torra[13]. Here, hesitant fuzzy process is used to minimize not only power generation cost but also to reduce emission level of load dispatch problem. The contributions of our work are as follows,

- A solution of the multi-objective Economic Emission Load Dispatch problem in uncertainty is formulated using a hesitant fuzzy programming framework.
- Several apprehensive membership functions are used to successfully elicit expert hesitation and decision criteria conflicts.
- An algorithmic nonlinear optimization problem is developed based on payoff matrices and additional parameters to get compromise solutions.
- The efficiency, viability, and soundness of the suggested dispatch plan are proven by using numerical and graphical analyses.

2. Multi-Objective Economic Emission Load Dispatch Model

The objective of the economic emission problem in energy system is to find out the power output of each thermal unit by decreasing the cost of fuel and emission of the system. The basic constraints are known and the target is to identify the optimal power produced by the plants so that the cost of generation and amount of emission emitted are minimum.

2.1 Fuel Cost Objective Function

The objective is expressed as the following equation.

$$Minimize\ \ Cost(P)=\sum_{i=1}^{n}\left(a_iP_i^2+b_iP_i+c_i\right) \tag{1}$$

Where a_i, b_i and c_i are the cost coefficients of the i^{th} generator, P_i is the generated power of i^{th} power plants and n is the number of generators.

2.2. Emission Objective Function

The emission objective is summarized in the following expression.

$$Minimize\ \ Em(P)=\sum_{i=1}^{n}\left(\alpha_iP_i^2+\beta_iP_i+\gamma_i\right) \tag{2}$$

Where α_i, β_i and γ_i are emission coefficients of the i^{th} generator and n is the number of generators.

Subject to

1. Power Balance Constraints

It can be formulated as

$$\sum_{i=1}^{n}P_i=(P_D+P_L) \tag{3}$$

where, P_D is the power demand

$$P_L=\sum_{i=1}^{n}\sum_{j=1}^{n}P_iB_{ij}P_j+\sum_{i=1}^{n}B_{0i}P_i+B_{00} \tag{4}$$

where, P_D and P_L are the total system demand and line loss respectively and B_{ij}, B_{0i} and B_{00} are the loss coefficients.

2. Generator Limit Constraints

It is given as

$$P_{\mathrm{i}}^{\min}\le P_i\le P_i^{\max}, i=1,2,3,...,n \tag{5}$$

where, P_i is the output power of i^{th} generator and $P_i^{\min}$ and $P_i^{\max}$ are the minimum and maximum generated power of i^{th} generator respectively.

3. Preliminaries

3.1. Fuzzy Set: Let X is a set (space), with a generic element of X denoted by x , that is $X(x)$ .Then a Fuzzy set (FS) is defined as $A=\{(x,\mu_A(x)):x\in X\}$

where $\mu_A:X\to[0,1]$ is the membership function of FS A. $\mu_A(x)$ is the degree of membership of the element x to the set A.

3.2. Intuitionistic Fuzzy Set: For a set X, an Intuitionistic fuzzy set (IFS) A in the sense of Atanassov is given by equation $A^{i} = \left\{\left\langle x, \mu_{A^i}(x), \upsilon_{A^i}(x)\right\rangle / x \in X\right\}$

where the function $\mu_{A^i}(x): X \to [0,1]$ and $\upsilon_{A^i}(x): X \to [0,1]$ with the condition $0 \le \mu_{A^i}(x) + \upsilon_{A^i}(x) \le 1$, $\forall x \in X$ .The numbers, $\mu_{A^i}(x) \in [0,1]$ and $\upsilon_{A^i}(x) \in [0,1]$ denote the degree of membership and the degree of non-membership of the element x to the set A, respectively. For each IFS A in X, the amount $\pi_{A^i}(x) = 1 - (\mu_{A^i}(x) + \upsilon_{A^i}(x))$ is called the degree of indeterminacy (hesitation part), which may cater to membership value, non-membership value or both.

3.3. Hesitant Fuzzy Set: Let X be a fixed set, a hesitant fuzzy set on X is in terms of a function that when applied to X returns a subset of [0,1]. Further, Xia and Xu [14] expressed it mathematically by: $A^H = \left\{\left(x, h_A(x)\right) \mid x \in X\right\}$, where h_A is set of some values in [0,1], is called the possible membership degree of the element $x \in X$.

4. Hesitant Fuzzy Nonlinear Programming Technique to solve Multi-objective Nonlinear Programming Problem:

A Multi-Objective Non-Linear Programming (MONPL) or Vector Minimization problem (VMP) may be taken in the following form:

$$Min\, f(x) = [f_1(x), f_2(x), ...f_k(x)]^T$$

(6)

Subject to

$$x \in X = \left\{x \in R^n : g_j(x) \le or = or \ge b_j \;\; for \;\; j = 1,2,3,....,m\right\}$$

and $l_i \le x_i \le u_i \; (i = 1,2,3,....,n)$.

Here $x = \left[x_1, x_2, ..., x_n\right]^T$ is an n-dimensional vector of decision variables, $f_1(x)$, $f_2(x)$,.... $f_k(x)$ are k distinct objective functions.

The steps of hesitant fuzzy optimization technique are as follows:

Computational algorithm

Step 1: Taking the first objective function from set of k objectives of the problem and solve it as a single objective subject to the given constraints. Find value of objective functions and decision variables.

Step 2: From the result of step-1, determine the corresponding values for every objective for at each derived ideal solution. Now, we construct pay off matrix for the values of all objectives at each ideal solution as follows:

$$\begin{array}{c} \\ x^1 \\ x^2 \\ \cdots \\ x^m \end{array} \begin{array}{c} \begin{array}{cccc} f_1(x) & f_2(x) & \cdots & f_k(x) \end{array} \\ \begin{pmatrix} f_1^*\left(x^1\right) & f_2\left(x^1\right) & \cdots & f_k\left(x^1\right) \\ f_1\left(x^2\right) & f_2^*\left(x^2\right) & \cdots & f_k\left(x^2\right) \\ \cdots & \cdots & \cdots & \cdots \\ f_1\left(x^m\right) & f_2\left(x^m\right) & \cdots & f_k^*\left(x^m\right) \end{pmatrix} \end{array}$$

Step 3: Here, we denote and define upper and lower bounds by $U_K^{\mu} = \max\left(Z_K\left(X_r\right)\right) and\, L_K^{\mu} = \min\left(Z_K\left(X_r\right)\right), 1 \le r \le k$ Respectively for each uncertain and imprecise objective functions of Multi-Objective Optimization problems.

Step 4: In this step, we present uncertain and imprecise objectives of Multi-Objective problem by using the following non- linear hesitant membership functions $\mu_k^{E_1}\left(f_k\left(x\right)\right)$:

$$\mu_k^{E_1}\left(f_k\left(x\right)\right)=\begin{cases} 1 & ,if\ f_k\left(x\right)\le L_K^{\mu} \\ \alpha_1\left(\dfrac{\left(f_k\left(x\right)\right)^t-\left(L_K^{\mu}\right)^t}{\left(U_K^{\mu}\right)^t-\left(L_K^{\mu}\right)^t}\right) & ,if\ L_K^{\mu}\le f_k\left(x\right)\le U_K^{\mu} \\ 0 \quad , & if\ f_k\left(x\right)\ge U_K^{\mu} \end{cases}$$

$$\mu_k^{E_2}\left(f_k\left(x\right)\right)=\begin{cases} 1 & ,if\ f_k\left(x\right)\le L_K^{\mu} \\ \alpha_2\left(\dfrac{\left(f_k\left(x\right)\right)^t-\left(L_K^{\mu}\right)^t}{\left(U_K^{\mu}\right)^t-\left(L_K^{\mu}\right)^t}\right) & ,if\ L_K^{\mu}\le f_k\left(x\right)\le U_K^{\mu} \\ 0 & f\ f_k\left(x\right)\ge U_K^{\mu} \end{cases}$$

.....

.....

$$\mu_k^{E_n}\left(f_k\left(x\right)\right)=\begin{cases} 1 & ,if\ f_k\left(x\right)\le L_K^{\mu} \\ \alpha_n\left(\dfrac{\left(f_k\left(x\right)\right)^t-\left(L_K^{\mu}\right)^t}{\left(U_K^{\mu}\right)^t-\left(L_K^{\mu}\right)^t}\right) & ,if\ L_K^{\mu}\le f_k\left(x\right)\le U_K^{\mu} \\ 0 & ,if\ f_k\left(x\right)\ge U_K^{\mu} \end{cases}$$

Where, $0\le\alpha_1,\alpha_2,......,\alpha_n\le 1$

Step 5: Now the hesitant fuzzy operation method for Multi-Objective Optimization problem (6) with non linear membership functions gives an equivalent non- linear programming problem as:

$$Maximize\left(\frac{\alpha_1+\alpha_2+........+\alpha_n}{n}\right) \qquad (7)$$

$$Subject\ to\ \mu_k^{E_1}\left(f_k\left(x\right)\right)\ge\alpha_1,\ for\ all\ k$$
$$\mu_k^{E_2}\left(f_k\left(x\right)\right)\ge\alpha_2,\ for\ all\ k$$
....
$$\mu_k^{E_n}\left(f_k\left(x\right)\right)\ge\alpha_n,\ for\ all\ k$$
$$g_j\left(x\right)\le 0,\ j=1,2,.....,q$$
$$x\ge 0;0\le\alpha_1,\alpha_2,......,\alpha_n\le 1,$$

where, $\mu_k^{E_1}\left(f_k\left(x\right)\right)$ membership degrees are given by 1st expert, $\mu_k^{E_2}\left(f_k\left(x\right)\right)$ membership degrees are given by 2nd expert, $\mu_k^{E_n}\left(f_k\left(x\right)\right)$ membership degrees are given by nth expert.

Step 6: The above non- linear programming problem (7) can be easily solved by the any suitable techniques or some optimizing software packages.

5. Computational Algorithm for Multi-Objective Economic Emission Problem Using Hesitant Fuzzy Optimization Technique:

Step 1: Taking the first objective function from set of objectives of the problem (2) and solve it as a single objective subject to the given constraints. Find the value of objective functions and decision variables.

Step 2: Repeat the Step 1 for remaining objective functions. After that according to step 2 pay-off matrix formulated as follows:

$$\begin{array}{c} \\ P^1 \\ P^2 \end{array} \begin{array}{cc} Cost(P) & Em(P) \\ \left(\begin{array}{c} Cost^*(P^1) \\ Cost^*(P^2) \end{array} \right. & \left. \begin{array}{c} Em^*(P^1) \\ Em^*(P^2) \end{array} \right) \end{array}$$

The bounds are $U_C = \max\left\{Cost(P^{1*}), Cost(P^{2*})\right\}$, $L_C = \min\left\{Cost(P^{1*}), Cost(P^{2*})\right\}$ for cost function $(L_C \le Cost(P) \le U_C)$ and the bounds of objective are $U_E = \max\{Em(P^{1*}), Em(P^{2*})\}$, $L_E = \min\left\{Em\left(P^{1*}\right), Em\left(P^{2*}\right)\right\}$ for emission function $Em(P)$ (where $L_E \le Em(P) \le U_E$) are identified.

Step 3: Since the objective functions are of minimization types and the satisfaction level of experts or decision makers increases if the value of the objective function tends towards it's lower bound. Thus the truth hesitant membership, indeterminacy hesitant membership and falsity membership functions of the lower bound can be represented as follows :

The hesitant-membership functions for $Cost(P)$:

$$\mu^{h^{-1}}{}_{E_1}\left(Cost(P)\right) = \begin{cases} 1 & \text{if } Cost(P) \le L_C \\ \alpha_1 \left(\dfrac{(U_C)^t - (Cost(P))^t}{(U_C)^t - (L_C)^t} \right) & \text{if } L_C \le Cost(P) \le U_C \\ 0 & \text{if } Cost(P) \ge U_C \end{cases}$$

$$\mu^{h^{-1}}{}_{E_2}\left(Cost(P)\right) = \begin{cases} 1 & \text{if } Cost(P) \le L_C \\ \alpha_2 \left(\dfrac{(U_C)^t - (Cost(P))^t}{(U_C)^t - (L_C)^t} \right) & \text{if } L_C \le Cost(P) \le U_C \\ 0 & \text{if } Cost(P) \ge U_C \end{cases}$$

$$\mu^{h^{-1}}{}_{E_n}\left(Cost(P)\right) = \begin{cases} 1 & \text{if } Cost(P) \le L_C \\ \alpha_n \left(\dfrac{(U_C)^t - (Cost(P))^t}{(U_C)^t - (L_C)^t} \right) & \text{if } L_C \le Cost(P) \le U_C \\ 0 & \text{if } Cost(P) \ge U_C \end{cases}$$

The hesitant-membership functions for $Em(P_i)$:

$$\mu^{h^{-1}}{}_{E_1}\left(Em(P)\right) = \begin{cases} 1 & \text{if } Em(P) \le L_E \\ \alpha_1 \left(\dfrac{(U_E)^t - (Em(P))^t}{(U_E)^t - (L_E)^t} \right) & \text{if } L_E \le Em(P) \le U_E \\ 0 & \text{if } Em(P) \ge U_E \end{cases}$$

$$\mu^{h^{-1}}{}_{E_2}\left(Em(P)\right)=\begin{cases}1 & \text{if } Em(P)\le L_E\\ \alpha_2\left(\dfrac{(U_E)^t-(Em(P))^t}{(U_E)^t-(L_E)^t}\right) & \text{if } L_E\le Em(P)\le U_E\\ 0 & \text{if } Em(P)\ge U_E\end{cases}$$

$$\mu^{h^{-1}}{}_{E_n}\left(Em(P)\right)=\begin{cases}1 & \text{if } Em(P)\le L_E\\ \alpha_n\left(\dfrac{(U_E)^t-(Em(P))^t}{(U_E)^t-(L_E)^t}\right) & \text{if } L_E\le Em(P)\le U_E\\ 0 & \text{if } Em(P_i)\ge U_E\end{cases}$$

Step 4: Now the hesitant fuzzy programming technique for Multi-Objective Economic Emission Load Dispatch Optimizationwith the help of auxiliary parameters modelcan be transformed into the following form,

$$Max\left(\frac{a_1+a_2+.....+a_n}{n}\right) \tag{8}$$

Subject to $\mathrm{T}_{\mathrm{E_1}}^{\mathrm{h^{-1}}}\left(Cost(P)\right)\ge a_1, \mathrm{T}_{\mathrm{E_2}}^{\mathrm{h^{-1}}}\left(Cost(P)\right)\ge a_2,....., \mathrm{T}_{\mathrm{E_n}}^{\mathrm{h^{-1}}}\left(Cost(P_i)\right)\ge a_n$

$\mathrm{T}_{\mathrm{E_1}}^{\mathrm{h^{-1}}}\left(Em(P)\right)\ge a_1, \mathrm{T}_{\mathrm{E_2}}^{\mathrm{h^{-1}}}\left(Em(P)\right)\ge a_2,....., \mathrm{T}_{\mathrm{E_n}}^{\mathrm{h^{-1}}}\left(Em(P)\right)\ge a_n,$

$$\sum_{i=1}^{n}P_i-(P_D+P_L)=0$$

$$P_{\mathrm{i}}^{\min}\le P_i\le P_i^{\max}$$

$$Cost(P)=\sum_{i=1}^{n}\left(a_iP_i^2+b_iP_i+c_i\right)$$

$$Em(P)=\sum_{i=1}^{n}\left(\alpha_iP_i^2+\beta_iP_i+\gamma_i\right)$$

$$a_n\in(0,1);\forall n.$$

Step 5: The above non- linear programming problem (8) can be easily solve an appropriate mathematical programming algorithm.

6. Numerical Illustration:

In this section a system consisting of 3 thermal units is considered. The cost coefficient, emission coefficient and generating limits of the 3-generator system are given as follows:

$P_D=700$.

Table 1: Input data for cost coefficient, generating limits of the 3-generator system

Unit	a_i	b_i	c_i	$P^{\min}$	$P^{\max}$
1	0.03546	38.30553	1243.53110	35	210

2	0.02111	36.32782	1658.56960	130	325
3	0.01799	38.27041	1356.65920	125	315

Table 2: Input data for emission coefficient of the 3-generator system

Unit	α_i	β_i	γ_i
1	0.00683	-0.54551	40.26690
2	0.00461	-0.51160	42.89553
3	0.00461	-0.51160	42.89553

B- Coefficients of 3-generator system are as follows

$$B_{ij} = \begin{bmatrix} 0.000071 & 0.000030 & 0.000025 \\ 0.000030 & 0.000069 & 0.000032 \\ 0.000025 & 0.000032 & 0.000080 \end{bmatrix}$$

where B_{0i} and B_{00} are considered as zero.

Solution: According to step 2 pay off matrix is formulated as follows:

$$\begin{matrix} & Cost(P) & Em(P) \\ P^1 & 35424.44 & 660.7492 \\ P^2 & 35473.32 & 651.4851 \end{matrix}$$

Here $U_C = 35473.32$, $L_C = 35424.44$, $U_E = 660.7492$, $L_E = 651.4851$.

$L_C \le Cost(P) \le U_C$ $L_E \le Em(P) \le U_E$ .Here linear membership functions for the objective functions $Cost(P)$ and $Em(P)$ are defined as follows

$$\mu_C(Cost(P)) = \begin{cases} 1 & \text{if } Cost(P) \le 35424.44 \\ 0.98\left(\dfrac{(35473.32)^t - (Cost(P))^t}{(35473.32)^t - (35424.44)^t}\right) & \text{if } 35424.44 \le Cost(P) \le 35473.32 \\ 0 & \text{if } Cost(P) \ge 35473.32 \end{cases}$$

$$\mu_C(Cost(P)) = \begin{cases} 1 & \text{if } Cost(P) \le 35424.44 \\ 0.99\left(\dfrac{(35473.32)^t - (Cost(P))^t}{(35473.32)^t - (35424.44)^t}\right) & \text{if } 35424.44 \le Cost(P) \le 35473.32 \\ 0 & \text{if } Cost(P) \ge 35473.32 \end{cases}$$

$$\mu_C(Cost(P)) = \begin{cases} 1 & \text{if } Cost(P) \le 35424.44 \\ 1\left(\dfrac{(35473.32)^t - (Cost(P))^t}{(35473.32)^t - (35424.44)^t}\right) & \text{if } 35424.44 \le Cost(P) \le 35473.32 \\ 0 & \text{if } Cost(P) \ge 35473.32 \end{cases}$$

$$\mu_E(Em(P)) = \begin{cases} 1 & \text{if } Em(P) \le 651.4851 \\ 0.98\left(\dfrac{(660.7492)^t - (Em(P))^t}{(660.7492)^t - (651.4851)^t}\right) & \text{if } 651.4851 \le Em(P) \le 660.7492 \\ 0 & \text{if } Em(P) \ge 660.7492 \end{cases}$$

$$\mu_E(Em(P)) = \begin{cases} 1 & \text{if } Em(P) \le 651.4851 \\ 0.99\left(\dfrac{(660.7492)^t - (Em(P))^t}{(660.7492)^t - (651.4851)^t}\right) & \text{if } 651.4851 \le Em(P) \le 660.7492 \\ 0 & \text{if } Em(P) \ge 660.7492 \end{cases}$$

$$\mu_E(Em(P)) = \begin{cases} 1 & \text{if } Em(P) \le 651.4851 \\ 1\left(\dfrac{(660.7492)^t - (Em(P))^t}{(660.7492)^t - (651.4851)^t}\right) & \text{if } 651.4851 \le Em(P) \le 660.7492 \\ 0 & \text{if } Em(P) \ge 660.7492 \end{cases}$$

Table 3: Solution based on proposed algorithm taking $t = 2$

P_1	P_2	P_3	$Cost(P)$	$Em(P)$
170.1108	279.3570	274.0626	35436.66	653.8003

7. Results and Discussion:

This section is a report of the numerical findings that were achieved with the proposed hesitant fuzzy programming model in addressing the multi-objective Economic Emission Load Dispatch (EELD) problem. The findings are addressed and elaborated with respect to Figures 1 - 4 to draw out the trade-off nature of the cost and emission goals as well as to support the feasibility of the proposed approach in practice.

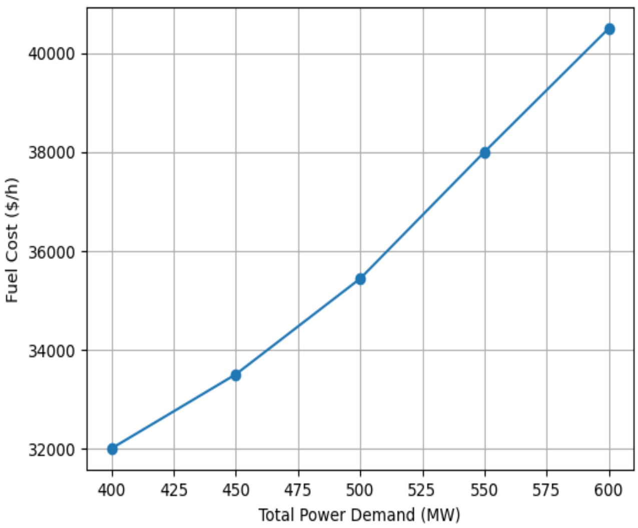

Figure. 1: Fuel cost vs Total power demand

Figure 1 depicts total fuel cost variation with respect to the power demand of the system. The fuel cost also increases in non-linear fashion with the load demand because the cost of thermal generators is quadratic in nature. Such action is indicative of the greater use of more expensive units of generation and running to nearer levels of capacity. The continuous and monotonic rise in the fuel cost reflects that the suggested hesitant fuzzy-based dispatch plan distributes the generation in an economical way and fulfills the demand and operational limitations. The curve proves that the optimization model is cost effective in various loading conditions.

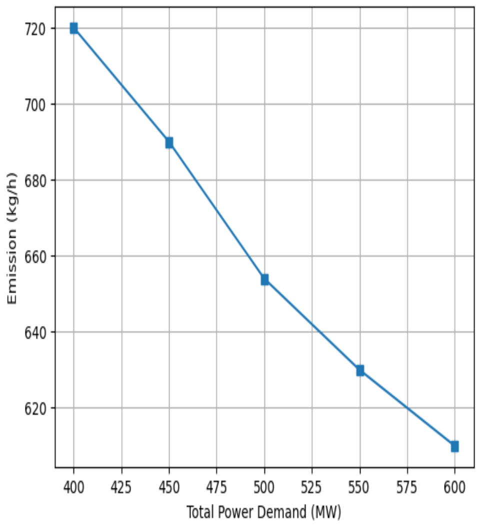

Figure. 2: Emission vs Total power demand

Figure 2 shows the correlation between the total emission and the power demand of the system. The emissions tend to grow with the increasing load, however, the slope of the curve is moderate, which proves the usefulness of the multi-objective optimization in regulating the levels of emissions. The hesitant fuzzy programming methodology is used to achieve this by adding emission as a conflicting goal to the fuel cost and thus the environmentally cleaner generators will play a greater role wherever possible. This finding underscores the ability of the suggested approach to strike a balance between economic and environmental targets in different demand conditions.

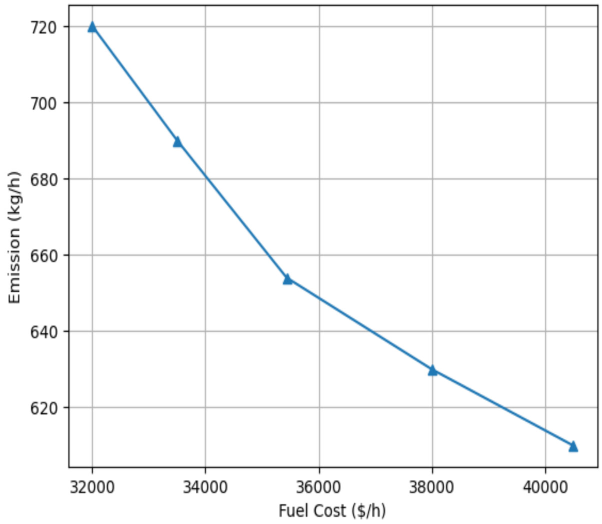

Figure. 3: Fuel cost vs Emission

Figure 3 illustrates the curve of cost-emission trade-off, which is the Pareto-type relationship between the two opposing goals. All the points on the curve give a possible compromise solution which has been achieved by the method of hesitant fuzzy optimization. The negative correlation that exists between the cost of fuel and the emission is a clear indication of the conflict that exists between the economic and environmental objectives. The fact that the curve is smooth means that convergence is stable and easily handles uncertainty, therefore decision-makers can choose an operating point following a desired priority of cost-emission.

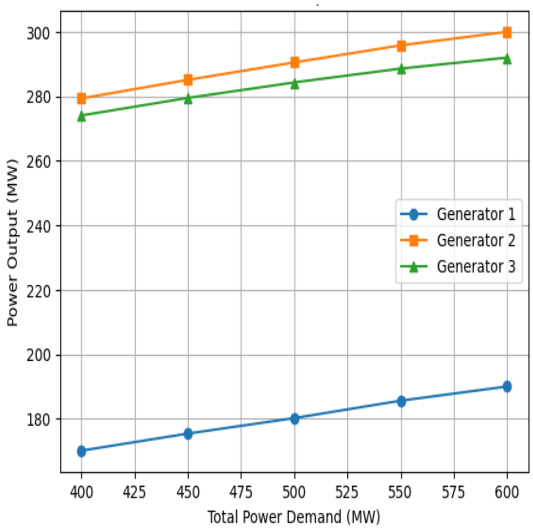

Figure. 4: Power output vs Total power demand

Figure 4 will indicate how the three generating units distribute the power outputs given different power demand. The figure justifies that every generator is working within the stipulated limit of minimum and maximum. Units which are cheaper and have a lower emission contribute a greater portion of the total demand whereas more conservative dispatch is done on the higher-cost units. Such a balanced involvement justifies the possibility of the received solution and indicates that the hesitant fuzzy programming methodology gives the guaranteed load sharing and, at the same time, achieves the cost and emission targets optimization.

8. Conclusions:

A fuzzy programming-based optimization approach that is hesitant approach of the multi-objective Economic Emission Load Dispatch problem has been effectively performed in this work. The suggested model is quite efficient in resolving the dilemma of minimizing fuel cost and reducing emission and apprehension and reluctance of expert evaluation. Compared to classical fuzzy and intuitionistic fuzzy models, the hesitant fuzzy model has several degrees of membership, which gives it a more complex and accommodating model of the preferences of decision-makers. The EELD problem was established as non-linear constrained optimization and the structured hesitant fuzzy algorithm was created to give an optimal compromise solution. The results of numerical simulations of a three-generator thermal power system prove that the proposed approach can provide economically efficient and environmentally friendly dispatch solutions, as well as meet all constraints of the system. The graphical outcomes affirm smooth cost-demand, as well as emission-demand, features, cost-emission trade-off, and even distributed participation of generators in the limits of operation. These results confirm the soundness and high usefulness of the hesitant fuzzy programming model to the real world power dispatch problems. In general, the suggested approach increases the ability to make decisions in the multi-objective optimization of the power system and offer a valid alternative to the traditional solutions. Future directions can involve expanding the model to large scale systems, use of renewable energy, and incorporation of the use of data-driven or artificial intelligence-based optimization procedures to dynamic smart grid contexts.

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