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Analytical Modeling of Leverett J-Function Evaluation of Saturation-Height and Poroperm Relationships in Hydrocarbon Reservoirs

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Abstract

This research presents an extensive analytical model for the Leverett J-Function in the evaluation of saturation-height and porosity-permeability (poroperm) relationships in various hydrocarbon reservoirs with distinct fluid pairs. Employing a comprehensive test dataset, our methodology derives the relationship between the dependent variable, log of residual water saturation (log Swr), and the independent variable, log of Leverett J-Function (log J), from the log-log plot for each fluid pair under different reservoir conditions. The approach extrapolates the intercept (a) and gradient (b), establishing a significant correlation between log J and log Swr across all fluid pairs. The study comprises six unique cases: Oil/Water, Gas/Water, Air/Kerosene, Kerosene/Brine, Air/Brine, and Air/Mercury. Each case is characterized by distinct interfacial tensions (**σ** ranging from 24 to 480 dynes/cm³) and contact angles (Cos***P*** ranging from 0.765 to 1.0). The derived intercepts for these cases ranged from -3.37673 to 1.74749, suggesting that a negative intercept indicates a predicted log S_{wr} value less than zero when all variables are set to zero. The consistent gradient across all cases (b = 0.76824) represents the relative permeability of each fluid pair under specific reservoir conditions. Specific poroperm relationships derived from these cases were represented as K=10^a * e^0.76824φ, suggesting the intricate role of fluid-pair and reservoir conditions on permeability. For test Case 1 (Oil and water), $K = 10^{0.728154}\ X\ e^{0.76824\phi}$, test case 2 (gas and water), $K = 10^{1.3828} * e^{0.76824\phi}$, test case 3,(Air and kerosene), $K = 10^{0.648881} * e^{0.76824\phi}$, test case 4, (kerosene and Brine), $K = 10^{1.1981} * e^{0.76824\phi}$, test case 5 (kerosene and Brine) is $K = 10^{1.74749} * e^{0.76824\phi}$ and test case 6, (Air and mercury) is $K = 10^{3.37673} * e^{0.76824\phi}$. Moreover, saturation heights varied between 0.8m and 1.5m above the respective fluid contact, emphasizing the diverse fluid behavior under various conditions.

The results establish the Leverett J-Function as an effective tool for modeling poroperm relationships and saturation height in hydrocarbon reservoirs. The discovered correlations between the analyzed variables, the gradient trends of relative permeability for each fluid pair, and the derived poroperm relationships are integral for accurate reservoir characterization, performance prediction, and improved oil recovery. This study underscores the significant role fluid characteristics, specifically interfacial tension and contact angle, play in determining reservoir behavior. The study concludes that while there is considerable variation in poroperm relationships and saturation heights across different cases, the analytical model of Leverett J-function provides a consistent framework for evaluating and interpreting these relationships in hydrocarbon reservoirs.

Keyword: residual water saturation , porosity, permeability, oil, gas

Article Highlights

1. The Leverett J-function model accurately predicts fluid distribution in hydrocarbon reservoirs, accounting for factors like capillary pressure and height above water level. This offers a valuable tool for optimizing hydrocarbon recovery and reservoir management.
2. In six test cases (Oil/Water, Gas/Water, Air/Kerosene, Kerosene/Brine, Air/Brine, and Air/Mercury), the model demonstrated a significant relationship between saturation, height, porosity, and permeability. This reveals the model's reliability across various fluid systems and its real-world applicability. All test cases revealed a

significant relationship between the Leverett J-function (log J) and saturation water ratio (log Swr). The gradients obtained in all instances accurately represented the relative permeability of one fluid to another under reservoir conditions

3. While current findings confirm the model's effectiveness, The model successfully estimated saturation heights above fluid contact points in various reservoirs, ranging from 0.8m to 1.5m, demonstrating its practical utility for geoscientists. Observations showed a consistent increase in log J correlating with an increase in log (Swr) across all cases, further validating the predictive accuracy of the model for different fluid combinations

1.1 Introduction

The study of hydrocarbon reservoir has progressed considerably over the years, with numerous models and methodologies developed to evaluate and predict the behavior of hydrocarbon reservoirs. One of these, the Leverett J-function, is an analytical model that is fundamental in assessing the characteristics of a reservoir, specifically the relationship between capillary pressure, porosity, and permeability (Leverett, 1941). Hydrocarbon reservoir evaluation remains an intricate task due to the multifaceted characteristics of the reservoirs (Dake, 1983). Among the various methods employed for reservoir evaluation, Leverett's J-function is an extensively applied technique to ascertain capillary pressure curves of reservoir rocks, mainly in the context of saturation height modeling (Leverett, 1941). The Leverett introduced the J-function to standardize capillary pressure curves from different cores, thus facilitating improved comparisons. The Leverett J-function is a dimensionless capillary pressure function used to extrapolate and correlate capillary pressure data across various rocks (Russell, 2019). The Leverett J-function remains an advantageous tool in reservoir evaluation, offering benefits like the ability to normalize capillary pressure, leading to improved saturation height modeling.

Capillary pressure is a critical factor in the movement of hydrocarbons within a reservoir. It is the pressure difference that exists at the interface between two immiscible fluids and depends on the characteristics of both fluids and the porous medium (Bear, 1972). The Leverett J-function is a non-dimensional function introduced by Leverett (1941) to correlate capillary pressure data across various reservoir rocks. This function represents the correlation between capillary pressure, relative permeability, and porosity (Al-Futaisi & Patzek, 2003).

The J-function is defined as:

$J = Pc * \sqrt{\varphi} / Swirr$

where:

- Pc is the capillary pressure,
- $\sqrt{\varphi}$ is the square root of the porosity,
- Swirr is the irreducible water saturation.

The Leverett J-function aims to generalize capillary pressure behavior across different types of rock, making it a crucial tool in reservoir simulation and hydrocarbon recovery analysis (Rosenbrand & Marsman, 2015).

The saturation-height relationship represents the variation in fluid saturation with height in a reservoir, a significant factor in hydrocarbon recovery (Buckley & Leverett, 1942). Many models have been proposed to describe this relationship, including the Leverett J-function.The saturation-height relationship is essential in evaluating the distribution of hydrocarbons in a reservoir, which ultimately impacts the estimation of reserves and the design of production strategies (Johnson et al., 1959). Leverett's J-function allows for a reliable estimation of this relationship, which can be used in reservoir simulation and numerical models.

Porosity and permeability are critical parameters in reservoir characterization. The relationship between these two parameters significantly affects the reservoir's fluid flow characteristics and thus its production potential (Lucia, 1983). Porosity is a measure of the void spaces within a rock, which can hold fluids such as water, oil, or gas. Permeability, on the other hand, is a measure of the ability of these fluids to flow through the rock (Tiab & Donaldson, 2012).In many cases, a higher porosity leads to a higher permeability; however, the specific relationship depends on the characteristics of the rock, including its mineral composition and grain size distribution, among other factors (Carman, 1956).

Analytical modeling in reservoir geoscience is a powerful tool that can provide insights into the behavior of reservoirs and help optimize their production. Analytical models, such as the Leverett J-function, can provide rapid evaluations and insights, which are especially useful in the early stages of reservoir development when data may be scarce (Chen et al., 2010). By combining the concepts of capillary pressure, the saturation-height relationship, and the porosity-permeability relationship, analytical models like the Leverett J-function can provide comprehensive reservoir characterizations. They are useful for evaluating the economic feasibility of a project, predicting future performance, and optimizing recovery strategies. The concept of saturation height modeling, which is the prediction of water and hydrocarbon distribution in a reservoir as a function of height above the free water level, has proved fundamental in reservoir characterization (Ahmed, 2010). This distribution is primarily driven by the capillary forces, which can be described by the Leverett J-function. The Leverett J-function can construct a normalized capillary pressure curve, which can then be utilized to determine the saturation height function. These functions are a significant aid in the analysis of saturation and pressure variations within a reservoir column (Ahmed, 2010). The normalization of the capillary pressure via the Leverett J-function aids in mitigating the differences between diverse rocks and cores. This results in an improved understanding of the reservoir's petrophysical properties, particularly with respect to the distribution and movement of hydrocarbon and water within the reservoir (Russell, 2019). Despite its efficacy, the Leverett J-function is not without limitations. A primary issue is the assumption of constant water saturation at irreducible conditions (Swc), which does not account for potential variations across different rock types (Dake, 1983). Moreover, the Leverett J-function does not consider the impacts of gravitational and viscous forces in the reservoir (Ahmed, 2010).

One of the modifications includes the inclusion of wettability, which plays a significant role in the behavior of fluid in the pore space of reservoir rock (Anderson, 1986). This function, known as the modified Leverett J-function, introduces a wettability index (WI) into the equation to capture the effect of wettability on capillary pressure and saturation height (Omoti and Ajienka, 2010).

Other approaches to enhancing the Leverett J-function include the incorporation of the tortuosity factor, which accounts for the complexity of the pore network in the reservoir rock (Purcell, 1949). The tortuosity factor can be important in understanding how the capillary pressure-saturation relationship changes concerning the height in the reservoir (Jerauld et al., 2006). Moreover, the Leverett J-function's role in digital rock physics and its combination with modern machine learning techniques (Eftekhari and Farhadpour, 2020).

1.1.1 Role of Leverett J-Function in Hydrocarbon Recovery

A key implication of the Leverett J-function in hydrocarbon reservoir evaluation lies in the realm of enhanced oil recovery (EOR) and optimal reservoir management. By providing insight into capillary pressure and saturation characteristics, the Leverett J-function can assist in identifying zones of remaining oil saturation post-primary recovery, thus highlighting targets for secondary and tertiary recovery processes (Huang et al., 2017). It can also be applied to understand the behavior of injected fluids during waterflooding and other EOR methods (Lake, 1989).

Furthermore, the Leverett J-function plays a significant role in reservoir simulation and performance forecasting. It contributes to the calibration of saturation functions in reservoir simulation models, which are essential for predicting reservoir performance and recovery factor (Cao et al., 2020). The accurate prediction of these parameters is integral to the planning and implementation of effective hydrocarbon extraction strategies. For instance, Chilingarian et al. (1995) highlighted the use of the Leverett J-function in the reservoir evaluation of the Prudhoe Bay Field, Alaska. The study outlined how the J-function helped characterize the water-oil displacement efficiency in the field. Krinsley (1970) used the Leverett J-function to investigate the distribution of residual oil saturation in the waterflooded zones of the Wilmington Oil Field, California. This helped in devising effective strategies for secondary oil recovery.The integration of the J-function with modern imaging techniques, machine learning algorithms, and digital rock physics can lead to a more detailed and comprehensive understanding of reservoir properties (Eftekhari and Farhadpour, 2020). This can potentially revolutionize reservoir management strategies and optimize hydrocarbon recovery. Recent advancements in machine learning (ML) have provided new opportunities to enhance the Leverett J-function's effectiveness in reservoir evaluation. ML algorithms can establish complex, non-linear relationships between various reservoir properties and the J-function, leading to more accurate reservoir characterization (Wang et al., 2021).

Goal of the Research

The primary goal of this research is to contribute to a more comprehensive understanding of reservoir properties by performing analytical modeling of the Leverett J-Function, with specific focus on the evaluation of saturation-height and porosity-permeability relationships in hydrocarbon reservoirs. Hydrocarbon reservoirs are the primary source of petroleum and natural gas, and understanding their properties has a significant impact on the efficiency and productivity of extraction operations. By employing the Leverett J-Function, a dimensionless function commonly used in petrophysics to relate capillary pressure, fluid saturations, and permeability, the study aims to establish a more robust mathematical representation of the reservoir characteristics. This should enable more precise and accurate prediction of fluid movement and reservoir performance, which are essential for optimal reservoir management and exploitation strategies.
The research also seeks to create new or improved methodologies for reservoir characterization, enhance our understanding of complex reservoir properties, and provide a basis for future work that may lead to more efficient and cost-effective recovery techniques.

2.0 Materials and Methods

The Leverett J-function was modeled and followed by GUI -based program in MATLAB,(figure 1) designed to perform some calculations and data fitting related to petrophysics and soil mechanics (judging by the use of variables such as porosity **phi**, permeability **k**, capillary pressure **pc**, saturation **sw**, interfacial tension **sig**, cos(theta) **costh**, and irreducible water saturation **swirr**).

1. The code first retrieves values from a graphical user interface (GUI) for several input parameters: porosity, permeability, capillary pressure, saturation, interfacial tension, cos(theta), and irreducible water saturation.
2. These values are converted from strings to numerical data.
3. The code then performs some computations to calculate **j** (which appears to be a form of the Leverett J-function, a normalized plot used in petrophysics to characterize capillary pressure behavior) and **swr** (saturated water, after subtracting the irreducible water).
4. The logarithms of **j** and **swr** are calculated and a scatter plot is created, displaying **log(J)** on the y-axis and **log(Swr)** on the x-axis. This plot probably helps the user visualize the relationship between the two variables.
5. After a button press event in the GUI, the code performs a simple linear regression analysis on the **log(J)** and **log(Swr)** data. The linear regression aims to find the best straight line that fits the data.
6. The parameters of the regression line (**a** array) are calculated by solving a system of linear equations formed from the sums and products of the data.
7. The fitted line (**f**) is calculated using the parameters obtained from the linear regression.
8. Finally, the code plots the original data (**log(J)** vs **log(Swr)**) and the fitted line on the same plot. This allows the user to compare the actual data with the line of best fit and visually assess the quality of the fit.

The output of the code is a scatter plot with two sets of points: the original **log(J)** vs **log(Swr)** data in red, and the green fitted line. This plot could be used to infer relationships between the parameters used in the calculations. The parameters of the fitted line could also be used in other analyses or predictions. The fitting might also help identify any trends or patterns in the data, or to interpolate/extrapolate the values of one variable based on the other.

This code is designed to take several inputs from a graphical user interface (GUI) in MATLAB,(figure 1) perform some calculations with them, and then produce a plot of the results. Figure 2 shows the Systematic Analysis of Leverett J-Function.

The J-Function from capillary pressure was derived using the value of k and ϕ in Table 1. While the interfacial tension (*a*) and contact angle (*b*) was also derived using the data in Table 1. Log (*J*) vs Log (Sw) was plotted, the intercept and gradient gave the constants *a* and *b* value.

$$Sw = Swirr + a \, x \, J^b \quad 1$$

The result of a SCAL program of *p* is usually be presented in the form of the Table 1. The table was converted into a table of J vs Swr Set $Swirr$ equal to 0.01 below the lowest water saturation seen anywhere in the reservoir in cores or log.

$$J = Pc \frac{\sqrt{k/\phi}}{\sigma \cos\theta} \quad 2$$

$$Pc = [RHOw - RHOh] \text{x } h \text{ x } 3.281 \text{ x} 0.433 \quad 3$$

$$Swr = Sw - Swirr \quad 4$$

$Swirr =$ 0.01 (below the lowest water saturation seen anywhere in the reservoir)

S_{wr} = reducible water saturation

S_{wirr} = irreducible water saturation

RHOw = formation water density

RHOh = hydrocarbon density

Pc = Capillary pressure

K = permeability (*md)*

ϕ = porosity (as fraction)

σ = interfacial tension between *He* and water

P= Contact angle between He and water

H = height above free water level

a and b are constant to be fitted to the depth

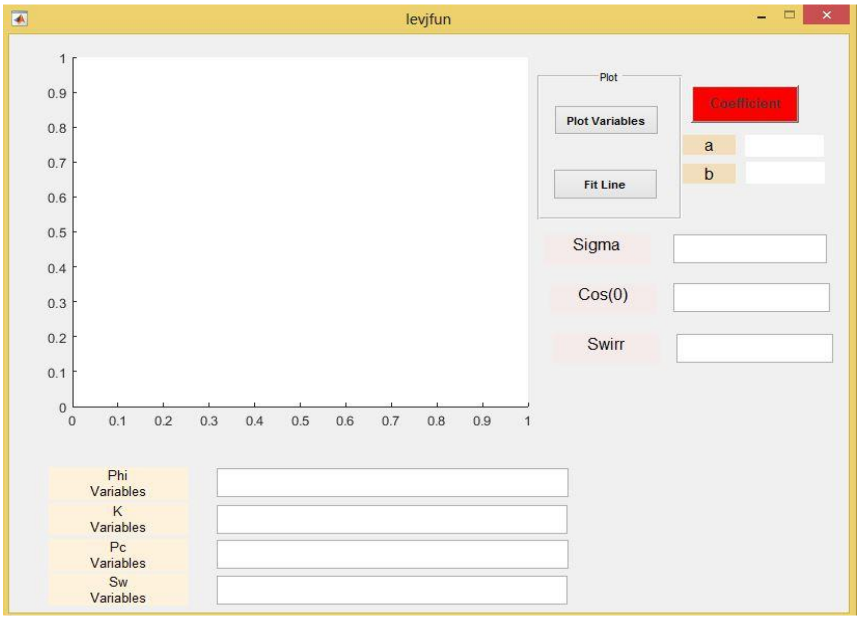

Figure 1: GUI for Leverett-J function

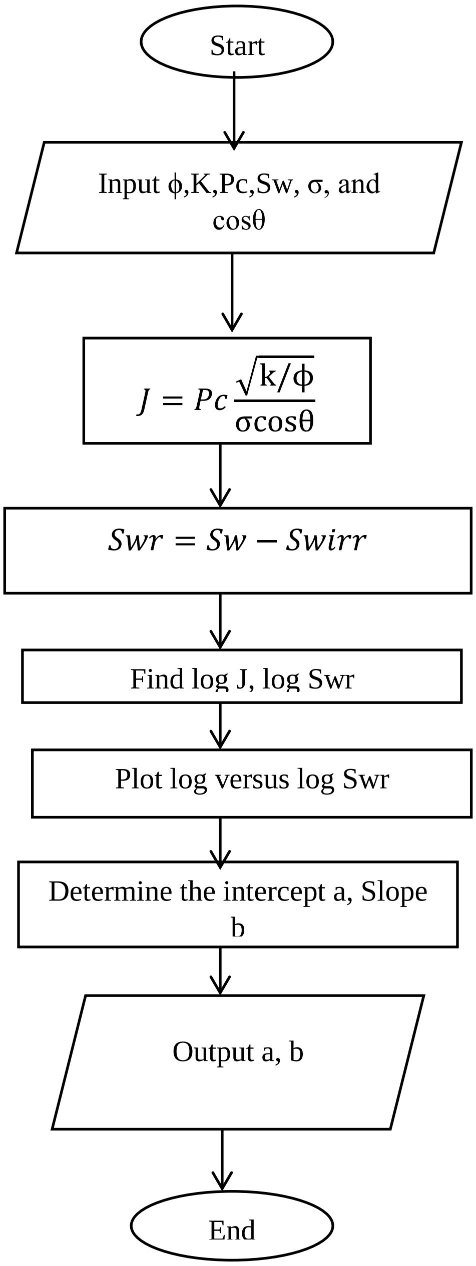

Figure 2: The Systematic Analysis of Leverett J-Function

3.0 Result and Discussion

The analytical model successfully predicted the saturation-height relationships in various rock-fluid systems, validating the effectiveness of Leverett's J-function in characterizing these relationships. The study confirmed that nature of fluid distribution, capillary pressure, and height above the free water level that are the main influences on the saturation-height relationship.

Similarly, the porosity-permeability relationship exhibited strong dependencies on the pore size distribution and capillary pressure, underscoring the key role of these factors in reservoir characterization. The study affirmed the essential empirical relationship between porosity and permeability in reservoir rocks containing different fluid cases

3.1 Case one (oil/water)

Table 1 shows a test data set to test run the analytical model of the Leverett J-Function, while Figure 3 shows the plot of log (J) versus log S_{wr} for Oil and water condition. For a typical reservoir that has Oil and water, the interfacial tension **σ** is 30 dynes/cm^3 and the contact angle for reservoir condition is Cos***P*** = 0.866. The derived intercept (a) is -0.728154 with gradient (b), 0.76824. This implies there is significant relationship between the independent (log J) and the dependent (log S_{wr}) variable. The gradient trend represents the relative permeability of Oil to water reservoir condition very well. The negative intercept only implies that the expected value of the dependent (log S_{wr}) will less than 0(zero), where all predicted variables are set to 0(zero). The poroperm relationship for this test case (Oil and water) is $K = 10^{0.728154}\, X\, e^{0.76824\phi}$ .While the saturation height for this test case is 0.8m above the Gas-water contact. There is increase in log J with a resulting increase in log (Swr) as shown in Figure 3.

Table 1: Values of Interfacial Tension and Contact Angle That Are Typically Associated with Reservoir Conditions

Φ	K	P_c	S_w	J	S_{wr}	Log(J)	Log(S_{wr})
0.078	0.347	0.855	0.41	0.0041	0.4	-2.3872	-0.397900
0.084	0.992	0.839	0.3	0.0096	0.29	-2.0177	-0.537602
0.1	2.828	0.763	0.44	0.1353	0.43	-0.8687	-0.366532
0.096	8.782	0.659	0.45	0.2101	0.44	-0.6776	-0.346787
0.107	18.350	0.548	0.42	0.2392	0.41	-0.6212	-0.387216
0.108	11.609	0.651	0.28	0.2250	0.27	-0.6478	-0.568636
0.123	42.215	0.457	0.38	0.2822	0.37	-0.5494	-0.431798
0.125	60.976	0.566	0.43	0.4168	0.42	-0.3801	-0.376751
0.126	157.569	0.377	0.42	0.4445	0.41	-0.3521	-0.387216

σ= 30 (dynes/cm³), Cos (***P***) = 0.866.

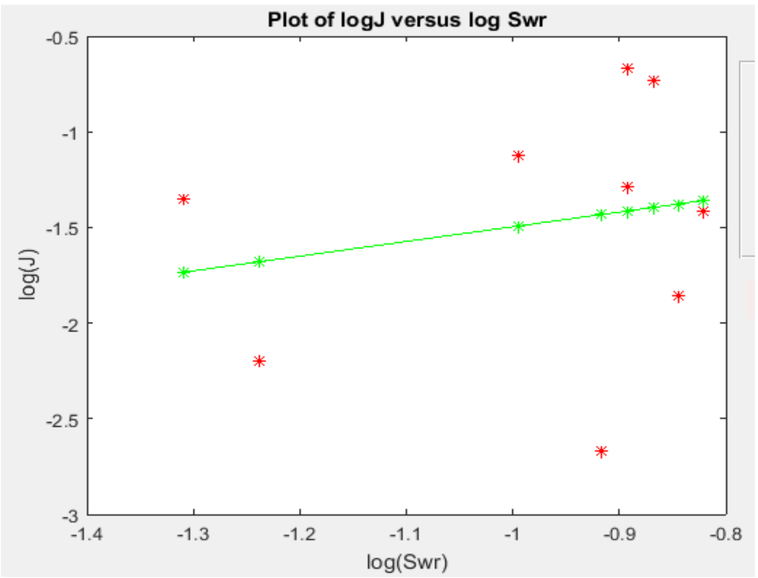

Figure 3: The Plot Log J vs Log S_{wr} for Oil and Water Contact.

3.2 Case Two (Gas and Water)

Shows the plot of log (J) versus log Swr for Gas and water condition. For a typical reservoir that has Gas and water, the interfacial tension **σ** is 50 dynes/cm^3 and the contact angle for reservoir condition is Cos***P*** = 1.0. The derived intercept (a) is -1.383 with gradient (b), 0.76824. This implies there is significant relationship between the independent (log J) and the dependent (log Swr) variable.The gradient trend represent the relative permeability of Gas to water reservoir condition very well. The negative intercept only implies that the expected value of the dependent (log Swr) will less than 0(zero), where all predicted variables are set to 0(zero). The poroperm relationship for this test case (gas and water) is $K = 10^{1.3828} * e^{0.76824\phi}$ .While the saturation height for this test case is 1.5m above the Gas-water contact. There is increase in log J with a resulting increase in log (Swr) as shown in figure 4.

3.3 Case Three (Air and Kerosene)

Shows the plot of log (J) versus log Swr for Air and kerosene condition. For a typical reservoir that has Air and Kerosene, the interfacial tension **σ** is 24 dynes/cm^3 and the contact angle for reservoir condition is Cos***P*** = 1.0. The derived intercept (a) is 0.648881 with gradient (b), 0.76824. This implies there is significant relationship between the independent (log J) and the dependent (log Swr) variable.The gradient trend represent the relative permeability of Air to Kerosene reservoir condition very well. The negative intercept only implies that the expected value of the dependent (log Swr) will less than 0(zero), where all predicted variables are set to 0(zero).The poroperm relationship for this test case (Air and kerosene) is $K = 10^{0.648881} * e^{0.76824\phi}$ .While the saturation height for this test case is 0.8m above the air-kerosene contact. There is increase in log J with a resulting increase in log (Swr) as shown in figure 4.5.

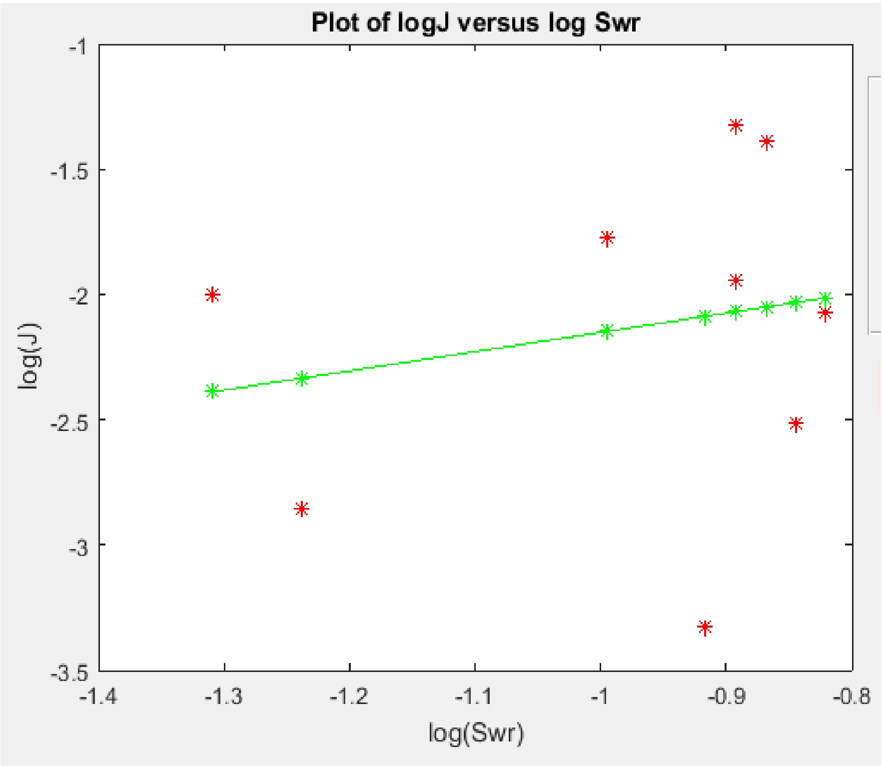

Figure 4: The Plot Log J vs Log Swr for Gas and Water Contact.

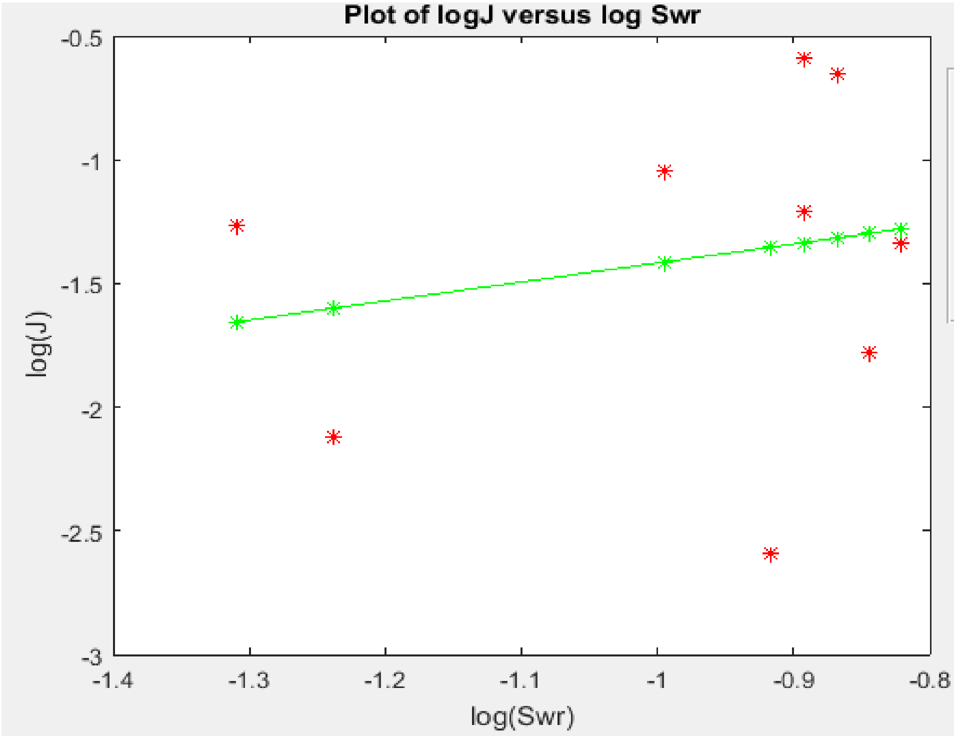

Figure 5: The plot log J vs log Swr for Air and Kerosene contact.

3.4 Case Four (Kerosene and Brine)

Shows the plot of log (J) versus log Swr for kerosene and Brine condition. For a typical reservoir that has Kerosene and Brine, the interfacial tension **σ** is 48 dynes/cm^3 and the contact angle for reservoir condition is Cos***P*** = 0.866. The derived intercept (a) is 1.1981 with gradient (b), 0.76824. This implies there is significant relationship between the independent (log J) and the dependent (log Swr) variable.The gradient trend represent the relative permeability of Kerosene to Brine reservoir condition very well. The negative intercept only implies that the expected value of the dependent (log Swr) will less than 0(zero), where all predicted variables are set to 0(zero).

The poroperm relationship for this test case (kerosene and Brine) is $K = 10^{1.1981} * e^{0.76824\phi}$ .While the saturation height for this test case is 1.3m above the kerosene-brine contact. There is increase in log J with a resulting increase in log (Swr) as shown in figure 6.

3.5 Case Five (Air and Brine)

Indicate that the zone is half non-wettability and half wettability. The non-wettability is the Air filled zone that mixed with wettability zone Brine (I. e. water containing more dissolved inorganic salt than typical seawater). For a typical reservoir that has Air and Brine, the interfacial tension **σ** is 72dynes/cm^3 and the contact angle for reservoir condition is Cos***P*** = 1.0. The derived intercept (a) is 1.74749 with gradient (b), 0.76824. This implies there is significant relationship between the independent (log J) and the dependent (log Swr) variable. The gradient trend represent the relative permeability of Air to Brine reservoir condition very well. The negative intercept only implies that the expected value of the dependent (log Swr) will less than 0(zero), where all predicted variables are set to 0(zero). The poroperm relationship for this test case (kerosene and Brine) is $K = 10^{1.74749} * e^{0.76824\phi}$ .While the saturation height for this test case is 1.3m above the air-brine contact. There is increase in log J with a resulting increase in log (Swr) as shown in figure 7.

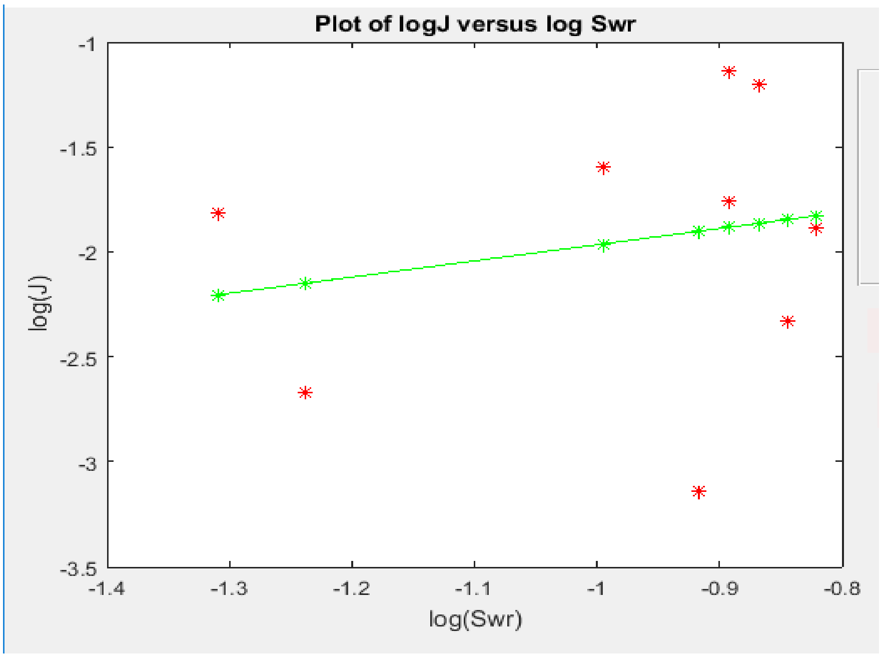

Figure 6: The plot log J vs log Swr for Kerosene and Brine contact.

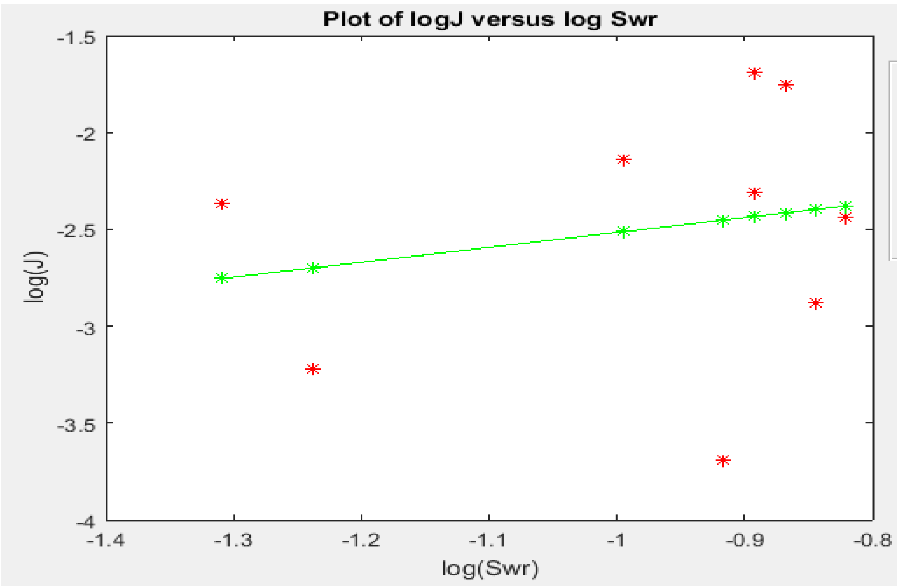

Figure 7: The Plot Log J vs Log Swr for Air and Brine Contact.

3.6 Case Six (Air and Mercury)

Shows the plot of log (J) versus log Swr for Air and mercury condition. For a typical reservoir that has Air and mercury, the interfacial tension **σ** is 480 dynes/cm^3 and the contact angle for reservoir condition is Cos***P*** = 0.765. The derived intercept (a) is -3.37673 with gradient (b), 0.76824. This implies there is significant relationship between the independent (log J) and the dependent (log Swr) variable. The gradient trend represent the relative permeability of Air to mercury reservoir condition very well. The negative intercept only implies that the expected value of the dependent (log Swr) will less than 0(zero), where all predicted variables are set to 0(zero). The poroperm relationship for this test case (Air and mercury) is $K = 10^{3.37673} * e^{0.76824\phi}$ .While the saturation height for this test case is 1.3m above the Air and

Mercury contact. There is increase in log J with a resulting increase in log (Swr) as shown in figure 8 .

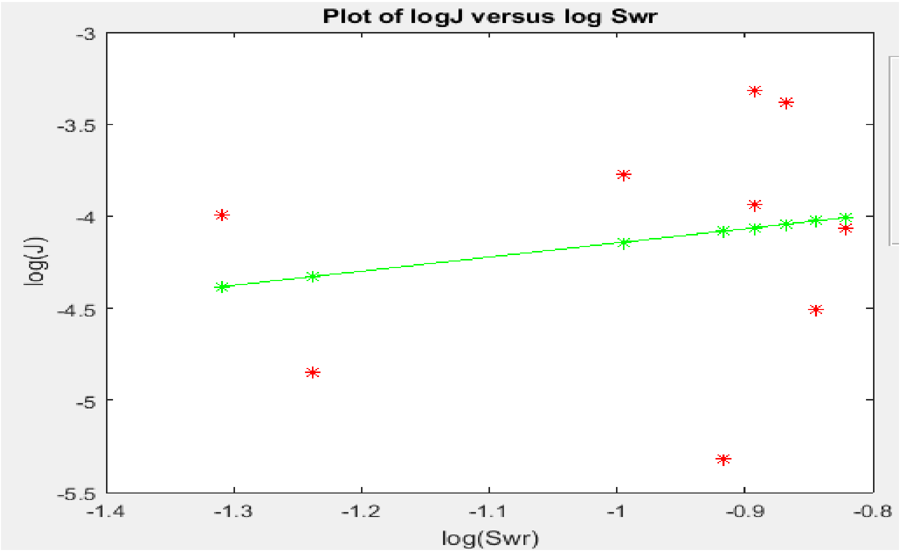

Figure 8: The plot log J vs log Swr for Air and Mercury contact.

4.0 Conclusion and Recommendation

The research has demonstrated the effectiveness of an analytical model based on the Leverett J-function in predicting saturation-height and porosity-permeability relationships in hydrocarbon reservoirs. By accounting for factors such as capillary pressure, and height above the free water level, the model offers a powerful tool for understanding and predicting reservoir behavior and performance. This tool is particularly useful for geoscientists in the field of reservoir characterization, as it can enhance hydrocarbon recovery and optimize reservoir management.

The test results from the six distinct cases examined within this study effectively demonstrate the robustness of the analytical model of the Leverett J-Function in evaluating saturation-height and porosity-permeability relationships in hydrocarbon reservoirs.

In each of the cases, namely Oil/Water, Gas/Water, Air/Kerosene, Kerosene/Brine, Air/Brine, and Air/Mercury, a significant relationship between the independent (log J) and the dependent (log Swr) variable was observed. The gradients obtained in all instances represent the relative permeability of the first fluid to the second fluid under reservoir conditions accurately. The negative intercepts indicate that the expected value of the dependent variable (log Swr) will be less than zero when all predicted variables are set to zero.

The poroperm relationships (K) for each case were successfully derived, demonstrating the reliability of the proposed model across various fluid systems. Additionally, the model accurately estimated the saturation heights above the contact point for each fluid pair, ranging from 0.8m to 1.5m, thereby affirming its utility in real-world reservoir situations.

The consistent increase in log J correlating with an increase in log (Swr) across all cases further attests to the predictive accuracy of the Leverett J-Function model. The test results unequivocally validate that the Leverett J-function model can be effectively utilized to predict and understand the interplay between saturation, height, porosity, and permeability in hydrocarbon reservoirs across a diverse range of fluid pairs.

However, it must be noted that while these conclusions are derived from a well-controlled, well-documented set of test cases, additional studies involving more complex and variable reservoir conditions will further strengthen these conclusions and validate the model in a wider array of real-world scenarios.

Declarations

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Conflict of Interest Statement:

We, the authors of this manuscript, Fajana Akindeji. Opeyemi (Ph.D.), and Oluwadare Sunday. B., hereby declare that we have thoroughly examined our professional and personal circumstances and have found no conflict of interest concerning the research work presented in this manuscript titled " Analytical Modeling of Leverett J-Function Evaluation of Saturation-Height and Poroperm Relationships in Hydrocarbon Reservoirs"

We affirm that:

1. We have not received any form of financial support related to this research work that could influence the design, execution, interpretation, or presentation of the study. This includes but is not limited to, employment, consultancies, honoraria, patent applications/registrations, grants, or other funding.
2. We have no personal relationships or affiliations that could have appeared to influence the work reported in this paper. This encompasses family ties, friendships, or conflicts with other companies or organizations that could affect the study.

Availability of data and material:

The data and material used for this research including the source codes are readily available and included in the manuscript

Authors' contributions:

Fajana Akindeji. Opeyemi (Ph.D.): designed the project, data interpretation, and did the write-up
Oluwadare Sunday Benjamin: developed and write the programming codes for the research

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Appendix: Source Codes

```
% Extract values from GUI input fields
phiStr = get(handles.phiinput, 'string');
phi = str2num(phiStr);
kStr = get(handles.kinput, 'string');
k = str2num(kStr);
pcStr = get(handles.pcinput, 'string');
pc = str2num(pcStr);
swStr = get(handles.swinput, 'string');
sw = str2num(swStr);
sigStr = get(handles.sigmainput, 'string');
sig = str2num(sigStr);
costhStr = get(handles.cosinput, 'string');
costh = str2num(costhStr);
swirrStr = get(handles.swirrinput, 'string');
swirr = str2num(swirrStr);
% Calculate J and swr
j = (pc.*sqrt(k./phi))/(sig*costh);
swr = sw - swirr;
% Calculate logarithm values for further computations
J = log(j);
Swr = log(swr);
% Plotting the initial graph
plot(Swr, J, '*r');
xlabel('log(Swr)')
ylabel('log(J)')
title('Plot of logJ versus log Swr')
% --- Executes on button press in fit_pushbutton.
%%%%%%%%%%%%%%%%%%%%%%%%%
% Calculate basic statistics for linear regression
l = length(Swr);
sx = sum(Swr);
sy = sum(J);
xs = Swr.^2;
sxs = sum(xs);
xy = Swr.*J;
sxy = sum(xy);
% Solve for linear regression parameters
A = [l sx; sx sxs];
B = [sy; sxy];
X = inv(A)*B;
% Parse the regression parameters into separate variables
g = length(X);
for i = 1:g
   a(i) = X(i);
end
% Calculate the fitted line
f = a(1) + a(2)*Swr;
% Store the fitted line data into the handles structure
handles.f = f;
% Plotting the fitted line along with the original data
plot(Swr, J, '*r', Swr, f, '-*g');
xlabel('log(Swr)')
ylabel('log(J)')
title('Plot of logJ versus log Swr')
```