Journal of Harbin Engineering University
ISSN: 1006-7043
Vol 44 No. 7
July 2023

Modelling of Blood Flow Through Stenosed Tube with Permeable Walls in the Presence of Magnetic Field and Nanoparticles, Considering Blood as Elastic Viscous Fluid

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Abstract-This research focuses on the mathematical analysis of nanoparticles in blood flow via constrained vessels with permeable walls under the effect of an external magnetic field. The effects of the magnetic field on the effects of nanoparticles are investigated in this paper. Unfavourable results could arise from the beginning of nanoparticles in blood. The governing equations of the hypothetical fluid are resolved by the Laplace and Finite Hankel transforms. The formulas for the temperature, flow rate, and velocity distributions in the confined region of tubes are determined analytically. The results are shown graphically for various parametric values using MATLAB. It is found that magnetic fields improve blood flow in confined tubes and, in addition, that the velocity of fluid is reduced for nanoparticles with higher volume fractions. For medical professionals, this study would be useful to understand the circumstances of nanoparticle delivery in blood flow.

Keywords: Elastic-viscous fluid, Nanoparticles, Viscosity, Constrained vessel, Magnetic field, Permeability.

1. Introduction

Blood rheology and its characteristics are observed to be closely linked to cardiovascular system constriction, which results in an abundance of deaths across borders. Therefore, it is crucial to comprehend and to keep a watchful eye on health difficulties that result from heart conditions. Heart disease and stroke are two persistent diseases in the modern world. Cardiovascular disease is the leading cause of death. Acute myocardial infarction (AMI), sometimes known as a heart attack, affects around 1.1 million people annually and causes nearly half of all deaths in the United States. Numerous theoretical and numerical studies have been conducted globally to investigate the root causes and effects of the advancement of stenosis in the arteries that surround the heart muscle. Over 75% of deaths are thought to be caused by this disorder, with "Arteriosclerosis" being one of the main contributors. Predominantly, atherosclerosis refers to an accumulation of low-density lipoproteins along their walls. This aggregation points to expensive angioplasty and by-pass surgery procedures. Since atherosclerosis is the primary cause of cardiovascular disorders, there is an urgent need to develop techniques for early identification and acute treatment of the condition in order to stop it from progressing to a heart attack. Heart attacks are brought on by obstructions in the coronary arteries, which carry oxygen-rich blood to the myocardium, the muscle wall of the heart. The "Stenosis" model for aberrant arterial lumen expansion, which is a result of intravascular plaques, is shown in Figure 1. A non-Newtonian model of the flow of blood via confined arteries was invented by Misra et al. [1]. We require a deeper comprehension of the elements that influence blood flow in such restricted configurations because authentic atherosclerotic lesions are both symmetric and uneven. Ji-Huan He's Homotopy perturbation method methodology is further explained [2]. Given that blood is a non-Newtonian couple stress

fluid, Pralhad and Schultz explored the modelling of artery stenosis and applied it to blood disorders [3]. In general, experimental and computational models of the flow processes become more complex due to the stenosis' surface flaws. A significant amount of scientific work was earlier put into examining the features of blood flow across blocked arteries in light of these complications [4,6,7]. By conceptualising of the blood as a two-fluid model, assessed the impact of body acceleration and slip velocity at the wall. The multiple stenoses effects for the power law fluid model with viscosity variation [5]. The movement of blood through arteries has been researched by numerous researchers. In 2013, how blood flow in a stenosed artery behaved in non-Newtonian fluids was investigated [8]. When examining blood as a two-fluid model in 2014, Roy and Sinha evaluated the impact of body acceleration and slide velocity against the wall. Viscosity fluctuation in a power law fluid model with multiple stenosis [9]. The transport of MHD couple stress fluid through peristaltic contraction in a porous media under the impact of heat transfer and slip effects was covered by Sankad and Nagathan [11]. Using a magnetic field and a slip velocity, Kumari et al. [12] investigated the peristaltic flow properties of blood through a stenotic artery. Under the impact of a chemical reaction and activation energy, Ellahi et al. [13] described the peristaltic blood flow of a couple stress fluid suspended with nanoparticles. Liu and Liu [14] investigated the 2D pulsatile blood flow through a tapered artery with stenosis using a non-Newtonian fluid model and assessed the blood flow in tapered stenosed arteries under the impact of heat and mass transfer. The research of blood flow in the situation of blood clots and arterial elasticity in the presence of stenosis has been shed new light by Asha and Srivastava [15]. The geometry and techniques for solving the model under various circumstances were examined by Awad [16]. A perturbation technique was used to formulate coupled partial differential equations, make them dimensionless, and reduce them to ordinary differential equations. For the blood flow and lipid concentration profiles, respectively, the nonlinear ordinary differential equations were analytically solved with some relevant parameters. Typically, metals, oxides, carbides, or carbon nanotubes are employed as the basis for the nanoparticles in nanofluids. Investigation of recent articles reveals that non-Newtonian nanofluid investigations have received little attention over the past few decades, despite the significance of the nanofluid issue. Numerous difficult diseases have already begun to spread as a result of the blood's streaming nanoparticles. The introduction of nanoparticles has revolutionised our understanding of the progression of atherosclerosis along tube walls. As a result, it is now important for scientific research to analyse the blood flow in stenosed arteries using nanoparticles. The purpose of the study is to examine how a magnetic field affects the introduction of nanoparticles into blood flow through stenosed tubes with porosity. In this work, we have created a model of blood under pair tension moving through a narrow tube. It has been noted that nanotechnology has gained acceptance as a multidisciplinary discipline in which all of the stack-holders have contributed significantly to the development of novel therapies, treatments, and diagnostic methods. Blood flow that is magnetohydrodynamic (MHD) under the influence of a magnetic field has been researched. The erratic flow of blood as an incompressible and elasto-viscous fluid is precisely solved with the aid of Laplace and Hankel Transforms. Additionally, a theoretical mathematical model was carried out to investigate the blood flow properties through a stenotic artery with a magnetic field when nanoparticles experiencing temperature effects are present.

2. Formulation Of The Problem

It is hypothesised that the blood is a non-Newtonian Elastico-viscous fluid with nanoparticles of variable viscosity μ and density ρ moving through an artery with a cylindrical form and length l_0. The cylindrical polar coordinate system (r, θ, z) is considered with the z-axis parallel to the artery's axis. Here, r = 0 taken as the tube's symmetric axis. The stenosis in an arterial section with symmetric form is represented by the following equation,

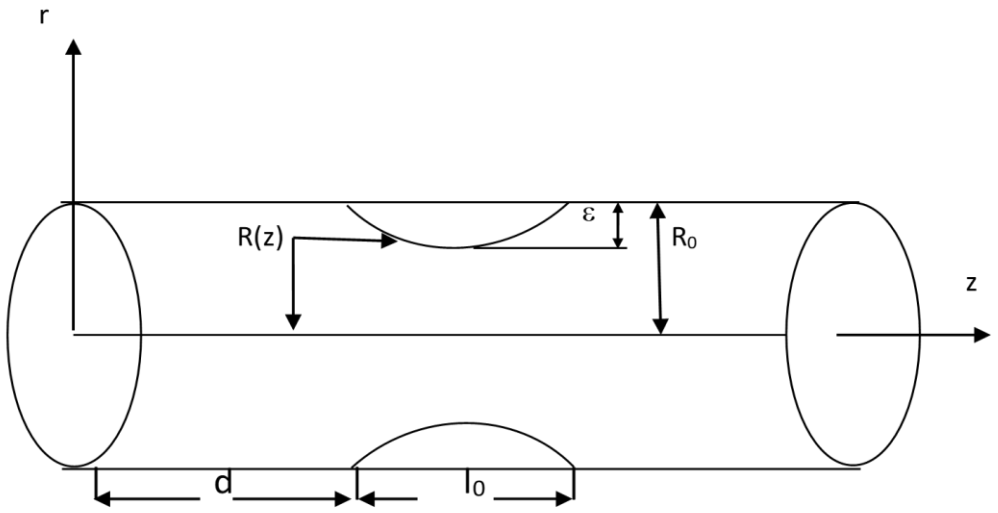


Fig. 1 Haldar and Ghosh (1994) [4] characterise the geometry of stenosis.

$$\frac{R(z)}{R_0} = 1 - A[l_0^s(z-d) - (z-d)^s],$$
$$d \le z \le d + l_0 \qquad (1)$$

where

$$A = \frac{\epsilon}{R_0 l_0^s} \frac{s}{\overline{s(s-1)}} \frac{}{s-1}$$

and $s \ge 2$ being stenosis shape parameter, d represents position of stenosis, l_0 as length of stenosis, maximum height of stenosis is located at $z = d + \frac{l_0}{2}$, $R(z)$ represents radius of stenosed vessel, R_0 as radius of normal or unconstructed artery and $\frac{\varepsilon}{R_0} \ll 1$.

The symbols and notations used for the above model and the followings equations is given in the table below:

Symbols	**Description of symbols**
$R(z)$	Radius of artery in stenosed vessel *(m) (6.31· 10^{-3})*
R_0	Radius of normal or unconstructed artery *(m)*
d	Position of stenosis
l_0	Length of stenosis
ε	Maximum height of stenosis
s	Stenosis shape parameter *(s=2)*
μ	Viscosity$(kg/m.s)$ *(5.1 · 10^{-3})*
ρ_{nf}	Density (kg/m^3)*(1050)*
a_0	Constant amplitude
a_1	Amplitude of pulsatile component
f	Heart pulse frequency *(Hz)*
μ_1	Elastic-viscous coefficient
w	Axial velocity component (m/s)
σ	Electrical conductivity
K	Permeability constant
θ	Non-dimensional Temperature profile
$(\rho\gamma)_{nf}$	heat capacitance
μ_f	fluid's constant viscosity
G_r	local temperature Grashof number
g	Acceleration due to gravity (m/s^2) *(9.8)*
Q_0	constant heat absorption or heat generation
τ	Shear Stress *(Pa)*
t	Time (sec)*(0.917)*
T	Temperature (K)
Q	Volumetric flow rate

ζ	Womersley parameter
r	Radial coordinate
t'	Time variable *(sec)*
J_0	Bessel's function of first kind and zeroth order
J_1	Bessel's function of first kind and first order

2.1 Governing Equations

The governing equations assumed for the axial velocity and temperature of nanoparticles in pulsatile blood flow in stenotic tube under the influence of magnetic field through porous medium:

$$\left(\mu + \mu_1 \frac{\partial}{\partial t'}\right)\nabla^2 w - \sigma\,\beta_0^2 w - \nabla p - \frac{\mu}{K} w + g(\rho\gamma)_{nf}(T - T_\circ) = \rho_{nf}\frac{\partial w}{\partial t'} \quad (2)$$

$$\frac{1}{(\rho c_p)_{nf}} Q_0 + \alpha_{nf}\,\frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)\right] = \frac{\partial T}{\partial t'} \quad (3)$$

where $\Delta^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)$, is the linear operator, $-\nabla p = a_0 + a_1 Cos\,\omega t$, is the pressure gradient, a_0 being constant amplitude & a_1 being amplitude of pulsatile component, $\omega = 2\pi f$, f is the heart pulse frequency, w is the axial velocity component, t' is the time variable, T is the temperature, $(\rho\gamma)_{nf}$ is the heat capacitance, Q_0 is constant heat absorption or heat generation, K is permeability constant, ρ_{nf} is density. and r is radial coordinate.

2.2 Boundary conditions

$\frac{\partial T}{\partial r} = 0$ at $r = 0$ and $T = T_0$ at $r = R(z)$

Defining, $T = [\theta + 1]T_o$; $t' = tt_o$; $r = yR_o$

As stated by, the thermophysical characteristics are

$\nabla p = a_0 + a_1 Cos(\omega t)$, where a_0 is the steady state part of pressure gradient and a_1 is the amplitude of oscillatory part, $\omega = 2\pi f$, f is the heart pulse frequency; $(\rho\gamma)_{nf} = (1-\phi)(\rho c_p)_{nf} + \phi\left(\rho c_p\right)_s$; $\alpha_{nf} = \frac{k_{nf}}{(\rho\gamma)_{nf}}$;

$\frac{\mu_{nf}}{\mu_f} = \frac{1}{(1-\phi)^{2.5}}$; μ_f, the fluid's constant viscosity.

$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_{fs}$; ρ_f is the base fluid's density.

2.3 Non-dimensional form

The equations [2,3] in non-dimensional form are

$$\left(1 + \Omega\frac{\partial}{\partial t}\right)\nabla^2 w - L^2 w + N(a_o + a_1\cos t) - \frac{R_0^2}{K} w + G_r\theta = \zeta^2\frac{\partial w}{\partial t} \quad (4)$$

$$\frac{1}{y}\frac{\partial}{\partial y}\left(y\frac{\partial\theta}{\partial y}\right) + \beta = \zeta_1^2\frac{\partial\theta}{\partial t} \quad (5)$$

where

$\Omega = \frac{\mu_1}{\mu_{nf} t_0}$; $\beta = \frac{R_0^2 Q_0}{k_{nf} T_0}$; $\zeta_1^2 = \frac{R_0^2}{\alpha_{nf} T_0 t_0}$; $L^2 = \frac{\sigma\beta_0^2 R_0^2}{\mu_{nf}}$;

$\zeta^2 = \frac{R_0^2\rho_{nf}}{t_0\mu_{nf}}$; $N = \frac{R_0^2}{\mu}$;

$G_r = (\rho\gamma)_{nf}\frac{R_0^2 T_0}{\mu}$, local temperature Grashof number; L is the permeability parameter;

ζ is the Womersley parameter.

Additionally, it is presumed that just the heart's pumping action exists for values of $t < 0$ and for $t = 0$, the "instant pressure gradient" is caused by arterial blood flow. i.e. $-\nabla p = a_0 + a_1$.

Boundary conditions are:

$\frac{d\theta}{dy} = 0$ where $y = 0$; $\theta = 0$ when $y = R(z)$

3. Methodology

Applying the algorithm of "Laplace and Hankel transform" on [4,5] with boundary conditions, we get the equations for axial velocity and temperature:

$$w(y,t) = \frac{2R}{R_0}\sum_{n=1}^{\infty}\left[\frac{a_0}{z_1}\left(\left(\frac{1}{m}\right)e^{-\frac{z_1 t}{m}}\right) + a_1\left(\frac{1}{mh_1}e^{-\frac{z_1 t}{m}} + \frac{1}{m^2 + z_1^2}\left(z_1\cos(t) - m\sin(t) - z_1 e^{-\frac{z_1 t}{m}}\right)\right) + G_r B\left\{\frac{1}{\lambda_n^2 m z_1}e^{-\frac{z_1 t}{m}} + \frac{1}{\zeta_1^2 m_1 z_1}\left(1 - e^{-\frac{z_1 t}{m}}\right) + \frac{(1-\zeta_1^2)}{\lambda_n^2(z_1 - m m_1)}\left(e^{-m_1 t} - e^{-\frac{z_1 t}{m}}\right)\right\}\right]\frac{J_0(\lambda_n y)}{J_1\left(\lambda_n\frac{R}{R_0}\right)} \quad (7)$$

$$\theta(y,t) = \frac{2R_0\beta}{R\zeta_1^2}\sum_{n=1}^{\infty}\frac{1}{\lambda_n^3}\left[\zeta_1^2 + (1-\zeta_1^2)e^{-\frac{\lambda_n^2}{\zeta_1^2}t}\right]\frac{J_0(\lambda_n y)}{J_1\left(\lambda_n\frac{R}{R_0}\right)} \tag{8}$$

In a similar manner, we may use an expression for volumetric flow rate:

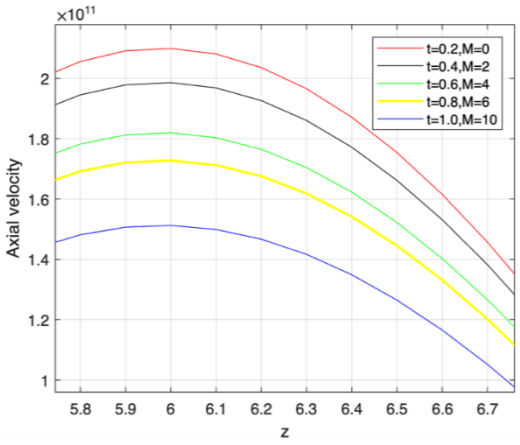
$$\begin{aligned} Q(y,t) &= 2\pi\int_0^a y.w(y,t)dy \\ &= \frac{4\pi R}{R_0}\sum_{n=1}^{\infty}\left[\frac{a_0}{z_1}\left(\left(\frac{1}{m}\right)e^{-\frac{z_1 t}{m}}\right) + a_1\left(\frac{1}{mh_1}e^{-\frac{z_1 t}{m}} + \frac{1}{m^2+z_1^2}\left(z_1\cos(t) - m\sin(t) - z_1 e^{-\frac{z_1 t}{m}}\right)\right)\right. \\ &\quad \left. + G_r B\left\{\frac{1}{\lambda_n^2 m z_1}e^{-\frac{z_1 t}{m}} + \frac{1}{\zeta_1^2 m_1 z_1}\left(1-e^{-\frac{z_1 t}{m}}\right) + \frac{(1-\zeta_1^2)}{\lambda_n^2(z_1-mm_1)}\left(e^{-m_1 t} - e^{-\frac{z_1 t}{m}}\right)\right\}\right]\frac{J_1(\lambda_n a)}{J_1\left(\lambda_n\frac{R}{R_0}\right)} \end{aligned}$$

As well, the shear stress is

$$\begin{aligned} \tau_{rz} &= \frac{2R_0\mu_{nf}}{R\mu_f}\sum_{n=1}^{\infty}\left[\frac{a_0}{z_1}\left(\left(\frac{1}{m}\right)e^{-\frac{z_1 t}{m}}\right) + a_1\left(\frac{1}{mz_1}e^{-\frac{z_1 t}{m}} + \frac{1}{m^2+z_1^2}\left(z_1\cos(t) - m\sin(t) - z_1 e^{-\frac{z_1 t}{m}}\right)\right)\right. \\ &\quad \left. + G_r B\left\{\frac{1}{\lambda_n^2 m z_1}e^{-\frac{z_1 t}{m}} + \frac{1}{\zeta_1^2 m_1 z_1}\left(1-e^{-\frac{z_1 t}{m}}\right) + \frac{(1-\zeta_1^2)}{\lambda_n^2(z_1-mm_1)}\left(e^{-m_1 t} - e^{-\frac{z_1 t}{m}}\right)\right\}\right] \end{aligned}$$

4. Results and Discussion

Through a porous media and a magnetic field, the flow of blood through a stenosed artery is examined while erythrocytes are present. The analytical expressions for axial velocity and shear stress are graphically displayed using the MATLAB platform with the parameters chosen for numerical computations being Hartmann number (M), permeability constant (K), and depending on hematocrit, concentration of nano particles φ, Grashof number G_r. This is done in consideration of the realistic model of blood streaming across stenosed tube. In the aforementioned model, $\rho = 1.05gm/ml$, $\mu = 0.35Poise$, $\omega = 7.4$, is taken into account for numerical calculations.. Stenosis is located at position z = 3 and $l_0 = 4$.



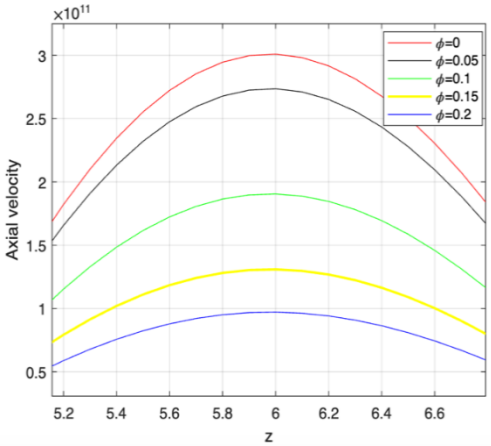
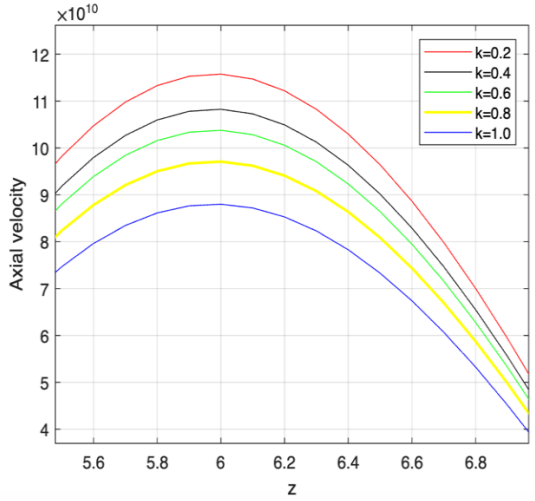


Fig. 2 Axial velocity varies along Z for Fig. 3 Axial velocity with various t & M. values of ϕ

Axial velocity is plotted in Figure 2 to demonstrate the effects of the magnetic field. Axial velocity along the tube's axis reduces when there is no magnetic field. However, the axial velocity decreases as long as a magnetic field is applied and grows progressively; beyond a certain point, it then bounces back. As the magnetic field increases, the rate of increase in axial velocity decreases. Figure 3 displays the change in axial velocity based on various nanoparticles volume fraction values, ϕ. Axial velocity decreases at the mouth of stenosis and then again increases. At the culmination of the stenosis, the axial velocity drops before increasing once more. The velocity profile significantly varies as nanoparticle concentration increases. The rate of change in axial velocity lowers as ϕ facilitates forward due to the existence of a magnetic field and the continuous dissipation of heat throughout the stenosed arteries. It decreases with an increase in value from ϕ = 0.05 to 0.2. The fluctuation in axial velocity regarding porosity K is shown in Figure 4. The permeability constant describes the emphasis in axial velocity.



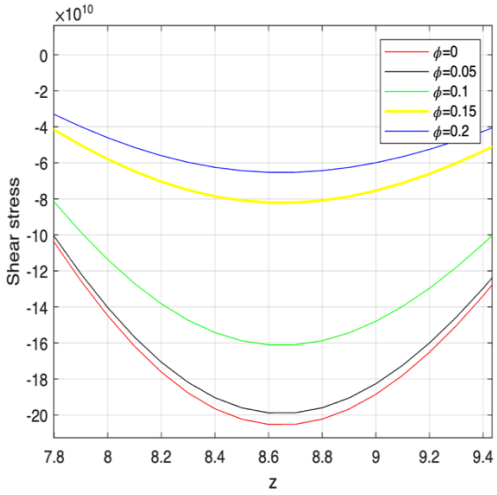
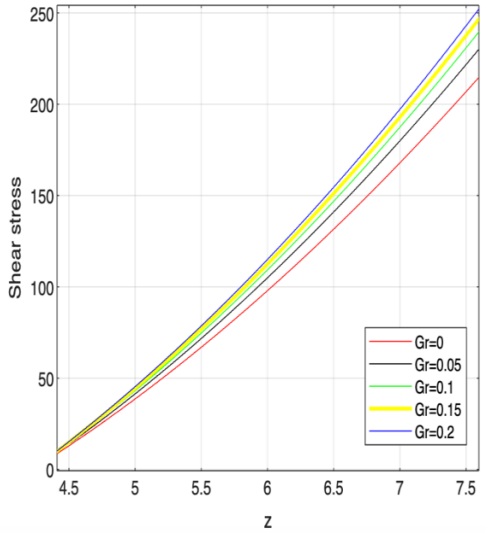


Fig. 4 Axial velocity varies along Z with
Fig. 5 Shear stress varies along

various values of K.
with values of ϕ.

The development of vascular disease is considerably impacted by the shear stress. Shear stress distribution in an artery section with stenosis is shown in Figures 5, 6, and 7. Figure 5 shows how the volume percentage of nanoparticles impacts the shear stress fluctuation. With an increase in nanoparticle concentration, shear stress rises. Figure 6 shows that when the Grashof number increases, shear stress increases. The fluctuation of shear stress with the Hartmann number is seen in Figure 7. More notably than the presence of nanoparticles in the blood flow, the magnetic field has an impact. With a growing magnetic field, the shear stress decreases.



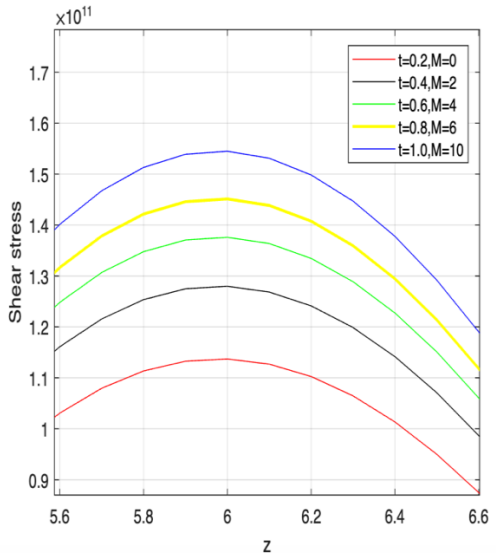


Fig. 6 Shear stress varies along Z for values of Gr.

Fig. 7 Shear stress with values for values of t & M.

5. Conclusion

The present work assessed the impact of an externally imposed magnetic field on the pulsatile blood flow in a constricted porous artery. The present framework offers the ability to calculate the impact of a variety of parameters on various flow characteristics and ensures the impact of various parameters for a more thorough understanding of blood circulation in the human body. The following are significant findings from the analysis:

1. The LDL impact continues as nanoparticle concentration rises. Therefore, the possibility of stenosis along an arterial wall is increased which causes a blocked artery and further leads to blood clot formations.
2. It turns out, the flow in restricted regions is enhanced by magnetic fields, and the viscosity of blood is reduced, thus seizures and cardiac arrest might ensue from this.
3. We can determine the maximum magnetic field intensity at which hypertensive individuals and those with artery blockages can have their blood flow controlled. When applied to pathological circumstances, high strength magnetic fields are detrimental.
4. A rise in blood pressure causes the size of nanoparticles to increase, which leads to quicker heartbeats and blood vessel constriction.
5. Increased shear stress and increased LDL accumulation on tube walls, which affects the liver and the majority of other organs, are brought on by high nanoparticle volume.

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