

# Convergence Analysis of Dynamic Response of Double-Drum Vibratory Roller under Random Road Excitation

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## Abstract

The main role of the vehicle suspension system is to achieve driving stability and passenger comfort regardless of the road surface. Usually, we solve the optimization problem for road surfaces with either regular or random undulations. This paper will consider the convergence property of the dynamic response of the system when considering the road surface as random excitation. The goal is that we will find the number of calculation cases corresponding to the dynamic parameters of interest to obtain the objective function value in the optimization process. The contribution of the present paper is shown with the calculation case of a double-drum vibratory roller.

**Keywords :** Convergence analysis, Dynamic characteristic, Random excitation, Vibratory roller

## 1.Introduction

The double-drum vibratory roller is a widely used machine in soil compaction work. The dynamic performance of the vibratory roller directly affects the quality and efficiency of compaction and consequently influences both the durability and progress of construction projects. Especially under current conditions, compaction work increasingly demands high standards in terms of quality, construction efficiency, and smooth operation and comfort for the operator [1, 2].

Numerous studies have investigated the dynamic response (i.e., natural frequency [3], amplitude, and acceleration [4]) of vibratory rollers via models with different degrees of freedom (DOF) [5-8] or experimental methods [9-11]. Typically, researchers represent the entire mass of the machine frame and drums as a lumped mass system. The suspension system and the compacted soil are commonly modeled as interconnected elements with stiffness and damping coefficients [6, 7, 9]. The model of the machine can be further detailed by including the seat mechanism or other concentrated masses. The soil model can account for elasto-plastic properties, surface roughness, and characteristics that vary over time or with the number of compaction passes [6, 7]. The random characteristics of compacted soil can influence the dynamic response of the system and the evaluation criteria of the compaction process [11]. In this work, the convergence property of the dynamic response of the system when considering the road surface as random excitation. The

goal is that we will find the number of calculation cases corresponding to the dynamic parameters of interest to obtain the objective function value for the optimization process.

## 2. Method and Model

Dynamic model of a typical double-drum vibratory roller is established based on the following assumptions: Only the vibration of the machine in the vertical plane along its longitudinal axis is considered; the compacted soil is characterized by constant parameters throughout the compaction process, namely  $k_{si}$  and  $c_{si}$ ; only the vertical displacement of the machine frame is taken into account; and continuous contact between the drums and the soil is maintained during the entire compaction process. The suspension system linking the frame and the compaction wheel is described by elements characterized by stiffness and damping coefficients ( $k_{di}$  and  $c_{di}$ ). The dynamic model, as illustrated in Fig. 1, consists of three masses (i.e., front drum  $m_1$ , rear drum  $m_2$ , and frame with mass  $m_3$  and moment of inertia  $I_3$ ) and four degrees of freedom corresponding to the generalized coordinates as  $y = [y_1, y_2, y_3, \phi_3]$ .

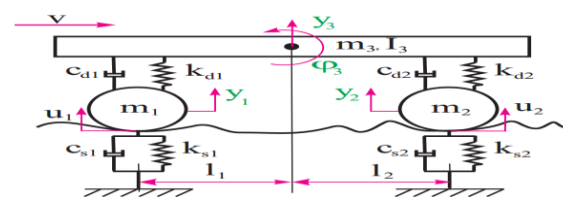


Fig. 1 Model for dynamic analysis of double-drum vibratory roller

Applying the Lagrange equation II for the above model, the system of differential equations describing the vehicle vibration is obtained as follows [8],

$$\begin{aligned} m_1\ddot{y}_1 &= F_{01}\sin(2\pi f_1 t) + [k_{d1}(y_3 + l_1\varphi_3 - u_1) + c_{d1}(\dot{z}_3 + l_1\dot{\varphi}_3 - \dot{u}_1)] - k_{s1}y_1 - c_{s1}\dot{y}_1, \\ m_2\ddot{y}_2 &= F_{02}\sin(2\pi f_2 t) + [k_{d2}(z_3 - l_2\varphi_3 - u_2) + c_{d2}(\dot{y}_3 - l_2\dot{\varphi}_3 - \dot{u}_2)] - k_{s2}y_2 - c_{s2}\dot{y}_2, \\ m_3\ddot{y}_3 &= -[k_{d1}(y_3 + l_1\varphi_3 - y_1) + c_{d1}(\dot{y}_3 + l_1\dot{\varphi}_3 - \dot{y}_1)] - \\ &\quad [k_{d2}(y_3 - l_2\varphi_3 - y_2) + c_{d2}(\dot{y}_3 - l_2\dot{\varphi}_3 - \dot{y}_2)], \\ I_3\ddot{\varphi}_3 &= [k_{d2}(y_3 - l_2\varphi_3 - y_2) + c_{d2}(\dot{y}_3 - l_2\dot{\varphi}_3 - \dot{y}_2)]l_2 - \\ &\quad [k_{d1}(y_3 + l_1\varphi_3 - y_1) + c_{d1}(\dot{y}_3 + l_1\dot{\varphi}_3 - \dot{y}_1)]l_1. \end{aligned} \quad (1)$$

In the above equations,  $u_1$  and  $u_2$  are road excitations according to the ISO 8064 [12], and  $F_{0i}$  ( $2\pi f_i t$ ) with  $i=1,2$  are the vibration forces of the roller drums.

In order to illustrate the present method, the configuration of the vibratory roller with the main parameters listed in Table 1 is selected.

Table 1 Input parameters of the investigated double-drum vibratory roller [8]

Parameters	Values	Parameters	Values	Parameters	Values
$m_1$ (kg)	2300	$k_{d2}$ (N/m)	$1803 \times 10^3$	$F_{01}$ (N)	$128 \times 10^3$
$m_2$ (kg)	2300	$c_{d2}$ (Ns/m)	3990	$F_{02}$ (N)	$96 \times 10^3$
$m_3$ (kg)	3270	$k_{s1}$ (N/m)	$1 \times 10^7$	$f_1$ (Hz)	48
$I_3$ (kg·m <sup>2</sup> )	7811	$c_{s1}$ (Ns/m)	$210 \times 10^3$	$f_2$ (Hz)	48
$k_{d1}$ (N/m)	$1803 \times 10^3$	$k_{s2}$ (N/m)	$1.2 \times 10^7$	$l_1$ (m)	1.80
$c_{d1}$ (Ns/m)	3990	$c_{s2}$ (Ns/m)	$280 \times 10^3$	$l_2$ (m)	1.80

The road excitation is given by [12]

$$u(x) = \sum_{i=1}^N \sqrt{2G_u(n_i) \Delta n_i} \cos(2\pi n_i x + \phi_i). \quad (2)$$

Herein, the road classification (i.e., namely by class A, B, C...) is introduced through the power spectral density of road profile  $G_u(n)$  for the very smooth to very rough level, see also four unpaved off-road classifications detailed in [7].

For optimizing suspension systems of vibration systems, multi-objective functions  $J$  often involve the characteristics of vibration responses with some weighting coefficients  $\beta$  of sub-objective functions as [2, 4, 8],

$$J = \sum_{i=1}^m \beta_{i1} \sqrt{\frac{1}{T} \int_0^T \ddot{y}_i^2(t) dt} + \sum_{j=1}^n \beta_{j2} \sqrt{\frac{1}{T} \int_0^T y_j^2(t) dt}. \quad (3)$$

The sub-objective functions related with acceleration response of the roller frame as:

$$J_y = \sqrt{\frac{1}{T} \int_0^T \ddot{y}_3^2(t) dt}, \quad J_\varphi = \sqrt{\frac{1}{T} \int_0^T \ddot{\varphi}_3^2(t) dt}. \quad (4)$$

### 3. Results and discussion

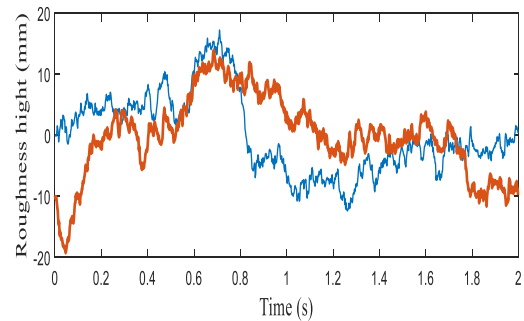


Fig. 2 Two random excitations of the class-C road within a length of 5 m

Herein, three random road profiles (e.g., class C, D, E) and a vehicle speed of  $v = 9$  km/h are considered. Fig. 2 illustrates two roughness configurations of a type C road surface over a length of 5 m, corresponding to a vehicle travel time of 2 seconds. From these road excitations, the dynamic responses of the machine are calculated, as present in Fig. 3. Then, the objective

functions of  $J_y = 1.3433/1.1927 \text{ m/s}^2$  and  $J_\phi = 0.8457/0.6963 \text{ rad/s}^2$  are estimated. The results of the dynamic characterization and the values of the evaluation parameters reveal a clear and significant difference.

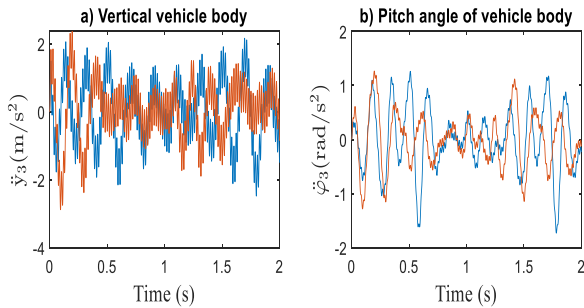


Fig. 3 Dynamic response of the frame with road excitations present in Fig. 2

As shown in Fig. 4 for 1000 dynamic responses of the vehicle, the average sub-objective functions are  $J_y = 1.2295 \text{ m/s}^2$  and  $J_\phi = 0.6572 \text{ rad/s}^2$ , while these values computed from the average response are  $J_y^* = 0.6673 \text{ m/s}^2$  and  $J_\phi^* = 0.0954 \text{ rad/s}^2$ . This indicates that the average dynamic response cannot represent the system's characteristics (as this value tends to zero when the number of configurations becomes sufficiently large); instead, the average evaluation indicators must be determined based on the metrics calculated from each individual response.

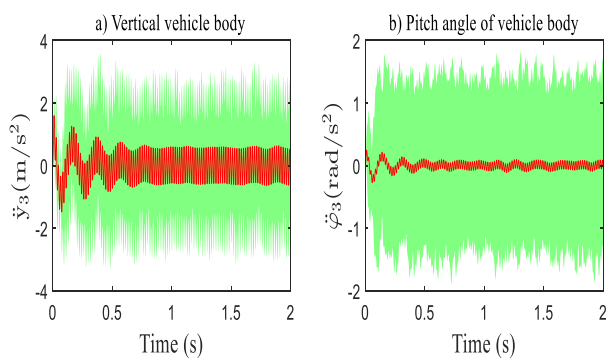


Fig. 4 Dynamic characteristics of the frame with a set of 1000 class-C road profiles.

The shaded region is bounded by the minimum and maximum values of all responses, while solid center line represents the mean value.

Among the 1000 random road configurations for each road type C/D/E, the corresponding variation ranges of the evaluation indicators are as follows:  $J_y = [0.7759 \ 1.8525]/[1.0811 \ 3.3114]/[1.9432 \ 6.4956] \text{ m/s}^2$  and  $J_\phi = [0.3503 \ 1.0368]/ [0.6300 \ 1.9139] [1.1523 \ 3.9806]$

$\text{rad/s}^2$ . This variation range shows that the deviation from the mean value can reach up to approximately 50% when only a single road surface excitation configuration is considered. The results in Figure 5 illustrate the deviation levels of the two evaluation indicators when approximated using 50 to 950 road surface configurations.

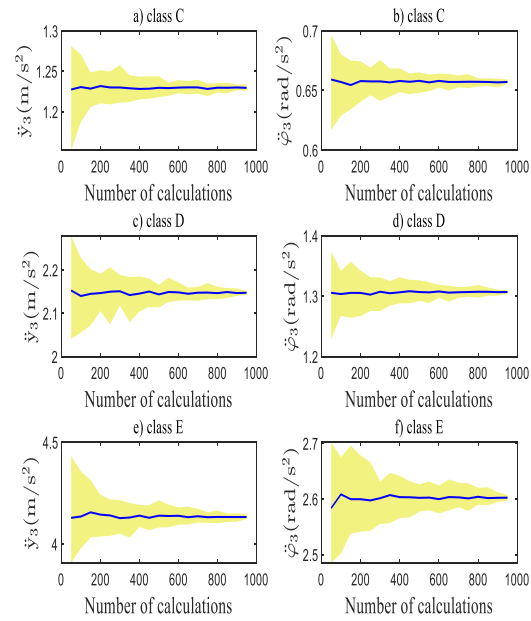


Fig. 5 Convergence analysis of dynamic response via number of calculations

It is evident that with a larger number of cases, the dynamic response characteristics exhibit a consistent convergence trend. Specifically, when only 50 road surface profiles are considered, the deviation in the vibration response values ranges from approximately 5% to 7%. In contrast, increasing the number of computed profiles to 950 reduces the deviation of the objective function to within 0.2% to 0.4%. However, it should be noted that each road surface configuration takes nearly 10 seconds to compute on average.

#### 4. Conclusion

In this work, the dynamic response of a two-drum vibratory roller subjected to random road surface roughness excitation is investigated. The study results indicate that, under the same roughness level, the system exhibits significantly different responses, leading to considerable variations in dynamic performance indicators and ride comfort. It can be

stated that the mean dynamic response does not accurately reflect the system characteristics—especially since it approaches zero as the number of configurations increases. Therefore, the average of evaluation indicators should be derived from the results of each individual response rather than from the averaged system response itself. For the considered machine configuration and the three types of random road surfaces, the survey results show that two key indicators — vertical displacement acceleration and angular acceleration of the frame — can see their deviation reduced from around 50% (when using a single road configuration) to 7%, or even below 0.5%, when approximated over 50 and 950 road surface configurations, respectively.

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#### **Conflict of Interest**

The authors declare no conflicts of interest

#### **Author Contribution**

The authors confirm contribution to the paper as follows: study conception and design: Autho VAN; data collection : Author DUONG; analysis and interpretation of results : Author VAN, Author DUONG; draft manuscript preparation : Author DUONG, Author VAN. All authors reviewed the results and approved the final version of the manuscript.

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