

Circular Regression for the Prediction of Ocean Wave Direction Using wrapped Double Lomax Neural Network

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Abstract

Ocean wave direction is a significant circular parameter in oceanographic and marine engineering applications and for coastal engineering, offshore structure design, navigation and marine environmental assessment, accurate prediction of wave direction is a crucial part of the process. The direction of a wave is a circular quantity as the directions (360^0) are the same direction. Conventional regression methods can thus generate "wrong" predictions and errors near the circular boundary. To solve this problem, this work introduces a new ocean wave peak direction prediction model named Wrapped Double Lomax Neural Network (WDLNN), which is based on a CNN architecture. The model proposed here uses the Wrapped Double Lomax activation function derived using wrapping method that was applied to Double Lomax Distribution and it was converted into circular probability model named as Wrapped Double Lomax Distribution(WDLD), which is a probability-based activation function that can be used to model the directionality, and is applied to the CNN framework. The peak direction is converted to its sinusoidal components, sine and cosine, so that the direction can be represented with a circular structure, but without it being discontinuous at 0^0 and 360^0 . The ocean wave observations in the coastal dataset from Mooloolaba is used to evaluate the model. Various combinations of the location and scale parameters of the Wrapped Double Lomax are explored to study the effect on prediction performance. Circular prediction error measures are used to evaluate model performance: Mean Squared Error (MSE), Root Mean Squared Error (RMSE), Mean Absolute Error (MAE) and a circular coefficient of determination (R^2). This proposed framework WDLNN shows a systematic application of circular probability distributions for deep learning and its ability to accurately predict the direction of the ocean waves.

Keywords: Ocean wave direction, Wrapped Double Lomax Distribution(WDLD), Wrapped Double Lomax Neural Network (WDLNN), peak direction.

1. Introduction

The ocean waves are essential elements of the marine environment and are essential for ocean transportation, coastal processes, offshore engineering, navigation and oceanographic research. For offshore and coastal structures design and operation, accurate analysis and prediction of ocean wave characteristics are therefore crucial. Wave direction is an important wave characteristic since it indicates the direction of wave energy movement and can be used to gain an understanding of wave propagation and coastal processes.

Wave direction is distinct from regular linear variables since it is a circular variable due to angular measures. A direction is measured on a circular scale as a number, typically in ranges from 0^0 to 360^0 and is considered the same direction if (0^0) and (360^0) are the same. This means that standard statistical and machine-learning approaches for linear variables might not capture the behavior of observations taken in a direction. For instance (359^0) and (1^0) differ by (358^0) when they are considered as linear quantities, but only by (2^0) when they are measured as angles. This boundary effect can result in wrong performance metrics of the error and a misrepresentation of predicting performance.

The use of circular statistics is an appropriate mathematical framework for analyzing directional data. In circular analysis, a unit circle is used to represent observations, and many trigonometric transformations are used to maintain the angular structure of the observations sine and cosine are among the most common. Additional, specially developed probability distributions for circular variables may offer better representations of directional observations. The main advantage of wrapped distributions is that linear probability distributions can be mapped onto a circular domain by summing its density over the integers of a circular period.

A widely considered probability distribution for the positive valued data is the Lomax distribution. The wrapped forms of the Lomax distribution offer flexible ways of modeling circular data. One of these is the Wrapped Double Lomax Distribution (WDL), a circular representation that is based on the Double Lomax distribution. It is flexible in describing directionality of behaviour, and is therefore an appropriate choice to be included in computational models for circular prediction.

The use of machine-learning and deep-learning techniques to solve problems in oceanographic and environmental prediction has been growing in the last few years. Originally designed for image-processing applications, Convolutional Neural Networks (CNNs) can also be employed to capture local patterns and relationships in structured numerical observations. Conventional neural-network activation functions, however, are typically intended for inputs that are in a continuous space of real numbers, and fail to explicitly consider the probabilistic properties of the circular domain. This is what encourages the design of neural architectures that have an activation function derived from a suitable circular probability model. In the technique of wrapped, an attempt is made to make a new circular probability model arise as Wrapped Double Lomax Distribution (WDL). To assess the characteristics of population of the new circular probability model by the trigonometric moments obtained from the characteristic function of WDL. After then a Wrapped Double Lomax Neural Network (WDLNN) is proposed in this study to predict the direction of

the ocean wave peak. The proposed method is based on CNN with a modified Wrapped Double Lomax activation function. The activation function itself is based on Double Lomax probability density, but given a value on the range $[0, 2\pi]$. This allows for the inclusion of a distribution based mechanism in the process of taking in the input features by the neural network. To resolve the circular situation of the prediction target, the observed peak direction (θ) is converted to two continuous components, sine and cosine. The CNN thus predicts the sine and cosine terms instead of the angle itself. The inverse tangent function is then used to reconstruct the predicted direction, and were translated into interval 0° to 360° . This representation does not suffer from the artificial discontinuity at $(0^\circ/360^\circ)$ like other representations do.

A dataset of ocean wave observations from the Mooloolaba coastal wave monitoring system is used to investigate the proposed WDLNN using an ocean wave dataset that contains observations for Peak Direction. All the available numerical variables within the data set are treat as predictor variables and Peak Direction as the circular response variable. Pre-processing is applied to normalize the input variables before feeding to the CNN. Input variables are normalized prior to being fed into the CNN. The effect of varying the parameters of the Wrapped Double Lomax are explored systematically.

The Mean Squared Error (MSE), Root Mean Squared Error (RMSE), Mean Absolute Error (MAE), and circular version of the coefficient of determination, $((R^2))$, are used to assess the performance of the proposed models. Apart from a numerical evaluation, the parameter combinations are also compared graphically and the parameter setting, which gives the best prediction performance, is determined.

This work's main contribution is to integrate a Wrapped Double Lomax probability-based activation function, as part of a CNN framework, for circular regression of ocean wave direction. The proposed approach is a hybrid of probability-based distributions for directional modeling and pattern learning ability of deep neural networks. The study thus presents a computational framework to capture the circular characteristics of ocean waves

and shows the capability of WDLNN to deal with direction prediction problems.

The rest of this paper is structured as follows. The ocean wave data and the ocean wave preprocessing and circular representation method are described in Section 2. The Wrapped Double Lomax distribution and the proposed WDLNN architecture is provided in Section 3. The experimental results are discussed and compared with the different (α) and (β) parameter combinations in the Section 4. The results and potential future research directions are given in Section 5.

2. Motivation

Jammalamadaka and Sengupta presented the fundamental concepts of circular statistics, focusing on statistical methods for data measured in angular form[1], [12]. Mardia and Jupp provided a comprehensive treatment of directional statistics, which deals with observations represented by angles, directions, and points on a circle or sphere.[2]. Yedlapalli et al. investigated the properties and applicability of the Semicircular Reflected Gamma Distribution distribution[3]. Dattatreya Rao et al. showed how linear probability distributions can be converted into circular distributions by wrapping their observations around a circle[4]. Rao et al. transformed the Gamma distribution into a circular distribution through the wrapping technique[5]. Rao and Girija given the fundamental concepts of angular statistics, including statistical methods for observations represented as angles[6]. Girija et al. proposed the Stereographic Lognormal Distribution, which is obtained by transforming the Lognormal distribution into a circular form[7]. Subrahmanyam et al. investigated the wrapped Exponentiated Inverted Weibull distribution for modeling circular data[8]. Sarma et al. studied the characteristic functions of the wrapped Lognormal and wrapped Weibull distributions[9]. Durgamamba et al. investigated a reliability test plan based on the size-biased Lomax distribution[10]. Kantam and Ravikumar studied a limited failure censored life-testing sampling plan for the Burr Type X distribution. The research focused on statistical procedures for life-testing experiments where complete failure information

may not be available.[11]. Rao et al. proposed the Wrapped Lomax Distribution (WLD) as a new circular probability model. The study transformed the Lomax distribution into a circular form using the wrapping technique[13]. Kishore et al. evaluated machine learning algorithms for classifying sleeping disorders and demonstrated the applicability of data-driven techniques to real-world classification problems[14].

Puscasu investigated the integration of artificial neural networks with operational ocean wave prediction models. The study showed that neural networks can be used to emulate complex nonlinear wave interactions and provide fast predictions. The work demonstrates the potential of artificial intelligence techniques for improving the efficiency and accuracy of ocean wave prediction systems[15]. Kim et al. proposed STG-OceanWaveNet, a spatio-temporal neural network for ocean wave prediction. The model incorporates geographic and temporal information to capture the complex spatial and temporal behavior of ocean waves. The study demonstrates the effectiveness of deep learning for improving ocean wave prediction and provides motivation for developing advanced data-driven wave forecasting models[16]. Bennett and Mulligan investigated the role of surface wind fields in predicting directional ocean wave spectra during Hurricane Sandy. Their study showed that accurate wind information is important for representing the direction and characteristics of ocean waves[17]. Jain and Deo reviewed the application of neural networks in ocean engineering, including their use in predicting ocean waves and other marine processes. The study demonstrated that neural networks can model complex nonlinear relationships without requiring explicit mathematical formulations[18]. Bindu and Sangita studied the Double Lomax distribution and its applications and investigated the statistical properties of the distribution and demonstrated its usefulness in modeling data with suitable lifetime and reliability characteristics[20]. Punathumparambath focused on obtaining statistical estimates for the probability that one random variable exceeds another when both variables follow the Double Lomax distribution[21]. Altawil introduced a Doubly Truncated Power Lomax Distribution and the model extends the

Power Lomax family by incorporating truncation, providing additional flexibility for modeling bounded or restricted data[22]. Rady et al. investigated the Power Lomax distribution properties and applications[23]. Aljohani examined the statistical behavior of the Lomax distribution and demonstrated its applicability to practical data analysis[24].

3. Methodology of Wrapping

The wrapped circular probability density function $h(\theta)$ corresponding to the density function f of a linear r.v. is defined as

$$h(\theta) = \sum_{k=-\infty}^{\infty} f(\theta + 2k\pi) \quad (3.1)$$

Where $\theta \in [0, 2\pi)$

3.1 Wrapped Double Lomax Distribution

The two parameter Double Lomax distribution with location parameter a_{sh} and as scale parameter b_{sc} defined as

$$f(x) = \frac{1}{2\sigma \left(1 + \left|\frac{x - a_{sh}}{b_{sc}}\right|\right)^2} \quad (3.2)$$

where $x \geq 0$, $a_{sh} > 0$, $b_{sc} > 0$

The cdf of the Double Lomax distribution is known as

$$F(x) = \begin{cases} \frac{1}{2b_{sc} \left(1 + \left|\frac{x - a_{sh}}{b_{sc}}\right|\right)}, & x \leq a_{sh} \\ 1 - \frac{1}{2b_{sc} \left(1 + \left|\frac{x - a_{sh}}{b_{sc}}\right|\right)}, & x > a_{sh} \end{cases} \quad (3.3)$$

The pdf of Wrapped Double Lomax model is derived by applying the wrapping technique to equation (3.2) and it is

$$h(\theta) = \sum_{k=-\infty}^{\infty} f(\theta + 2k\pi) = \sum_{k=-\infty}^{\infty} \frac{1}{2b_{sc} \left(1 + \left|\frac{\theta + 2k\pi - a_{sh}}{b_{sc}}\right|\right)^2} \quad (3.4)$$

The cdf of wrapped Double Lomax model is $H(\theta)$

$$= \begin{cases} \sum_{k=0}^{\infty} \frac{1}{2b_{sc} \left(1 + \left|\frac{\theta + 2k\pi - a_{sh}}{b_{sc}}\right|\right)}, & x \leq \mu \\ \sum_{k=0}^{\infty} 1 - \frac{1}{2b_{sc} \left(1 + \left|\frac{\theta + 2k\pi - a_{sh}}{b_{sc}}\right|\right)}, & x > \mu \end{cases} \quad (3.5)$$

Also, it is observed that the series is convergent.

3.2 Graphs

The graph of the probability density function of WDLD (Linear and Circular Representations) are depicted in Figure 3.1 and Figure 3.2 respectively

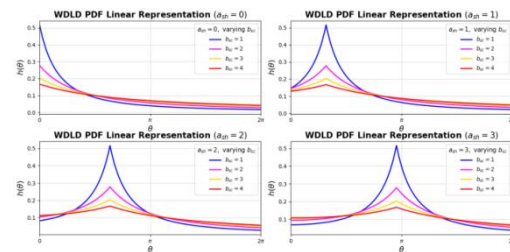


Figure 3.1: PDF of Wrapped Double Lomax distribution (Linear Representation)

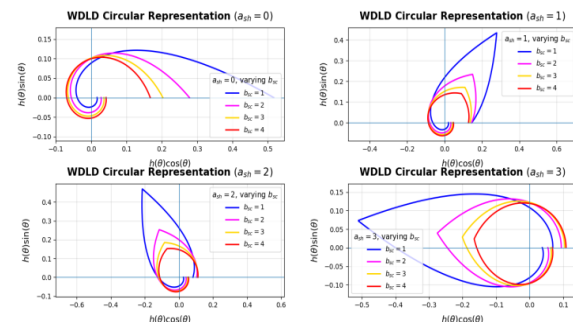


Figure 3.2: PDF of of Wrapped Double Lomax distribution (Circular Representation)

The characteristic function of Wrapped Double Lomax distribution(WDLD) is

$$\phi(t) = \int_0^{\infty} e^{itx} \frac{1}{2\sigma \left(1 + \left|\frac{x - a_{sh}}{b_{sc}}\right|\right)^2} dx \quad (3.6)$$

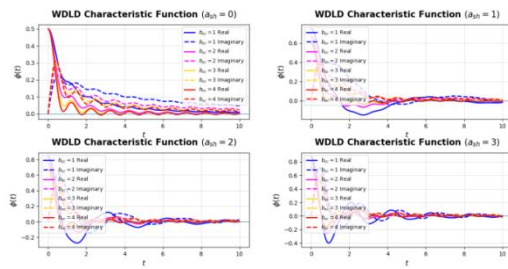


Figure 3.3: Characteristic function of WDL
The graphs for real and imaginary parts for different values of a_{sh} and b_{sc} of the characteristic function for WDL in Figure 3.3

3.3 Trigonometric Moments of WDL

The characteristics of WDL model are derived through the trigonometric moments obtained from the equation (3.6). The real and the imaginary parts of ϕ_p are defined as $\alpha_p = E(\cos p\theta)$ and $\beta_p = E(\sin p\theta)$, $p \in \mathbb{Z}$

4. Ocean Wave Direction Dataset Study

A dataset of ocean-wave observations (Mooloolaba) 01-2017 to 06 – 2019 that can be used to investigate relationships between wave characteristics and directional behavior, which contain wave and environmental parameters[19].

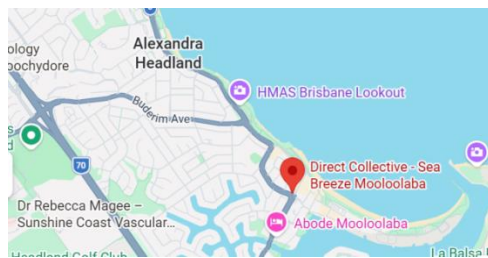


Figure 4.1: Geographical location of the the ocean wave observation at Mooloolaba study area, Queensland, Australia.

Pre-processing is applied to normalize the input variables before feeding to the CNN. Input variables are normalized prior to being fed into the CNN. The effect of varying the parameters of the Wrapped Double Lomax are explored systematically.

Attributes	Use
Hs(Significant Wave Height)	Average height of the highest one-third of waves. Usually measured in meters (m)
Hmax(Maximum Wave Height)	Height of the largest individual wave observed during the measurement period
Tz(Zero-Crossing Wave Period)	Average time between successive waves crossing the mean water level in the same direction. Usually seconds (s).
Tp(Peak Wave Period)	Wave period corresponding to the peak of the wave energy spectrum. Usually seconds (s)
Peak Direction (Peak Wave Direction)	Direction from which the waves with the maximum spectral energy are coming. Usually measured in degrees (°)
SST(Sea Surface Temperature)	Temperature of the ocean surface, usually measured in °C.

Table 4.2: Attributes of Dataset

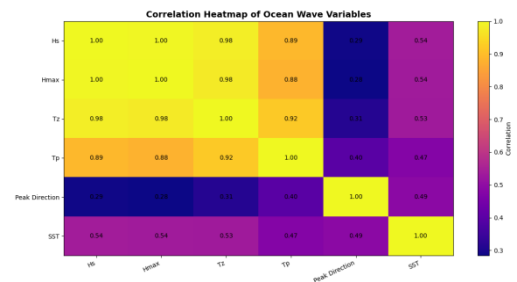


Figure 4.2: Heatmap of Ocean Wave Attributes

The correlation coefficients between the 6-variables of ocean wave and environment are shown in Figure 4.2. The variables wave height, wave maximum and wave period, wave mean and wave period are extremely well correlated with each other (0.88\), highlighted in bright yellow). In contrast, the correlations of Peak Direction with these parameters of height and period are very weak or slightly negative.

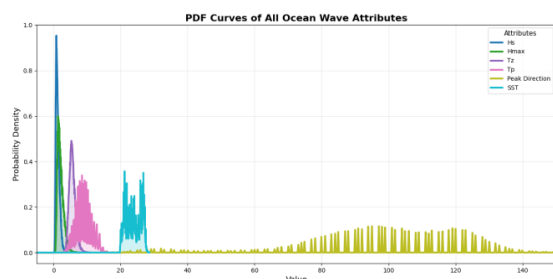


Figure 4.3: PDF Curves of Ocean Wave Attributes

Figure 4.3 reveal difference in the probability distributions and ranges of the ocean wave properties. Most of the wave heights are relatively small, with most of the Hs and Hmax concentrated at lower values, whereas the Tz and Tp show distributions over moderate values with distinct peaks. In contrast to SST, which is more narrowly distributed, Peak Direction is more broadly distributed, showing the variability in wave direction. The overall figure shows very large differences in the distribution, spread and variation of the ocean wave attributes with the largest range and variation in the distribution of Peak Direction.

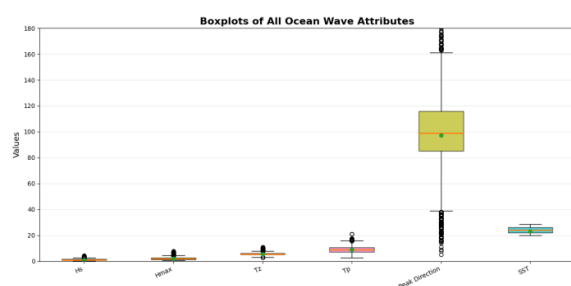


Figure 4.4: Boxplots of Ocean Wave Attributes

Figure 4.4 shows the distribution and variability in the ocean wave attributes. The ranges of the variables Hs, Hmax, Tz, and Tp are relatively small and several outliers suggest extreme wave conditions from time to time. Peak Direction is the most widely spread and has significant outliers, indicating high directional variability. SST is more tightly distributed with less outlier than other

variables, showing more consistency in sea-surface temperature. Directional distribution of Hs, Hmax, Tz, Peak Direction, SST and other wave parameters are shown in Figure 4.5.

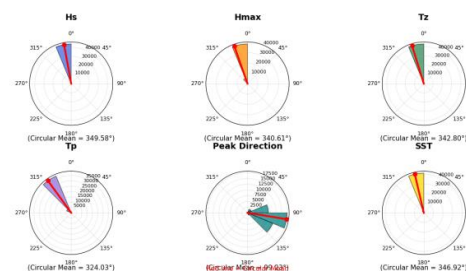


Figure 4.5: Rose Diagrams of Ocean Wave dataset Attributes

A dominant direction of occurrence is seen in the variables Hs, Hmax and Tz, with the main concentration being in the range of 315° to 345°. The Peak Direction scatter plot reveals a higher directionality spread with the observations ranging from about 225° to 315°, whereas SST has a significant concentration of observations in the 330°–350° directions. The majority of the wave and environmental parameters have a strong directionality towards the northwest sector with varying degrees of directionality between the parameters.

5. Wrapped Double Lomax Neural Network

A new activation based Neural Network: is introduced in this study, combination of CNN with Wrapped Double Lomax pdf as activation function called Wrapped Double Lomax Neural Networks(WDLNN). It combines the flexibility of a neural network with the Wrapped Double Lomax (WDL) distribution, which is suitable for modeling angular data. This model designed for circular regression problems, such as predicting ocean wave direction.

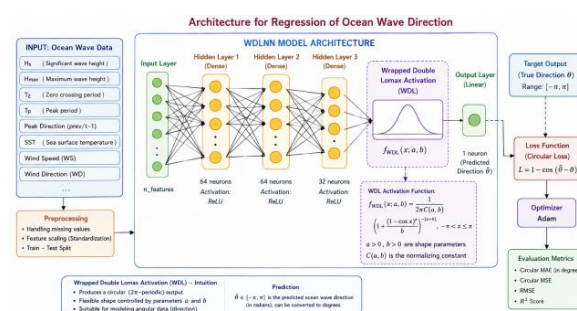


Figure 5.1: WDLNN Architecture

Steps involved in WDLNN are

Step1-Summation Function: If ground water quality parameter is x_j with weight w_j and bias is b_j

Then, for a neuron j ,

$$Nr_j = \sum_{j=1}^m w_j x_j + b_j \quad (5.1)$$

Step2-Activation Function: The Wrapped Double Lomax model p.d.f. (3.4) is the activation function. It is bounded, smooth and numerically stable due to no exponential function present in it.

The derivative required for back propagation is equation (3.5).

The dataset[19] is used to investigate the proposed WDLNN using an ocean wave dataset that contains observations for Peak Direction. All the available numerical variables within the data set are treat as predictor variables and Peak Direction as the circular response variable.

From Table 5.1, the results display the performance of 25 models tested for the different combinations of the parameters a_{sh} (1–5) and b_{sc} (1–5). The model performance varies across the combinations, with MSE ranging from 380.98 to 447.58, RMSE from 19.52 to 21.16, and MAE from 14.20 to 15.64. The R^2 values range from 0.394 to 0.484. The best performance is obtained for $a_{sh} = 3$ and $b_{sc} = 2$, with the lowest MSE (380.9794), RMSE (19.51869), MAE (14.20319), and highest R^2 (0.484088). Hence, the models with the combination $a_{sh} = 3$ and $b_{sc} = 2$ give the optimum prediction performance.

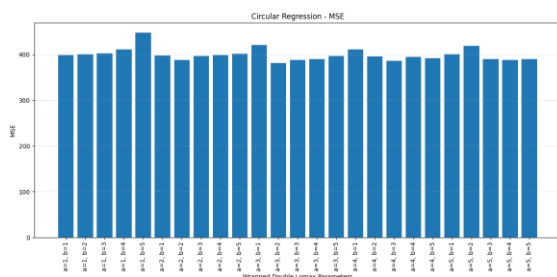


Figure 5.2: Comparison of MSE-Values for different values of a_{sh} and b_{sc} in WDLNN Circular Regression

From Figure 5.2, the relatively small variation in MSE demonstrates that the proposed WDLNN regression model provides consistent predictive performance across different paraThe circular

regression results shown in the figure indicate that the Mean Squared Error (MSE) remains relatively stable across the different Wrapped Double Lomax parameter combinations, with values approximately ranging from 380 to 450. The lowest MSE is observed for $a_{sh} = 2$ and $b_{sc} = 2$ indicating that this parameter combination provides the best

prediction performance among the evaluated models. In contrast, the highest MSE occurs around $a_{sh} = 1$, $b_{sc} = 5$ suggesting comparatively lower prediction accuracy for this combination. Overall, the relatively small variation in MSE demonstrates that the proposed Wrapped Double Lomax-based circular regression model provides consistent predictive performance across different parameter settings, while the $a_{sh} = 2$ and $b_{sc} = 2$ configuration can be considered the most suitable based on the minimum MSE.

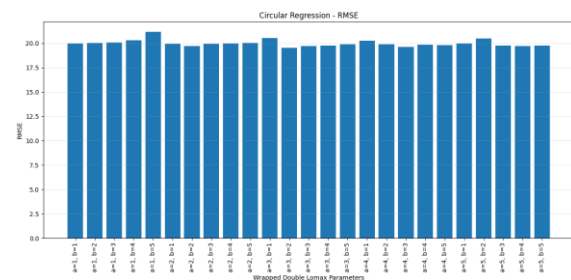


Figure 5.3: Comparison of RMSE-Values for different values of a_{sh} and b_{sc} in WDLNN Circular Regression.

The RMSE values for the wrapped double Lomax parameters a_{sh} and b_{sc} are fairly consistent, ranging from approximately 19.5 to 21.1. The lowest RMSE is around 19.5, while the highest is about 21.1. Since a lower RMSE indicates better prediction accuracy, the parameter combination with the lowest RMSE performs best, while the combination with the highest RMSE performs slightly worse.

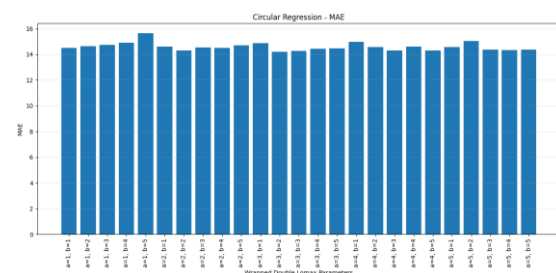


Figure 5.4: Comparison of MAE-Values for different values of a_{sh} and b_{sc} in WDLNN Circular Regression.

Figure 5.4 shows that the MAE values are generally consistent, mostly ranging from about 14 to 15 across the different wrapped double Lomax parameters. The highest MAE is around 15.5, while the lowest is about 14.1.

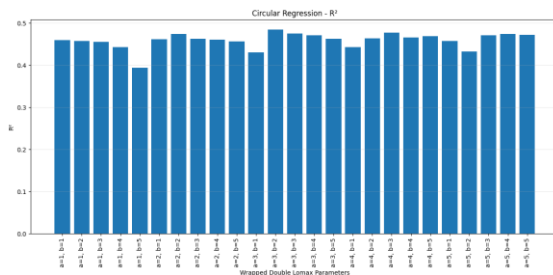


Figure 5.5: Comparison of R^2 Scores for different values of a_{sh} and b_{sc} in WDLNN Circular Regression

Most values are high, around 0.44–0.48 in Figure 5.5. The best R^2 is about 0.48, indicating a strong fit. The lowest is around 0.39.

6. Conclusion

Larger b_{sc} produce more angular dispersion and less concentration around the mean direction. a_{sh} mainly affects the direction, b_{sc} mainly controls the concentration/variability of the circular data. The WDLNN model performance varies across the combinations, with maximum MSE(447.58), RMSE(21.16), and MAE(15.64) The R^2 (0.484). The best performance is obtained for $a_{sh} = 3$ and $b_{sc} = 2$, with the lowest MSE (380.9794), RMSE (19.51869), MAE (14.20319), and highest R^2 (0.484088). Hence, the models with the combination $a_{sh} = 3$ and $b_{sc} = 2$ give the optimum prediction performance. MAE values are generally consistent and the differences are small, indicating similar prediction performance across the models. The best R^2 is about 0.48, indicating a strong fit. The models perform consistently well, with only small differences between parameters. The further scope of this probability model is to study about the stereographic projection and developing a neural network model.

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Circular_Mean							Resultant	
Parameters	Direction	Trigonometric Moments					Length (ρ)	Variance_V0
a_{sh}	b_{sc}	Deg	$\alpha 1$	$\alpha 2$	$\beta 1$	$\beta 2$		
0	1	0	0.37855	0.201958	0	0	0.37855	0.62145
0	2	0	0.201958	0.08323	0	0	0.201958	0.798042
0	3	0	0.124127	0.044167	0	0	0.124127	0.875873
0	4	0	0.08323	0.027006	0	0	0.08323	0.91677
0	5	0	0.059286	0.01809	0	0	0.059286	0.940714
1	1	57.29578	0.204532	-0.08404	0.318539	0.18364	0.37855	0.62145
1	2	57.29578	0.109118	-0.03464	0.169942	0.075681	0.201958	0.798042
1	3	57.29578	0.067066	-0.01838	0.104449	0.040161	0.124127	0.875873
1	4	57.29578	0.044969	-0.01124	0.070035	0.024556	0.08323	0.91677

1	5	57.29578	0.032032	-0.00753	0.049888	0.016449	0.059286	0.940714
2	1	114.5916	-0.15753	-0.13201	0.344215	-0.15284	0.37855	0.62145
2	2	114.5916	-0.08404	-0.0544	0.18364	-0.06299	0.201958	0.798042
2	3	114.5916	-0.05166	-0.02887	0.112868	-0.03343	0.124127	0.875873
2	4	114.5916	-0.03464	-0.01765	0.075681	-0.02044	0.08323	0.91677
2	5	114.5916	-0.02467	-0.01182	0.053909	-0.01369	0.059286	0.940714
3	1	171.8873	-0.37476	0.193914	0.053421	-0.05643	0.37855	0.62145
3	2	171.8873	-0.19994	0.079915	0.0285	-0.02326	0.201958	0.798042
3	3	171.8873	-0.12288	0.042408	0.017517	-0.01234	0.124127	0.875873
3	4	171.8873	-0.0824	0.02593	0.011745	-0.00755	0.08323	0.91677
3	5	171.8873	-0.05869	0.017369	0.008366	-0.00505	0.059286	0.940714
4	1	229.1831	-0.24744	-0.02938	-0.28649	0.199809	0.37855	0.62145
4	2	229.1831	-0.13201	-0.01211	-0.15284	0.082344	0.201958	0.798042
4	3	229.1831	-0.08113	-0.00643	-0.09394	0.043697	0.124127	0.875873
4	4	229.1831	-0.0544	-0.00393	-0.06299	0.026718	0.08323	0.91677
4	5	229.1831	-0.03875	-0.00263	-0.04487	0.017897	0.059286	0.940714
5	1	286.4789	0.10738	-0.16946	-0.363	-0.10987	0.37855	0.62145
5	2	286.4789	0.057288	-0.06984	-0.19366	-0.04528	0.201958	0.798042
5	3	286.4789	0.03521	-0.03706	-0.11903	-0.02403	0.124127	0.875873
5	4	286.4789	0.023609	-0.02266	-0.07981	-0.01469	0.08323	0.91677
5	5	286.4789	0.016817	-0.01518	-0.05685	-0.00984	0.059286	0.940714

Table 3.1: Trigonometric moments of WDLD for different values of a_{sh} and b_{sc}

The Table 3.1 shows how the a_{sh} and b_{sc} . The circular statistics are changed by the following parameters. When $a_{sh} = 0$, the direction of the circular mean is 0° . Progressively decreases direction from 57.3° to 286.5° . For all values of a_{sh} increasing b_{sc} . On its own, 1 to 5 will reduce the resulting length (ρ) of the data.) and enhances the spread of the data.

Parameters		Central Trigonometric moments						Circular_SD
a_{sh}	b_{sc}	α_1^*	α_2^*	β_1^*	β_2^*	γ_1^0	γ_2^0	σ_0
0	1	0.37855	0.201958	0	0	0	0.469765	1.393848
0	2	0.201958	0.08323	0	0	0	0.128073	1.788684
0	3	0.124127	0.044167	0	0	0	0.057263	2.042768
0	4	0.08323	0.027006	0	0	0	0.032075	2.229866
0	5	0.059286	0.01809	0	0	0	0.020428	2.377133

1	1	0.37855	0.201958	0	0	0	0.469765	1.393848
1	2	0.201958	0.08323	0	0	0	0.128073	1.788684
1	3	0.124127	0.044167	0	0	0	0.057263	2.042768
1	4	0.08323	0.027006	0	0	0	0.032075	2.229866
1	5	0.059286	0.01809	0	0	0	0.020428	2.377133
2	1	0.37855	0.201958	0	0	0	0.469765	1.393848
2	2	0.201958	0.08323	0	0	0	0.128073	1.788684
2	3	0.124127	0.044167	0	0	0	0.057263	2.042768
2	4	0.08323	0.027006	0	0	0	0.032075	2.229866
2	5	0.059286	0.01809	0	0	0	0.020428	2.377133
3	1	0.37855	0.201958	0	0	0	0.469765	1.393848
3	2	0.201958	0.08323	0	0	0	0.128073	1.788684
3	3	0.124127	0.044167	0	0	0	0.057263	2.042768
3	4	0.08323	0.027006	0	0	0	0.032075	2.229866
3	5	0.059286	0.01809	0	0	0	0.020428	2.377133
4	1	0.37855	0.201958	0	0	0	0.469765	1.393848
4	2	0.201958	0.08323	0	0	0	0.128073	1.788684
4	3	0.124127	0.044167	0	0	0	0.057263	2.042768
4	4	0.08323	0.027006	0	0	0	0.032075	2.229866
4	5	0.059286	0.01809	0	0	0	0.020428	2.377133
5	1	0.37855	0.201958	0	0	0	0.469765	1.393848
5	2	0.201958	0.08323	0	0	0	0.128073	1.788684
5	3	0.124127	0.044167	0	0	0	0.057263	2.042768
5	4	0.08323	0.027006	0	0	0	0.032075	2.229866
5	5	0.059286	0.01809	0	0	0	0.020428	2.377133

Table 3.2: Central Trigonometric moments of WDLD for different values of a_{sh} and b_{sc}

In Taable 3.2, Y_1^0 is Skewness and Y_2^0 is Kurtosis

a_{sh}	b_{sc}	MSE	RMSE	MAE	R^2
1	1	398.9231	19.97306	14.49787	0.459789
1	2	400.5058	20.01264	14.63271	0.457645
1	3	402.3857	20.05955	14.7308	0.4551
1	4	411.7642	20.29197	14.91256	0.4424
1	5	447.5834	21.15617	15.6376	0.393894
2	1	397.4596	19.93639	14.61436	0.461771

2	2	388.31	19.70558	14.29558	0.474161
2	3	396.9498	19.9236	14.54922	0.462461
2	4	398.6102	19.96523	14.5052	0.460212
2	5	401.2326	20.03079	14.69794	0.456661
3	1	420.9681	20.51751	14.86105	0.429936
3	2	380.9794	19.51869	14.20319	0.484088
3	3	388.0088	19.69794	14.25461	0.474569
3	4	390.4411	19.75958	14.43154	0.471275
3	5	396.5904	19.91458	14.4832	0.462948
4	1	411.3983	20.28296	14.95903	0.442895
4	2	395.7503	19.89347	14.57401	0.464085
4	3	385.8093	19.64203	14.3141	0.477547
4	4	394.9204	19.8726	14.60982	0.465209
4	5	392.3581	19.80803	14.29884	0.468679
5	1	400.3776	20.00944	14.5719	0.457819
5	2	419.4776	20.48115	15.02779	0.431954
5	3	390.4814	19.7606	14.37317	0.47122
5	4	388.556	19.71182	14.34753	0.473827
5	5	390.2612	19.75503	14.37126	0.471518

Table 5.1: Error Metrics-Values for different values of a_{sh} and b_{sc} in WDLNN Circular Regression