Journal of Harbin Engineering University
ISSN: 1006-7043

Vol 47 No. 01
January 2026

Neutrosophic Soft Weakly Regular Semi Continuous Functions

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Abstract

In this paper, we introduce the concept of neutrosophic soft weakly regular semi continuous, neutrosophic soft regular semi q- neighbourhood in neutrosophic soft topological spaces. Moreover, we investigate the relationship among neutrosophic soft weakly regular semi continuous and other existing continuous functions and some counter examples to show that these types of mappings are not equivalent. Finally, Neutrosophic soft retracts, neutrosophic soft regular semi retracts, neutrosophic soft regular semi quasi Urysohn space and neutrosophic soft regular semi Hausdorff spaces are introduced and studied.

Keywords: $NSWRSC$, $NSRS$ -q-nbd, NS -retracts, $NSRS$ -retracts, $NSRS$ -quasi Urysohn space, $NSRS$ -Hausdorff space.

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1. Introduction

Molodtsov [25], in the year 1999, introduced the soft set theory as a new mathematical tool. He has established the fundamental results of this new theory and successfully applied the soft set theory in to several directions, such as smoothness of functions, Operation research, Riemann integration, Game theory, Theory of probability and so on. Soft set theory has a wider application and its progress in very rapid in different fields. There is no need of membership function in soft set theory and hence very convenient and easy to apply practice. Shabir and Naz [29] introduced the notion of soft topological spaces which are defined over an initial universe with a fixed set of parameters. They also studied some of basic concepts of soft topological spaces. Later, Aygunoglu et al. [3], Zorlutuna et al. [36] and Hussain et al. [22] continued to study the properties of soft topological spaces. They got many important results in soft topological spaces.

Smarandache [30, 31, 32] introduced the notion of Neutrosophic set, it is classified into three independent functions namely, membership, indeterminacy and non membership function that are independently related. Later, Maji [23] has introduced a combined concept Neutrosophic soft set (NSS). Using this concept, several mathematicians have produced their research works in different mathematical structures for instance Arockiarani et al. [1, 2], Bera and Mahapatra [5], Deli [16, 17], Deli and Broumi [18], Maji [24], Broumi and Smarandache [13], Salama and Alblowi [27], Saroja and Kalaichelvi [28], Broumi [14], Sahin et al. [26]. Later, this concept has been modified by Deli and Broumi [19]. Accordingly, Bera and Mahapatra [5]-[12] have developed some algebraic structures over the neutrosophic soft set.

In general topology, the notion of regular semiopen set was introduced by Cameron [15] in 1978. Later, Vadivel and Elavarasan [20, 33] introduced the concept of soft regular semiopen sets and soft regular semi continuous in soft topological spaces. In this concept has been generalized to neutrosophic soft setting. Recently, Elavarasan et. al., [34], [35] introduced the concept of neutrosophic soft regular semiopen (semiclosed) sets and neutrosophic soft regular semi continuous functions in neutrosophic soft topological spaces. In this paper, we introduce and study the concept of neutrosophic soft weakly regular semi continuous, neutrosophic soft regular semi q -neighbourhood in neutrosophic soft topological spaces. Moreover, we investigate the relationship among neutrosophic soft weakly regular semi continuous and other existing continuous functions and some counter examples to show that these types of mappings are not equivalent. Finally, Neutrosophic soft retracts, neutrosophic soft regular semi retracts, neutrosophic soft regular semi quasi Urysohn space and neutrosophic soft regular semi Hausdorff spaces are introduced and studied.

2. Preliminaries

In this section, we recollect some relevant basic preliminaries about Neutrosophic soft sets and its operations in [30], [25], [23], [19], [7] .

Definition 2.1. [30] Let Z be a space of points (objects), with a generic element in Z denoted by *z*. A neutrosophic set R in Z is characterized by a truth-membership function T_A , an indeterminacy membership function I_A and a falsity-membership function F_A. T_A(z), I_A(z) and F_A(z) are real standard or non-standard subsets of $]^-0, 1^+[$. That is TA; IA; FA : Z →$]^-0, 1^+[$. There is no restriction on the sum of T_A(z), I_A(z), F_A(z) and so,$^-0 \le supT_A(z) + supI_A(z) + supF_A(z) \le 3^+$.

Definition 2.2. [25] Let *U* be an initial universe set and *E* be a set of parameters. Let *P(U)* denote the power set of *U* . Then for *A* ⊆ *E* , a pair (*F,A*) is called a soft set over *U* , where *F* : *A* → *P(U)* is a mapping.

Definition 2.3. [23] Let ***U*** be an initial universe set and *E* be a set of parameters. Let *NS(U)* denote the set of all Nss of *U* . Then for *A* ⊆ *E* , a pair (*F,A*) is called an Nss over *U* , where *F* : *A*→ *NS(U)* is a mapping.

This concept has been modified by Deli and Broumi [19] as given below.

Definition 2.4. [19] Let *U* be an initial universe set and *E* be a set of parameters. Let *NS(U)* denote the set of all Nss of *U* . Then, a neutrosophic soft set N over U is a set defined by a set valued function f_N representing a mapping f_N : *E* → *NS(U)* where f_N is called approximate function of the neutrosophic soft set *N* . In other words, the neutrosophic soft set is a parameterized family of some elements of the set *NS(U)* and therefore it can be written as a set of ordered pairs, N = $\{(e; \langle x;\ T_{f_{N(e)}}(x); I_{f_{N(e)}}(x); F_{f_{N(e)}}(x)\rangle: x \in U) : e \in E\}$ where $T_{f_{N(e)}}(x);\ I_{f_{N(e)}}(x);\ F_{f_{N(e)}}(x)$ [0;1] , respectively called the truth-membership, indeterminacy-membership, falsity-membership function of $f_{(N(e))}$. Since supremum of each *T*; *I*; *F* is 1 so the inequality $0 \le T_{f_{N(e)}}(x) + I_{f_{N(e)}}(x) + F_{f_{N(e)}}(x) \le 3$ is obvious.

Definition 2.5. [19] The complement of a neutrosophic soft set *N* is denoted by N^c and is defined by N^c = $\{(e; \langle x;\ F_{f_{N(e)}}(x); 1 - I_{f_{N(e)}}(x); T_{f_{N(e)}}(x)\rangle: x \in U) : e \in E\}$.

Definition 2.6. [19] Let N_1 and N_2 be two Nss's over the common universe (*U,E*) . Then N_1 is said to be the neutrosophic soft subset of N_2 if ∀$e \in E$ and ∀$x \in U$, $T_{f_{N_1(e)}}(x) \le T_{f_{N_2(e)}}(x)$, $I_{f_{N_1(e)}}(x) \le I_{f_{N_2(e)}}(x)$, $F_{f_{N_1(e)}}(x) \le F_{f_{N_2(e)}}(x)$. We write $N_1 \subseteq N_2$ and then N_2 is the neutrosophic soft superset of N_1.

Definition 2.7. [19] Let N_1 and N_2 be two Nss's over the common universe (*U,E*) . Then their union is denoted by $N_1 \cup N_2 = N_3$ and is defined as:

$N_3 = \{(e, \{\langle x, T_{f_{N_3(e)}}(x); I_{f_{N_3(e)}}(x); F_{f_{N_3(e)}}(x)\rangle : x \in U\}) : e \in E\}$ where

$T_{f_{N_3(e)}}(x) = max\{T_{f_{N_1(e)}}(x), T_{f_{N_2(e)}}(x)\}$,

$I_{f_{N_3(e)}}(x) = max\{I_{f_{N_1(e)}}(x), I_{f_{N_2(e)}}(x)\}$,

$F_{f_{N_3(e)}}(x) = max\{F_{f_{N_1(e)}}(x), F_{f_{N_2(e)}}(x)\}$.

Their intersection is denoted by $N_1 \cap N_2 = N_4$ and is defined as :

$N_4 = \left\{\left(e, \left\{\langle x, T_{f_{4(e)}}(x); I_{f_{N_4(e)}}(x); F_{f_{N_4(e)}}(x)\rangle : x \in U\right\}\right) : e \in E\right\}$ where

$T_{f_{N_4(e)}}(x) = min\{T_{f_{N_1(e)}}(x), T_{f_{N_2(e)}}(x)\}$,

$I_{f_{N_4(e)}}(x) = min\{I_{f_{N_1(e)}}(x), I_{f_{N_2(e)}}(x)\}$,

$F_{f_{N_4(e)}}(x) = min\{F_{f_{N_1(e)}}(x), F_{f_{N_2(e)}}(x)\}$.

Definition 2.8. [7] A neutrosophic soft set *N* over *(U,E)* is said to be null neutrosophic soft set if $T_{f_{N(e)}}(x) = 0$; $I_{f_{N(e)}}(x) = 0$; $F_{f_{N(e)}}(x) = 1$, $x \in U$, $e \in E$. It is denoted by 0_u . A neutrosophic soft set *N* over (*U,E*) is said to be absolute neutrosophic soft set if $T_{f_{N(e)}}(x) = 1$; $I_{f_{N(e)}}(x) = 1$; $F_{f_{N(e)}}(x) = 0$, $x \in U$, $e \in E$. It is denoted by 1_u . Clearly, ${0_u}^c = 1_u$ and ${1_u}^c = 0_u$.

Definition 2.9. [7] Let *Nss*(*U,E*) be the family of all neutrosophic soft sets over *U* via parameters in E and $\tau_u \subset$ *NSS(U,E)* . Then τ_u is called neutrosophic soft topology on *(U,E)* if the following conditions are satisfied.

(i) $0_u, 1_u \in \tau_u$.

(ii) the intersection of any finite number of members of τ_u also belongs to τ_u.
(iii) the union of any collection of members of τ_u belongs to τ_u.

Then the triplet ($U,E,\ \tau_u$) is called a neutrosophic soft topological space (**for short, nsts**). Every member of τ_u is called τ_u -open neutrosophic soft set (**for short, nsos**). An *Nss* is called τ_u-closed (**for short, nscs**) iff its complement is τ_u-open.

Definition 2.10. [7] Let ($U,E,\ \tau_u$) be a neutrosophic soft topological space over (*U,E*) and $M \in$ *NSS(U,E)* be arbitrary. Then the interior of *M* is denoted by M° or *nsint(M,E)* (resp. closure of *M* is denoted by M^- or *nscl(M,E)*) and is defined as : $M^\circ = \bigcup\{N : N$ is neutrosophic soft open and $N \subset M\}$. i.e., it is the union of all open neutrosophic soft subsets of *M* (resp. $M^- = \bigcap\{N : N$ is neutrosophic soft closed and $M \subset N\}$. i.e., it is the union of all closed neutrosophic soft supersets of *M*).

Definition 2.11. [4] Let *X* be a universe set. If r, t, s be real standard or on standard subsets of $]\,0^-, 1^+[$, then the neutrosophic soft set $x^{\mathrm{e}}_{\mathrm{r,t,s}}$ is called a neutrosophic soft point (briefly NSP) for every $x \in X$, $e \in E$ given by

$$x^{\mathrm{e}}_{\mathrm{r,t,st}}\ (e'\mathrm{y}) = \begin{cases} (\mathrm{r,t,s}), \text{if } e' = e and y = x \\ (0,0,1), \text{if } e' \neq e and y \neq x. \end{cases}$$

It is clear that every neutrosophic soft set is the union of its neutrosophic soft points.

Definition 2.12. [4] Let ($\tilde{A}$,*E*) be a Nss over *X* , let $x^{\mathrm{e}}_{\mathrm{r,t,s}}$ be NSP in X .
(i) $x^{\mathrm{e}}_{\mathrm{r,t,s}}$ is contained in ($\tilde{A}$,*E*) if $\mathrm{r} \leq T_{\tilde{A}(e)}(x)$, $\mathrm{t} \geq I_{\tilde{A}(e)}(x)$, $\mathrm{s} \geq F_{\tilde{A}(e)}(x)$.
(ii) $x^{\mathrm{e}}_{\mathrm{r,t,s}}$ is belong to ($\tilde{A}$,*E*) if $\mathrm{r} \leq T_{\tilde{A}(e)}(x)$, $\mathrm{t} \geq I_{\tilde{A}(e)}(x)$, $\mathrm{s} \geq F_{\tilde{A}(e)}(x)$.

Definition 2.13. A nsts ($U,E,\ \tau_u$), and a Nss ($\tilde{R}$,*E*) over (*U,E*) . Then ($\tilde{R}$,*E*) is called:
(1) neutrosophic soft regular open set (for short, nsros) [21] iff ($\tilde{R}$,*E*) = *nsint*(*nscl*($\tilde{R}$,*E*)).
(2) neutrosophic soft semi-open set (for short, nssos) [21] iff *($\tilde{R}$,E) ⊆ nscl(nsint(($\tilde{R}$,E)).*
(3) neutrosophic soft regular semi open set (for short,nsrsos) [34] if there exists a *nsros ($\tilde{S}$,E)* such that ($\tilde{S}$,*E*)⊆ ($\tilde{R}$,*E*) ⊆ *nscl*(($\tilde{S}$,*E*)).
(4) neutrosophic soft regular semi closed set (for short, nsrscs) [34] if there exists a *nsrcs* ($\tilde{S}$,*E*) such that *nsint*(($\tilde{S}$,*E*)⊆ ($\tilde{R}$,*E*) ⊆($\tilde{S}$,*E*).

The complement of nsros, nssos and nsrsos sets are called **nsrcs, nsscs and nsrscs** sets.

Definition 2.14. [34] Let ($X,\ E,\ \tau_u$) be a nsts over (*X,E*) and ($\tilde{R}$,*E*) ∈ *NSS*(*X,E*) be arbitrary. Then the
(1) neutrosophic soft regular interior of ($\tilde{R}$,*E*) is denoted by *nsrint*(($\tilde{R}$,*E*)) and is defined as: *nsrint*(($\tilde{R}$,*E*))= ⋃{($\tilde{S}$,*E*) :($\tilde{S}$,*E*) is nsros and ($\tilde{S}$,*E*) ⊆ ($\tilde{R}$,*E*)} . i.e., it is the union of all nsros of ($\tilde{S}$,*E*).
(2) neutrosophic soft regular closure of ($\tilde{R}$,*E*) is denoted by *nsrcl*(($\tilde{R}$,*E*)) and is defined as: *nsrcl*(($\tilde{R}$,*E*)) = ⋂{($\tilde{S}$,*E*) : ($\tilde{S}$,*E*) is nsrcs and ($\tilde{S}$,*E*) ⊒ ($\tilde{R}$,*E*)}. i.e., it is the union of all nsrcs of ($\tilde{R}$,*E*).
(3) neutrosophic soft regular semi interior of ($\tilde{R}$,*E*) is denoted by *nsrsint*(($\tilde{R}$,*E*)) and is defined as: *nsrsint*(($\tilde{R}$,*E*)) = ⋃{($\tilde{S}$,*E*) : ($\tilde{S}$,*E*) is nsrsos and ($\tilde{S}$,*E*) ⊆ ($\tilde{R}$,*E*)} . i.e., it is the union of all nsrsos of ($\tilde{S}$,*E*) .
(4) neutrosophic soft regular semi closure of ($\tilde{R}$,*E*) is denoted by *nsrscl*(($\tilde{R}$,*E*)) and is defined as: *nsrscl*(($\tilde{R}$,*E*)) = ⋂{($\tilde{S}$,*E*) : ($\tilde{S}$,*E*) is nsrscs and ($\tilde{S}$,*E*) ⊒ ($\tilde{R}$,*E*)} . i.e., it is the union of all nsrscs of ($\tilde{R}$,*E*).

Definition 2.15. [10] Let ($U,E,\ \tau_U$) and ($V,E,\ \tau_V$) be two neutrosophic soft topological spaces. Then (ϕ,ψ) : ($U,E,\ \tau_U$) → ($V,E,\ \tau_V$) is said to be
(1) a neutrosophic soft continuous mapping if for each (*N,E*) ∈ τ_V, the inverse image $(\phi,\psi)^{-1}$((*N,E*)) ∈ τ_U. i.e., the inverse image of each open Nss in (V,E,τ_V) is also open in ($U,E,\ \tau_U$).
(2) a neutrosophic soft open mapping if for each (*M,E*) ∈ τ_U , the image (ϕ,ψ)((*M,E*)) ∈ τ_V .
(3) a neutrosophic soft closed mapping if for each *(M,E)* ∈ τ_U, the image (ϕ,ψ)((*M,E*)) ∈ τ_V.

Definition 2.16. [10] Let ($U,E,\ \tau_U$) and ($V,E,\ \tau_V$) be two neutrosophic soft topological spaces. Then (ϕ,ψ) : ($U,E,\ \tau_U$) → ($V,E,\ \tau_V$) is said to be a neutrosophic soft semi continuous (for short Nss-Cts) if for each (*N,E*) ∈ τ_V , the inverse image $(\phi,\psi)^{-1}$((*N,E*)) is neutrosophic soft semiopen set.

Definition 2.17. [35] Let (X,E,τ) and (Y,E,τ) be any two nsts's. A map *f : (X,E, τ) →(Y,E,τ)* is neutrosophic soft regular semi continuous (for short, *NSRSC*) (neutrosophic soft regular semi irresolute (for short, *NSRSI*)) if the inverse image of every neutrosophic soft closed (neutrosophic soft regular semi closed) set in (Y,E,τ) is neutrosophic soft regular semiclosed (neutrosophic soft regular semi closed) set in (X,E,τ) .

Equivalently, A mapping *f : X → Y* is *NSRSC* iff for any *NSP* $x^{\mathrm{e}}_{(\mathrm{r,t,s})}$ in *X* and any nsos ($\tilde{B}$,*E*) in *Y* with $f\,(x^{\mathrm{e}}_{(\mathrm{r,t,s})}) \in$ ($\tilde{B}$,*E*),there exists ($\tilde{A}$,*E*) ∈ *NSRSO(X)* such that $x^{\mathrm{e}}_{(\mathrm{r,t,s})} \in$ ($\tilde{A}$,*E*) and *f* (($\tilde{A}$,*E*)) ⊆($\tilde{B}$,*E*).

Theorem 2.1. [35] If *f : X → Y* is *NSRSC* and *NSAO* , then *f* is *NSRSI* .

3 Neutrosophic soft weakly regular semi continuous functions

Definition 3.1. A function *f : (X,E,τ) → (Y,E,σ)* is called the :

(i) neutrosophic soft weakly regular continuous (for short, NSWRC) iff for any NSP $x^{e}_{(r,t,s)}$) in X and any nsos ($\tilde{B}$,E) in Y containing f ($x^{e}_{(r,t,s)}$), there exists an nsro set ($\tilde{A}$,E) containing $x^{e}_{(r,t,s)}$such that $f((\tilde{A},E)) \subseteq NSRCl((\tilde{B},E))$.

(ii) neutrosophic soft weakly regular semi continuous (for short, NSWRSC) iff for any NSP $x^{e}_{(r,t,s)}$ in X and any nsos ($\tilde{B}$,E) in Y containing f ($x^{e}_{(r,t,s)}$), there exists an nsrso set ($\tilde{A}$,E) containing $x^{e}_{(r,t,s)}$ such that f (($\tilde{A}$,E)) ⊆ *NSRSCl(($\tilde{B}$,E))* .

(iii) neutrosophic soft weakly semi continuous (for short, NSWSC) iff for any NSP $x^{e}_{(r,t,s)}$) in X and any nos ($\tilde{B}$,E) in Y containing f ($x^{e}_{(r,t,s)}$), there exists an nsso set $(\tilde{A},E)$ containing $x^{e}_{(r,t,s)}$ such that $f((\tilde{A},E)) \subseteq NSSCl((\tilde{B},E))$.

Remark 3.1.

(i) Every NSWRC functions is NSWRSC but not conversely.

(ii) Every NSWRSC functions is NSWSC but not conversely.

Example 3.1 Let $X = \{a,b\}$,$E = \{e\}$, $\tau = \{0_X, 1_X, (\tilde{A},E), (\tilde{B},E)\}$, $Y = \{a,b\}$ and $\sigma = \{0_N, 1_N, (\tilde{C},E)\}$, where $(\tilde{A},E)$ and $(\tilde{B},E)$ are Nss of X and $(\tilde{C},E)$ is Nss of Y , defined as follows:

$(\tilde{A},E) = \langle(e,(a,0.4,0.5,0.5),(b,0.5,0.6,0.5))\rangle$,
$(\tilde{B},E) = \langle(e,(a,0.4,0.5,0.5),(b,0.5,0.4,0.5))\rangle$,
$(\tilde{C},E) = \langle(e,(a,0.5,0.5,0.5),(b,0.5,0.6,0.5))\rangle$.

Clearly τ and σ are nsts on X and . If we define the identity function : $X \rightarrow Y$, then f is $NSWRSC$ but not $NSWRC$, for any NSP $x^{e}_{(0.5,0.6,0.6)}$ in X and a nsos $(\tilde{C},E)$ of Y containing $f(x^{e}_{(0.5,0.6,0.6)})$, there exists a nsrso$(\tilde{D},E)$ = $\langle(e,(a,0.5,0.5,0.5),(b,0.5,0.6,0.5))\rangle$ [$(\tilde{D},E)$ is nsrso set of X , since ∃ a nsro set $(\tilde{B},E)$ such that $(\tilde{B},E) \subseteq (\tilde{C},E) \subseteq$ NSCl((˜D,E))] containing $x^{e}_{(0.5,0.6,0.6)}$ such that $f((\tilde{D},E)) \subseteq NSRSCl((\tilde{C},E))$. But $(\tilde{D},E)$ is not nsro set.

Example 3.2 Let X = $\{a,b\}$, $E = \{e\}$, $\tau = \{0_X, 1_X, (\tilde{A},E), (\tilde{B},E)\}$, $Y = \{a,b\}$ and $\sigma = \{0_N, 1_N, (\tilde{C},E)\}$, where $(\tilde{A},E)$ and $(\tilde{B},E)$ are Nss of X and $(\tilde{C},E)$) is Nss of , defined as follows:

$(\tilde{A},E) = \langle(e,(a,0.3,0.5,0.5),(b,0.5,0.6,0.5))\rangle$,
$(\tilde{B},E) = \langle(e,(a,0.6,0.5,0.5),(b,0.5,0.5,0.5))\rangle$,
$(\tilde{C},E) = \langle(e,(a,0.4,0.5,0.5),(b,0.5,0.6,0.5))\rangle$.

Clearly τ and σ are nsts on X and . If we define the identity function $f : X \rightarrow Y$, then f is $NSWSC$ but not $NSWRSC$, for any NSP $x^{e}_{(0.4,0.5,0.6)}$ in X and a nsos $(\tilde{C},E)$ of Y containing $f(x^{e}_{(0.5,0.6,0.6)})$, there exists a nsrso $(\tilde{D},E)$ = $\langle(e,(a,0.4,0.5,0.5),(b,0.5,0.6,0.5))\rangle$ [$(\tilde{D},E)$is nsso set of X , since ∃ a nsos $(\tilde{B},E)$ such that $(\tilde{B},E) \subseteq (\tilde{C},E) \subseteq NSCl((\tilde{B},E))$] containing $x^{e}_{(0.4,0.5,0.6)}$ such that $f((\tilde{D},E)) \subseteq NSCl((\tilde{C},E))$. But $(\tilde{D},E)$ is not nsrso.

Definition 3.2. An Nss $(\tilde{A},E)$ is called an neutrosophic soft q-nbd (neutrosophic soft regular-q-nbd (for short, NSR-q-nbd), neutrosophic soft regular semi-q-nbd (for short, NSRS -q-nbd)) of an NSP $x^{e}_{(r,t,s)}$in an nsts (X,E,τ) iff there exists an nsos (nsro, nsrso) set $(\tilde{B},E)$ of X such that $x^{e}_{(r,t,s)}q(\tilde{B},E) \subseteq (\tilde{A},E)$.

Definition 3.3. An nsts (X,τ) is NSR-regular iff for each NSP $x^{e}_{(r,t,s)}$ in X and each NS -open-q-nbd $(\tilde{A},E)$ of $x^{e}_{(r,t,s)}$, there exists NSR-open-qnbd $(\tilde{B},E)$ of $x^{e}_{(r,t,s)}$ such that $NSRCl((\tilde{B},E)) \subseteq (\tilde{A},E)$.

Theorem 3.1. If Y is an NSR-regular space, then a mapping $f: X \rightarrow Y$ is $NSWRSC$ iff f is $NSRSC$.

Proof. The necessary part follows from Remark 3.1.. We prove only the sufficient part. Let f be $NSWRSC$ and Y be an NSR-regular space.

Let $x^{e}_{(r,t,s)}$ be any NSP of X and $(\tilde{B},E)$ be any nsro (it is nos) set in Y containing $(x^{e}_{(r,t,s)})$. Since Y is NSR-regular, there exists an NSR-q-nbd $(\tilde{C},E)$ of $f(x^{e}_{(r,t,s)})$ = $y^{e}_{(r,t,s)}$ (where y = f(x)) such that $((\tilde{C},E)) \subseteq (\tilde{B},E)$. Since f is $NSWRSC$ and $(\tilde{C},E)$ is an NSR-q-nbd of $f(x^{e}_{(r,t,s)})$, there exists $(\tilde{A},E)$ ∈ NSRSOS(X) with $x^{e}_{(r,t,s)}$ ∈ (A˜,E) such that $f((\tilde{A},E)) \subseteq NSRSCl((\tilde{C},E))$. By Theorem 2.1., $NSRSCl((C\tilde{},E)) \subseteq NSRSCl((\tilde{C},E))$ and so $((\tilde{A},E)) \subseteq NSRSCl((\tilde{C},E)) \subseteq NSCl((\tilde{C},E)) \subseteq (\tilde{B},E)$. Thus f is $NSRSC$ by Definition 3.1. and this completes the proof.

In the following theorems we give some characterization of $NSWRSC$ functions.

Theorem 3.2. A mapping $f: X \rightarrow Y$is $NSWRSC$ iff for each nsos $(\tilde{B},E)$ in Y, $f^{-1}((\tilde{B},E)) \subseteq NSRSInt(f^{-1}(NSRSCl((\tilde{B},E))))$.

Proof. Let f be $NSWRSC$ and $(\tilde{B},E)$ be any nsos set in Y. Let $x^{e}_{(r,t,s)}$ be an NSP in $f^{-1}((\tilde{B},E))$. Thus $f(x^{e}_{(r,t,s)}) \in (\tilde{B},E)$, f is $NSWRSC$ implies that there exists an $(\tilde{A},E) \in NSRSOS(X)$ such that $x^{e}_{(r,t,s)} \in (\tilde{A},E)$ and $f((\tilde{A},E)) \subseteq NSRSCl((\tilde{B},E))$, we have $(\tilde{A},E) \subseteq f^{-1}(NSRSCl((\tilde{B},E)))$. Hence $NSRSInt((\tilde{A},E)) \subseteq$

$NSRSInt(f^{-1}$(NSRSCl($(\tilde{B}\,,E)$))) and since $(\tilde{A}\,,E)$ is nsrso, $(\tilde{A}\,,E)\subseteq NSRSInt(f^{-1}(NSRSCl((\tilde{B}\,,E))))$. So $f^{-1}((\tilde{B}\,,E))\subseteq(\tilde{A}\,,E)\subseteq NSRSInt(f^{-1}(NSRSCl((\tilde{B}\,,E))))$.

Conversely, let $x^e_{(r,t,s)}$ be an NSP in X and $(\tilde{B}\,,E)$ be any nsos set in Y such that $f(x^e_{(r,t,s)})\in(\tilde{B}\,,E)$. By hypothesis, $f^{-1}((\tilde{B}\,,E))\subseteq NSRSInt(f^{-1}(NSRSCl((\tilde{B}\,,E)))=(\tilde{A}\,,E)$ (say). Hence $x^e_{(r,t,s)}\in f^{-1}((\tilde{B}\,,E))\subseteq(\tilde{A}\,,E)$, which implies that $(\tilde{A}\,,E)$ is an nsrso set in X containing $x^e_{(r,t,s)}$. So $(\tilde{A}\,,E)$ = $NSRSInt(f^{-1}(NSRSCl((\tilde{B}\,,E))))$ $f^{-1}(NSRSCl((\tilde{B}\,,E))$, i.e., f($(\tilde{A}\,,E)$) $\subseteq NRSCl((\tilde{B}\,,E))$. Hence f is $NSWRSC$ and this proves the result.

Theorem 3.3. A mapping $f: X\rightarrow Y$ is $NSWRSC$ if for each nsos $(\tilde{B}\,,E)$ in Y, $f^{-1}(NSRSCl((\tilde{B}\,,E)))\in NSRSOS(X)$.

Proof. Straightforward.

Theorem 3.4. If $f: X\rightarrow Y$ is $NSRSI$ and $g: Y\rightarrow Z$ is , then $g\circ f: X\rightarrow Z$ is $NSWRSC$.

Proof. Let $x^e_{(r,t,s)}$ be an NSP in X and $(\tilde{C}\,,E)$ be any nsos in Z containing $((g\circ f)(x^e_{(r,t,s)})=g(f(x^e_{(r,t,s)})$. Since g is $NSWRSC$ there exists an nsro $(\tilde{B}\,,E)$ in Y containing $f(x^e_{(r,t,s)})$ such that $(B)\subseteq NSRSCl((\tilde{C}\,,E))$. Also since f is $NSRSI$ and $(\tilde{B}\,,E)$ [every nsro set is nsos and every nsro set is nsrso] is nsos in Y, it follows that $f^{-1}((\tilde{B}\,,E))$ is nsrso in X. Let $(\tilde{A}\,,E)=f^{-1}((\tilde{B}\,,E))$. Now $(g\circ f)((\tilde{A}\,,E))=g(f((\tilde{A}\,,E)))\subseteq g((\tilde{B}\,,E))\subseteq NSRSCl((\tilde{C}\,,E))$. So $g\circ f$ is $NSWRSC$ and this completes the proof.

Corollary 3.1. If $f: X\rightarrow Y$ is $NSRSC$ and $NSRO$ and $g: Y\rightarrow Z$ is , then $g\circ f$ is $NSWRSC$.

Proof. The proof follows from Theorems 2.1. and 3.4..

4 Neutrosophic weakly regular semi continuous in terms of q -coincidence, q -neighborhoods and θ-cluster points

Definition 4.1. A Nss $(\tilde{A}\,,E)$ in an nsts (X,E,τ) is said to be an neutrosophic soft -θ-nbd (neutrosophic soft regular- θ-nbd (for short, NSR-θ-nbd), neutrosophic soft regular semi- θ-nbd (for short, NSRS - θ-nbd)) of an $NSP\ x^e_{(r,t,s)}$ iff there exists an NS -closed-q-nbd (NSR-closed-q-nbd, $NSRS$ -closed-q-nbd) $(\tilde{B}\,,E)$ of $x^e_{(r,t,s)}$such that $(\tilde{B}\,,E)\ \bar{q}\ (\tilde{A}\,,E)^c$, i.e., $(\tilde{B}\,,E)\subseteq(\tilde{A}\,,E)$.

Definition 4.2. A NSP $x^e_{(r,t,s)}$ in a nsts (X,E,τ) is called an neutrosophic soft regular semi cluster point (for short, NSRS-cluster point) of an Nss($\tilde{A}$,E) iff every NSRS-q-nbd of $x^e_{(r,t,s)}$ is q-coincident with ($\tilde{A}$,E) . The set of all NSRS -cluster points of an Nss ($\tilde{A}$,E) is called neutrosophic soft regular semi closure of ($\tilde{A}$,E) .

Definition 4.3. A NSP $x^e_{(r,t,s)}$ in a nsts (X,E,τ) is called an neutrosophic soft regular semi θ-cluster point (for short, $\boldsymbol{NSRS\theta}$-cluster point) of an Nss ($\tilde{A}$,E) iff every NSRS -open-q-nbd $(\tilde{B}\,,E)$ of $x^e_{(r,t,s)}$, *NSRSCl(*$(\tilde{B}\,,E)$) is q-coincident with ($\tilde{A}$,E) . The set of all NSRS θ-cluster points of an Nss ($\tilde{A}$,E) is called neutrosophic soft regular semi θ closure of ($\tilde{A}$,E) and it is denoted by $NSRS\theta$Cl(($\tilde{A}$,E)) .

Theorem 4.1. A mapping $f: X\rightarrow Y$ is NSWRSC iff corresponding to each NSR-open-q-nbd $(\tilde{B}\,,E)$ of $y^e_{(r,t,s)}$ in Y , there exists an NSRS -open-qnbd $(\tilde{A}\,,E)$ of $x^e_{(r,t,s)}$ in X such that $f((\tilde{A}\,,E))\subseteq NSRSCl((\tilde{B}\,,E))$, where $f(x^e_{(r,t,s)})=(f(x^e))_{(r,t,s)}=y^e_{(r,t,s)}$.

Proof. Let f be $NSWRSC$ and $(\tilde{B}\,,E)$ be an NSR-open-q-nbd of $y^e_{(r,t,s)}$, where $f(x)=y$ in Y . So, B(y) + (r, t, s) > 1_N . We can choose a positive is an NSR-open-nbd of $y^e_{(u,v,w)}$ in . Since f is , there exists an nsrso set $(\tilde{A}\,,E)$ containing $x^e_{(u,v,w)}$ such that $f((\tilde{A}\,,E))\subseteq NSRSCl((\tilde{B}\,,E))$. Now $(\tilde{A}\,,E)(x)\geq(u,v,w)$ implies $(\tilde{A}\,,E)(x)>1-(r,t,s)$, i.e., $(\tilde{A}\,,E)$(x) + (r, t, s) > 1_N . Thus $x^e_{(r,t,s)}$q$(\tilde{A}\,,E)$. So $(\tilde{A}\,,E)$ is an NSRS -open-q-nbd of $x^e_{(r,t,s)}$.

Conversely, let the condition of the theorem hold, i.e., let $x^e_{(r,t,s)}$ be an NSP in X and $(\tilde{B}\,,E)$ be an nos set in Y containing $y^e_{(r,t,s)}=(x^e_{(r,t,s)})$. So $x^e_{(r,t,s)}\in f^{-1}((\tilde{B}\,,E))$= $(\tilde{C}\,,E)$ (say). Hence $(\tilde{C}\,,E)(x)\geq(r,t,s)$. We can choose a (u, v,w) such that $(\tilde{C}\,,E)(x)\geq 1/(u,v,w)$. Put $(\mathrm{r,t,s})_n$ = 1 + $(1/n)-(\tilde{C}\,,E)(x)$, for any positive integer $n\geq(u,v,w)$. Clearly 0 < $(\mathrm{r,t,s})_n\subseteq 1$ for all $\geq(u,v,w)$. Now $(\tilde{B}\,,E)(y)$ + $(\mathrm{r,t,s})_n$ = $(\tilde{B}\,,E)(y)$ + 1 + (1/n) − $(\tilde{C}\,,E)(x)$ = 1 + (1/n) > 1 (Since $(\tilde{C}\,,E)(x)$ = $f^{-1}((\tilde{B}\,,E))(x)=(\tilde{B}\,,E)(f(x))=(\tilde{B}\,,E)$(y)). Hence $y^e_{(\mathrm{r,t,s})_n}$q$(\tilde{B}\,,E)$, i.e., $(\tilde{B}\,,E)$ is an NSR-open-q-nbd of $y^e_{(\mathrm{r,t,s})_n}$ for all $n\geq(u,v,w)$. So by hypothesis there exists an NSRS-open-q-nbd $(\tilde{A}\,,E)_n$ of $y^e_{(\mathrm{r,t,s})_n}$ such that $((\tilde{A}\,,E)_n)\subseteq NRSCl((\tilde{B}\,,E))$, for all $n\geq(u,v,w)$. Now $(\tilde{A}\,,E)=\bigcup_{n\geq(u,v,w)}(\tilde{A}\,,E)_n$ is nsrso in . It remains to show that $x^e_{(r,t,s)}\in(\tilde{A}\,,E)$. We have $(\tilde{A}\,,E)_n$(x) > 1−$(\mathrm{r,t,s})_n$ = $(\tilde{C}\,,E)(x)$−(1/n) for all $n\geq(u,v,w)$. Thus $(\tilde{A}\,,E)$(x) > $(\tilde{C}\,,E)(x)-(1/n)$ for all $n\geq(u,v,w)$. Since $x^e_{(r,t,s)}\in(\tilde{C}\,,E)(x)$ $(\tilde{A}\,,E)(x)\geq$

$C(x) \geq (r,t,s)$. So $(\tilde{A},E)$ is an nsrso set in X such that $((\tilde{A},E)) = f(\cup_{n\geq(u,v,w)} (\tilde{A},E)_n) = \cup_{n\geq(u,v,w)} f((\tilde{A},E)_n n) \subseteq NSRSCl((\tilde{B},E))$. Hence f is $NSWRSC$ and this completes the proof.

Lemma 4.1. For any two Nss's $(\tilde{A},E)$ and $(\tilde{B},E)$ in X, $(\tilde{A},E) \subseteq (\tilde{B},E)$ iff for each NSP $x^e_{(r,t,s)}$ in X , $x^e_{(r,t,s)} \in (\tilde{A},E)$ then $x^e_{(r,t,s)} \in (\tilde{B},E)$.

Lemma 4.2. Let $f : X \to Y$ be any neutrosophic soft function and $x^e_{(r,t,s)}$ be any NSP in X, then

(i) for $(\tilde{A},E) \subseteq X$ and $x^e_{(r,t,s)}$q$(\tilde{A},E)$, we have $f(x^e_{(r,t,s)}) q f((\tilde{A},E))$.

(ii) for $(\tilde{B},E) \subseteq Y$ and $f(x^e_{(r,t,s)}) q (\tilde{B},E)$, we have $x^e_{(r,t,s)} q f^{-1}((\tilde{B},E))$.

Theorem 4.2. If $f : X \to Y$ is an $WRSC$, then for each nsos $(\tilde{B},E)$ in Y, $NSRSCl(f^{-1}((\tilde{B},E)) \subseteq f^{-1}(NSRSCl((\tilde{B},E))$. XC

Proof. A Nss is the union of all of its NSP. Suppose that there is an NSP $x^e_{(r,t,s)} \in NSRSCl(f^{-1}((\tilde{B},E)))$ but $x^e_{(r,t,s)} \notin f^{-1} (NSRSCl((\tilde{B},E))$. Since f$(x^e_{(r,t,s)}) \notin NSRSCl((\tilde{B},E))$ there exists an nsro set $(\tilde{C},E)$ in Y with $f(x^e_{(r,t,s)}) \in (\tilde{C},E)$ such that $(\tilde{C},E) q NSRCl((\tilde{B},E))$. Thus $(\tilde{C},E) q (\tilde{B},E)$ and $((\tilde{C},E)) q (\tilde{C},E)$. Since f is $NSWRSC$, there exists an $(\tilde{A},E) \in NSRSOS(X)$ with $x^e_{(r,t,s)} \in (\tilde{A},E)$ such that $f((\tilde{A},E)) \subseteq NSRSCl((\tilde{C},E))$. Hence $((\tilde{A},E)) q (\tilde{B},E)$. Since $((\tilde{A},E)) \subseteq NSRSCl((\tilde{C},E)) \subseteq NSRCl((\tilde{C},E))$. But on the other hand, since each nsrso set is NSRS -q-nbd of each of its NSP $x^e_{(r,t,s)} \in NSRSCl(f^{-1}((\tilde{B},E)))$ and $(\tilde{A},E)$ is an NSRS -q-nbd of $x^e_{(r,t,s)}$. By Definition 4.2., $(\tilde{A},E) q f^{-1}((\tilde{B},E))$. By Lemma 4.2.(1), $f((\tilde{A},E)) q f(f^{-1}((\tilde{B},E)))$, and hence $f((\tilde{A},E)) q (\tilde{B},E)$ which is a contradiction. This completes the proof of the theorem.

Theorem 4.3. Let $f : X \to Y$ be an $NSRO$ and $NSWRSC$ mapping. Then $(NSRSCl((\tilde{A},E)) \subseteq NSRCl(f((\tilde{A},E)))$, for each nsro set $(\tilde{A},E)$ in X .

Proof. Let $(\tilde{A},E)$ be an nsro (it is nsos) set in X and let $((\tilde{A},E)) = (\tilde{B},E)$. Since f is NSRO , we see that $(\tilde{B},E)$ is an nsro (it is also nsos) set in Y. Hence $(\tilde{A},E) \subseteq f^{-1}(f((\tilde{A},E))) = f^{-1}((\tilde{B},E))$. Since f is NSWRSC, we have from Theorem 4.2., $NSRSCl(f^{-1}((\tilde{B},E))) \subseteq f^{-1}(NSRCl((\tilde{B},E))$. Thus $NSRSCl((\tilde{A},E)) \subseteq f^{-1}NSRCl((\tilde{B},E))$, i.e., $f(NSRSCl((\tilde{A},E))) \subseteq NSRCl((\tilde{B},E)) = NSRCl(f((\tilde{A},E)))$ and this proves the result.

Theorem 4.4. A function $f : X \to Y$ is NSWRSC iff for each nsos $(\tilde{B},E)$ in Y, $x^e_{(r,t,s)} f^{-1}((\tilde{B},E))$ implies $x^e_{(r,t,s)} q f^{-1} (NSRSInt(NSRSCl((\tilde{B},E)))$ for each NSP $x^e_{(r,t,s)}$ in .

Proof. Let f is C . Let $x^e_{(r,t,s)}$ be NSP in X and $(\tilde{B},E)$ be any nsro (nsos) set in Y such that $x^e_{(r,t,s)}$q$f^{-1}((\tilde{B},E))$. Then $(x^e_{(r,t,s)}) q (\tilde{B},E)$. Since f is NSWRSC by Theorem 4.1., there exists an nsrso set $(\tilde{A},E)$ in X such that $x^e_{(r,t,s)}$q$(\tilde{A},E)$ and $f((\tilde{A},E)) \subseteq NSRSCl((\tilde{B},E))$. We have $(\tilde{A},E) \subseteq f^{-1}(NSRSCl((\tilde{B},E)))$. Since $(\tilde{A},E) \in NSRSOS(X)$, $(\tilde{A},E) \subseteq f^{-1}(NSRSInt(NSRSCl((\tilde{B},E)$. So $x^e_{(r,t,s)} q f^{-1}(NSRSInt(NSRSCl((\tilde{B},E)))$.

Converesely, let the condition given in the statement hold. Let $x^e_{(r,t,s)}$ be an NSP in X and $(\tilde{B},E)$ be an NSR-open-q-nbd of f$(x^e_{(r,t,s)})$ such that $(x^e_{(r,t,s)}) q f^{-1} ((\tilde{B},E))$. By hypothesis, $(x^e_{(r,t,s)}) q f^{-1}(NSRSInt(NSRSCl((\tilde{B},E)$. Put $(\tilde{A},E) = f^{-1} (NSRSInt(NSRSCl((\tilde{B},E)))$. Hence $(\tilde{A},E) \in NSRSOS(X)$. $(x^e_{(r,t,s)}) q (\tilde{A},E)$ implies that $(\tilde{A},E)$ is an NSRS -open-q-nbd of $x^e_{(r,t,s)}$. Also $f(f^{-1}(NSRSInt(NSRSCl(\tilde{B},E))))) \subseteq f(f^{-1}(NSRSCl((\tilde{B},E))))$, *i.e.*, $f((\tilde{A},E)) \subseteq NSRSCl((\tilde{B},E))$. Thus f is NSWRSC and this completes the proof.

Theorem 4.5. If $f : X \to Y$ is NSWRSC then for each NSP $x^e_{(r,t,s)}$ in X and each NSR- θ -nbd $(\tilde{B},E)$ of f$(x^e_{(r,t,s)})$, $f^{-1} ((\tilde{B},E))$ is an NSRS -q-nbd of $x^e_{(r,t,s)}$.

Proof. Let $f : X \to Y$ is NSWRSC function and $x^e_{(r,t,s)}$ be an NSP in X. Let $NSRSCl((\tilde{B},E))$ be an NSR- θ-nbd of $f(x^e_{(r,t,s)})$. So there is an NSR-open-q-nbd $(\tilde{C},E)$ of $f(x^e_{(r,t,s)})$ such that $((\tilde{C},E)) q (\tilde{B},E) c$, *i.e.*, $NSRCl((\tilde{C},E)) \subseteq (\tilde{B},E)$. Since $(\tilde{C},E)))$ is an NSR-open-q-nbd of f$(x^e_{(r,t,s)})$, by Theorem 4.1., there is an NSRS -open-q-nbd $(\tilde{A},E)))$ of $x^e_{(r,t,s)}$ such that $f((\tilde{A},E)) \subseteq NSRSCl((\tilde{C},E))))$ and thus $f((\tilde{A},E)) \subseteq NSRSCl((\tilde{C},E)))) \subseteq NSRCl((\tilde{C},E)) \subseteq (\tilde{B},E)$. *So* $(\tilde{A},E) \subseteq f^{-1} ((\tilde{B},E))$. Hence $f^{-1} ((\tilde{B},E))$ is an NSRS -q-nbd of $x^e_{(r,t,s)}$ and this completes the proof.

Theorem 4.6. If $f : X \to Y$ is a function such that for each NSP $x^e_{(r,t,s)}$ in X and each NSRS - θ -nbd $(\tilde{B},E)$ of

$f(x^{e}_{(r,t,s)})$ in Y , f^{-1} (($\tilde{B}$, E)) is NSRS -q-nbd of $x^{e}_{(r,t,s)}$ in X , then f is NSWRSC .

Proof. Let $x^{e}_{(r,t,s)}$) be an NSP in X and ($\tilde{B}$, E) be any NSR-open-q- nbd of $f(x^{e}_{(r,t,s)})$. We note that ($\tilde{B}$, E) is an NSRS -open-q-nbd of ($x^{e}_{(r,t,s)}$) . So NSRSCl(($\tilde{B}$, E)) is an NSRS - θ -nbd of $f(x^{e}_{(r,t,s)})$. By hypothesis, f^{-1}(NSRSCl(B)) is an NSRS -q-nbd of $x^{e}_{(r,t,s)}$. So there exists an nsrso set ($\tilde{A}$, E) in X such that $x^{e}_{(r,t,s)}qA \subseteq f^{-1}(NRSCl((\tilde{B}\,,E)))$, i.e., $((\tilde{A}\,,E)) \subseteq NSRSCl((\tilde{B}\,,E))$. Hence f is NSWRSC and this completes the proof.

Theorem 4.7. If $f : X \to Y$ is C , then
(i) $f(NSRSC((\tilde{A}\,,E)) \subseteq NSRS\theta NSCl(f((\tilde{A}\,,E)))$ for each Nss ($\tilde{A}$, E) in X ,
(ii) $f(NSRSCl(f^{-1}(NSRSCl(NSRSInt((\tilde{B}\,,E)))))) \subseteq NSRS\theta Cl((\tilde{B}\,,E))$ for each Ns ($\tilde{B}$, E) in Y.

Proof. (i) Let $x^{e}_{(r,t,s)} \in NSRSCl((\tilde{A}\,,E))$ and ($\tilde{S}$, E) be an NSR-closed-q-nbd of ($x^{e}_{(r,t,s)}$) . Then there exists an NSR-open-qnbd ($\tilde{V}$, E) of $f(x^{e}_{(r,t,s)})$ such that ($\tilde{V}$, E) ⊆ ($\tilde{S}$, E) . Since f is C , by Theorem 4.1. there exists an NSRS -open-q-nbd ($\widetilde{U}$, E) of $x^{e}_{(r,t,s)}$ such that $f((\widetilde{U}\,,E)) \subseteq NSRSCl((\tilde{V}\,,E))$. Since $x^{e}_{(r,t,s)} \in NSRSCl((\tilde{A}\,,E))$, by Definition 4.2., $x^{e}_{(r,t,s)}$ is an NSRS -cluster point of ($\tilde{A}$, E) . Hence $(\widetilde{U}\,,E)q(\tilde{A}\,,E)$ and also $((\widetilde{U}\,,E)qf((\tilde{A}\,,E))$. Since $((\widetilde{U}\,,E)) \subseteq NSRSCl((\tilde{V}\,,E))$, $NSRSCl((\tilde{V}\,,E))qf((\tilde{A}\,,E))$. Again since, ($\tilde{S}$, E) is nsrc (it is nscs), we have NSRSCl$\tilde{V}$, E)) ⊆ NSRCl(($\tilde{V}$, E)) ⊆($\tilde{S}$, E). Therefore $(\tilde{S}\,,E)qf((\tilde{A}\,,E))$. By Definition 4.3., f($x^{e}_{(r,t,s)}$) $\in NSRS\theta Cl(f((\tilde{A}\,,E)))$, i.e., $x^{e}_{(r,t,s)} \in f^{-1}(NSRS\theta Cl(f((\tilde{A}\,,E)))$. Therefore, $NSRSCl((\tilde{A}\,,E)) \subseteq f^{-1}\,(NSRS\theta Cl(f((\tilde{A}\,,E))))$; which implies $f(NSRSCl((\tilde{A}\,,E))) \subseteq NSRS\theta Cl(f((\tilde{A}\,,E)))$, proving(i).

(ii) Let ($\tilde{B}$, E) be an Nss in Y and $x^{e}_{(r,t,s)}$ be an NSP in X such that $x^{e}_{(r,t,s)} \in NSRSCl(f^{-1}(NSRSCl(NSRSInt((\tilde{B}\,,E)))))$. Let ($\tilde{V}$, E) be any NSR-open-q-nbd of f($x^{e}_{(r,t,s)}$) . By Theorem 4.1., there exists NSRS-open-q-nbd ($\widetilde{U}$, E) of $x^{e}_{(r,t,s)}$ such that $((\widetilde{U}\,,E)) \subseteq NSRSCl((\tilde{V}\,,E))$.
Since $NSRSCl(f^{-1}(NSRSInt((\tilde{B}\,,E)))) \subseteq NSRSCl(f^{-1}((\tilde{B}\,,E)))$, we have $x^{e}_{(r,t,s)} \in NSRSCl(f^{-1}(NSRSCl(NSRSInt((\tilde{B}\,,E)))))$. By Definition 4.3., $(\widetilde{U}\,,E)qf^{-1}((\tilde{B}\,,E))$, i.e., $f((\widetilde{U}\,,E))q(\tilde{B}\,,E)$. Thus $l((\tilde{V}\,,E))q(\tilde{B}\,,E)$, which implies $f(x^{e}_{(r,t,s)}) \in NSRS\theta Cl((\tilde{B}\,,E))$. So $(NSRSCl(f^{-1}(NSRSCl(NSRSInt((\tilde{B}\,,E)))))) \subseteq NSRS\theta Cl((\tilde{B}\,,E))$, proving(ii).

5 Neutrosophic weakly regular semi continuous functions and neutrosophic retracts

Definition 5.1. Let X be a nsts and $A \subseteq X$. Then the subspace (crisp) ($\tilde{A}$, E) of X is called a neutrosophic soft retract (for short, NS -retract) of X if there exists a neutrosophic soft continuous function $r : X \to A$ such that $r(a) = a$ for all $a \in A$. In this case r is called a neutrosophic soft retraction. ($\tilde{A}$, E) is called an NSO -retract (NSRO -retract, NSRSC -retract) of X if r is NS-open function (NSRO -function, NSRSC -function). Similarly ($\tilde{A}$, E) is called NSWRSC -retract of X if r is NSWRSC .

Theorem 5.1. If ($\tilde{A}$, E) is an NSRO -retract, NSRSC -retract of the nsts X then for every nsts, any NSWRSC function $g : A \to Y$ can be extended to an NSWRSC function of X into .

Proof. Let Y be an arbitrary nsts and $g : A \to Y$ be an NSWRSC function. By Corollary 3.1., $g \circ r : X \to Y$ is NSWRSC and $g \circ r(a) = g(r(a)) = g(a)$ for all $a \in A$, where $r : X \to A$ is a NSRO -retract, NSRSC -retraction. Hence $g \circ r$ is an NSWRSC extension of g to X and this completes the proof.

Theorem 5.2. If ($\tilde{A}$, E) is an NSRO -retract, NSRSC -retract of X and ($\tilde{B}$, E) is an NSWRSC -retract of ($\tilde{A}$, E) then ($\tilde{B}$, E) is an NSWRSC - retract of X .

Proof. Let $r : X \to A$ be an NSRO and NSRSC mapping such that $r(a) = a$ for all $a \in A$, Let $s : A \to B$ be an NSWRSC retraction of ($\tilde{A}$, E) such that $s(b) = b$ for all $b \in B$. By corollary 3.1., $s \circ r : X \to B$ is NSWRSC and $s \circ r(b) = b$ for all $\in B$. Hence ($\tilde{B}$, E) is NSWRSC retract of X and this proves the result.

Definition 5.2. An nsts X is said to be NSR-quasi Urysohn space if for any two distinct NSP's $x^{e}_{(r,t,s)}$ and $y^{e}_{(r,t,s)}$, there exist nsro sets ($\widetilde{U_1}$,E) and ($\widetilde{U_2}$,E) in X such that $x^{e}_{(r,t,s)}q(\widetilde{U_1},E)$, $y^{e}_{(r,t,s)}q(\widetilde{U_2},E)$ and $NSCl((\widetilde{U_1},E)) \cap NSCl((\widetilde{U_2},E)) = 0_{(X\,E)}$.

Definition 5.3. An nsts X is said to be NSRS quasi Hausdorff if distinct NSP in X have disjoint NSRS -q-nbds, i.e., if $x^{e}_{(r,t,s)}$ and $y^{e}_{(r,t,s)}$are distinct NSP's in X , then there exist NSRS -q-nbds ($\widetilde{V_1}$,E) and ($\widetilde{V_2}$,E) such that $x^{e}_{(r,t,s)}q(\widetilde{V_1},E)$, $x^{e}_{(r,t,s)}q(\widetilde{V_2},E)$ and $(\widetilde{V_1},E) \cap (\widetilde{V_2},E) = 0_{(X\,E)}$.

Theorem 5.3. If Y is an NSR-quasi Urysohn space and $f : X \to Y$ is an NSWRSC injection, then X is a NSRS quasi Hausdorff space.

Proof. Let $x^e_{(r,t,s)}$ and $y^e_{(r,t,s)}$ be two distinct NSP's in . f being injective, $f(x^e_{(r,t,s)})$ and $f(y^e_{(r,t,s)})$ are distinct NSP's in Y . Since Y is NSR quasi Urysohn, there exists nsro sets ($\widetilde{V_1}$,E) and ($\widetilde{V_2}$,E) in Y such that $f(x^e_{(r,t,s)}q(\widetilde{V_1}, E)$, $f(y^e_{(r,t,s)})q(\widetilde{V_2}, E))$ and $NSCl((\widetilde{V_1}, E)) \cap NSCl((\widetilde{V_2}, E)) = 0_{(Y\ E)}$, i.e., $f^{-1}($ $NSRSInt(NSCl((\widetilde{V_1}, E)))\cap f^{-1}(NSRSInt(NSCl((\widetilde{V_2}, E))) = 0_X$, By Theorem 3.2., $x^e_{(r,t,s)}qf^{-1}((\widetilde{V_1},E))$ $\subseteq f^{-1}(NSRSInt(NSRSCl((\widetilde{V_2},E))))$ $\subseteq$ $f^{-1}(NSRSInt(NSCl((\widetilde{V_1},E)))$. Similarly $y^e_{(r,t,s)}qf^{-1}$ (V_2) $\subseteq f^{-1}(NSRSInt(NSRSCl((\widetilde{V_1},E))))$ $\subseteq f^{-1}$ $(NSRSInt(NSCl((\widetilde{V_2},E)))$.

So, f^{-1} $(NSRSInt(NSCl((\widetilde{V_1},E)))$ and f^{-1} $(NSInt(NSCl((\widetilde{V_2},E)))$ are disjoint NSRS -q-nbds of $x^e_{(r,t,s)}$ and $y^e_{(r,t,s)}$, respectively. So X is NSRS quasi Hausdorff and this proves the result.

Definition 5.4. A function $f : (X, E, \tau) \to (Y, E, \sigma)$ is said to be:
(i) neutrosophic soft weakly regular open (for short, NSWRO) if $f(R) \subseteq NSRInt(f(NSCl((\tilde{R}, E))))$ for each nsos ($\tilde{R}$,E) of X .
(ii) neutrosophic soft weakly regular semi open (for short, NSWRSO) if $f((\tilde{R}, E)) \subseteq NSRSInt(f(NSCl((\tilde{R}, E))$ for each nsos ($\tilde{R}$,E) of X .
(iii) neutrosophic soft weakly semi open (for short, NSWSO) if $f((\tilde{R},E)) \subseteq NSSInt(f(NSCl((\tilde{R}, E))))$ for each nsos ($\tilde{R}$,E) of X .

Remark 5.1. Clearly, every NSWRO function is NSWRSO and every NSWRSO function is also NSWSO .

Theorem 5.4. Let $f : (X, E, \tau) \to (Y, E, \sigma)$ be a function. Then the following statements are equivalent:
(i) f is NSWRSO .
(ii) For each NSP $x^e_{(r,t,s)}$ in X and each nsos ($\tilde{B}$,E) of X containing $x^e_{(r,t,s)}$, there exists a nsrso set ($\tilde{C}$,E) containing $f(x^e_{(r,t,s)})$ such that $(\tilde{C}, E) \subseteq f(NSCl((\tilde{B}, E)))$.

Proof. (i)→ (ii): Let $x^e_{(r,t,s)} \in X$ and ($\tilde{B}$,E) be a nsos in X containing $x^e_{(r,t,s)}$. Since f is NSWRSO $f((\tilde{B}, E)) \subseteq NSRSInt(f(NSCl((\tilde{B}, E))))$ .Let ($\tilde{C}$,E) = $SInt(f(NSCl((\tilde{B}, E))))$ .Hence $(\tilde{C}, E) \subseteq f(NSCl((\tilde{B}, E)))$, with T containing $f(x^e_{(r,t,s)})$.

(ii)→ (i): Let ($\tilde{B}$,E) be a nsos in X and let $y^e_{(r,t,s)} \in f((\tilde{B},E))$. It following from (ii) $(\tilde{C},E) \subseteq f(NSCl((\tilde{B},E)))$ for some nsrso ($\tilde{C}$,E) in Y containing $y^e_{(r,t,s)}$. Hence we have, $y^e_{(r,t,s)} \in (\tilde{C},E) \subseteq NSRSInt(f(NSCl((\tilde{B},E))))$. This shows that $f((\tilde{B}, E)) \subseteq NSRSInt(f(NSCl((\tilde{B}, E))))$, i.e., f is a NSWRSO functions.

Theorem 5.5. $f : (X, E, \tau) \to (Y, E, \sigma)$ be a bijective function. Then the following statements are equivalent:
(i) f is NSWRSO .
(ii) $NSRSCl(f((\tilde{A}, E))) \subseteq f(NSCl((\tilde{A}, E)))$ for each nsos ($\tilde{A}$,E) in X .
(iii) $NSRSCl(f(NSInt((\tilde{B}, E)))) \subseteq f((\tilde{B}, E))$ for each nscs ($\tilde{B}$,E) in X .

Proof. (i)→ (iii): Let ($\tilde{B}$,E) be a nscs in X. Then we have $f((\tilde{B}, E)^c)$ = $(f(\tilde{B},E)) \subseteq NSRSInt\ (f(NSCl((\tilde{B}, E))^c))$ and so $(f((\tilde{B}, E)))^c \subseteq (NSRSCl(f(NSInt((\tilde{B}, E)))))^c$. Hence $SCl(f(NSInt((\tilde{B}, E)))) \leq f((\tilde{B}, E))$.

(iii)→ (ii): Let ($\tilde{A}$,E) be a nsos in X. Since $NSCl((\tilde{A}, E))$ is a nscs and $R \subseteq NSInt(NSCl((\tilde{A}, E)))$ by (iii) we have $SCl(f((\tilde{A}, E))) \subseteq$ $NSRSCl(f(NSInt(NSCl((\tilde{A}, E))))) \subseteq$ $f(NSCl((\tilde{A}, E)))$.

(ii)→ (iii): Similar to (iii)→ (ii).

(iii)→ (i): Clear.

Definition 5.5. Two non-null Nss ($\tilde{A}$,E) and ($\tilde{B}$,E) in a nsts X are said to be NSRS -separated if $(\tilde{A}, E)qNSRSCl((\tilde{B}, E))$ *and* $(\tilde{B}, E)qNSRSCl((\tilde{A}, E))$ or equivalently if there exist two nsrso sets ($\tilde{C}$,E) and ($\tilde{D}$,E) such that $(\tilde{A}, E) \subseteq (\tilde{C}, E), (\tilde{B}, E) \subseteq (\tilde{D}, E), (\tilde{A}, E)\ q(\tilde{D}, E)$ and ($\tilde{B}$,E)q($\tilde{C}$,E) .

Definition 5.6. A nsts X which can not be expressed as the union of two NSRS -separated sets is said to be a NSRS -connected space.

Theorem 5.6. If $f : (X, E, \tau) \to (Y, E, \sigma)$ is a NWRSO of a space X onto a NSRS -connected space , then X is NSRS -connected.

Proof. If possible, let X be not NSRS -connected. Then there exist NSRS -separated sets ($\tilde{C}$,E) and ($\tilde{D}$,E) in X such that $= (\tilde{C}, E) \cup (\tilde{D}, E)$. Since ($\tilde{C}$,E) and ($\tilde{D}$,E) are NSRS -separated, there exist two nsro sets $(\tilde{A}, E)$ and

$(\tilde{B}, E)$ such that $(\tilde{C}, E) \subseteq (\tilde{A}, E), (\tilde{D}, E) \subseteq (\tilde{B}, E), (\tilde{C}, E) q (\tilde{B}, E)$ and $(\tilde{D}, E) q (\tilde{A}, E)$. Hence we have $f((\tilde{C}, E)) \subseteq f((\tilde{A}, E)), f((\tilde{D}, E)) \subseteq f((\tilde{B}, E))$, $f((\tilde{C}, E)) q f((\tilde{B}, E))$ and $((\tilde{D}, E)) q f((\tilde{A}, E))$. Since f is $NSWRSO$, we have $f((\tilde{A}, E)) \subseteq NSRSInt(f(NSCl((\tilde{A}, E))))$ and $f((\tilde{B}, E)) \subseteq NSRSInt(f(NSCl((\tilde{B}, E))))$ and since $(\tilde{A},E)$ and $(\tilde{B},E)$ are nsrso and also nsrsc by proposition ??, we have $f(NSRSCl((\tilde{A}, E))) = f((\tilde{A}, E)), f(NSRSCl((\tilde{B}, E))) = f((\tilde{B}, E))$. Hence $f((\tilde{A}, E))$ and $f((\tilde{B}, E))$ are nsrso in . Therefore, $f((\tilde{C}, E))$ and $f((\tilde{D}, E))$ are NSRS - separated sets in Y and $Y = f(X) = f\big((\tilde{C}, E\big) \cup (\tilde{D}, E)) = f((\tilde{C}, E)) \cup f((\tilde{D}, E))$. Hence this contrary to the fact that Y is NSRS -connected. Thus X is NSRS -connected.

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