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Comparison of Fuzzy Queue Models

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Abstract-Queuing theory is the study and modeling of people who wait in lines. Uncertainty is created when fuzziness gravels a perplexing item in regard to hazy information. Queuing systems are used in a wide range of daily contexts, including media transmission systems, computer networks, industrial firms, traffic control, organizations, and other industries and halls of fame. The M/M/1 and M/M/2 fuzzy queue models are compared in the extended scenario. Rate of Arrival andrate of service are taking Fuzzy numbers in triangular form. In order to quantify performance in queuing systems, this study proposes a technique for building the membership function utilizing DSW algorithms. An example is provided to compare the model's validity.

Keywords-Queuing theory, performance measures, α-cut, Triangular Fuzzy numbers DSW algorithm.

INTRODUCTION

Earlier, all models in queuing theory have assumed to follow poission incoming and exponential outgoing times. In various real world scenario, Conventions could be fairly restrictive, particularly the idea that service times are spread exponentially. Compared to the more common crisp queues, fuzzy queues are substantially more accurate.In reality, the terms quick, slow, or moderate, which are used to define the arrival rate and service rate. Many researcher described queuing models under fuzzy like Buckley[1], Negi and Lee[6], ;Li and Lee[5] .

Currently, Chan[2][3] developed (FM/FM/1) &(FM/FM/K) on basis of Zadeh's extension principle, Li and Lee[5] works on two fuzzy queuing system. This paper indicates to follow α-cut method to compare two fuzzy queue models.DSW algorithm is used for various values of α. The solution derive the membership functions of the fuzzy queues by using DSW algorithm..

Definitions

α-cut: If a fuzzy set A is defined on X, for any $\alpha \in [0,1]$, the α- cut of the fuzzy set A is represented by

$A_\alpha = \{x/\mu_A(x) \geq \propto, x \in Z\} = \{L_A(\propto), U_A(\alpha)\}$, $A_\propto$ is a non-empty bounded interval contained in Z , $l_A(\propto)$ and $U_A(\propto)$ represent the lower and upper bound of ∝-cut of A respectively .

Triangular Fuzzy numbers:

$$\mu_{\tilde{A}}(x) = \begin{cases} x-a_1/a_2-a_1 & \text{for } x<a_1 \\ a_3-x/a_3-a_2 & \text{for } a_1 \leq x \leq a_2 \\ 0 & \text{otherwise} \end{cases}$$

Fuzzy interval arithmetic:

Let Z_1 and Z_2 be two interval numbers defined by ordered pairs of real numbers with lower and upper bounds.

$Z_1=[x,y]$

$Z_2=[u,v]$

General property of arithmetic with symbol *, where *=[+,-,×,÷] symbolically. The operation

$Z_1 * Z_2=[x,y]*[u,v]$ represents another interval.

$Z_1+Z_2=[x+u ,y+v]$

$Z_1-Z_2=[x-v ,y-u]$

$Z_1 \times Z_2=[\min.(xu,xv,yu,yv),\max.(xu,xv,yu,yv)]$

$Z_1 \div Z_2=[x,y][1/v,1/u]$ provide $uv \neq 0$

$$\alpha[x,y] = \begin{cases} \alpha x,\alpha y & \alpha>0 \\ \alpha y,\alpha x & \alpha<0 \end{cases}$$

DSW ALGORITHM:

One of the suitable techniques for using intervals at different α -cut levels in establishing membership functions is the (Dong, Shah, and Wong) algorithm[23,24,25]. The modification of the extension principle for continuous value fuzzy parameters is simplified by the DSW method. On the real line, such hazy numbers are specified[10,11]. By applying discrimination to each domain of the fuzzy variable, it avoids irregularity in the output membership function and can stop the typical interval analysis approach from spreading the resultant functional expression[26]. Any membership function that has a continuous α-cut curve from α =0 to 1 is allowed. Assume that the membership function for the chosen -cut level needs to be extended for a single input mapping given by y=f(x) .

The following phases make up the DSW algorithm.

1. Select the value of the α- cut in the range [0,1].
2. Locate the membership function intervals in the input that match this.
3. Determine an interval for the outcome of the membership function for the chosen α -cut level using conventional binary interval operations.
4. Repeat steps 1-3 for different values of to finish the solution's α-cut representation.

SOLUTION PROCEDURE

Integrating the M/M/1 queuing model with fuzzy logic requires redefining rate of arrival and rate of service as fuzzy numbers[12,13]. Fuzzy numbers are commonly represented as triangular or trapezoidal numbers, due to their mathematical and computational simplicity. Suppose that we have an M/M/1 queuing system, where both the arrival rate (λ) and service rate (μ) are represented as fuzzy triangular numbers[14,15]. A triangular fuzzy number A can be represented as (a_1, a_2, a_3), where a_2 is the most probable value (mode), and [a_1, a_3] is the range of possible values[16,17]. Let's say that λ is represented as a triangular fuzzy number (2,3,4) customers/minute, and μ is represented as a triangular fuzzy number (6,7,8) customers/minute[18,19.20]. To calculate the performance measures for this fuzzy M/M/1 queuing system, we would perform fuzzy arithmetic operations on λ and μ using the extension principle or alpha-cut method [21,22].

The mathematical formulation of an M/M/1 queue system incorporating fuzzy logic would require fuzzy arithmetic and the concept of alpha-cut representation of fuzzy numbers.

Let's consider λ and μ to be fuzzy triangular numbers. We can represent $\lambda = (\lambda_1, \lambda_2, \lambda_3)$ and $\mu = (\mu_1, \mu_2, \mu_3)$.

The alpha-cut representation for a triangular fuzzy number A = (a_1, a_2, a_3) is given as:

$A^\alpha = [a_1 + \alpha(a_2 - a_1), a_3 - \alpha(a_3 - a_2)]$ for $0 <= \alpha <= 1$

The alpha-cuts of λ and μ can be represented as:

$\lambda^\alpha = [\lambda_1 + \alpha(\lambda_2 - \lambda_1), \lambda_3 - \alpha(\lambda_3 - \lambda_2)]=[\alpha+2,4-\alpha]$

$\mu^\alpha = [\mu_1 + \alpha(\mu_2 - \mu_1), \mu_3 - \alpha(\mu_3 - \mu_2)]=[\alpha+6,8-\alpha]$

Using these, we can compute the alpha-cut of ρ = λ / μ (utilization) as:

$$\rho^\alpha = \left[\frac{\lambda_1+\alpha(\lambda_2-\lambda_1)}{\mu_3-\alpha(\mu_3-\mu_2)}, \frac{\lambda_3+\alpha(\lambda_3-\lambda_2)}{\mu_1+\alpha(\mu_2-\mu_1)}\right]$$

Similarly, the alpha-cut of L_s, L_q, W_s, W_q

$$L^\alpha{}_s = \frac{\rho^\alpha}{1-\rho^\alpha} = \frac{\lambda^\alpha}{\mu^\alpha-\lambda^\alpha}$$

$$L^\alpha{}_q = \frac{(\lambda^\alpha)^2}{\mu^\alpha(\mu^\alpha-\lambda^\alpha)}$$

$$W^\alpha{}_q = \frac{L_q{}^\alpha}{\lambda^\alpha} = \frac{\lambda^\alpha}{\mu^\alpha(\mu^\alpha-\lambda^\alpha)}$$

$$W_s^\alpha = \frac{L_s^\alpha}{\lambda^\alpha} = \frac{1}{\mu^\alpha-\lambda^\alpha}$$

The following results obtained below in table:

∝	λ	μ	L_s	L_q	W_s	W_q
0	[2.0,4.0]	[6.0,8.0]	[0.33,2]	[0.083,1.33]	[0.082,1]	[0.0207,0.665]
0.2	[2.2,3.8]	[6.2,7.8]	[0.39,1.58]	[0.11,0.967]	[0.10,0.72]	[0.028,0.44]
0.4	[2.4,3.6]	[6.4,7.6]	[0.46,1.28]	[0.145,0.723]	[0.13,0.53]	[0.040,0.301]
0.8	[2.8,3.2]	[6.8,7.2]	[0.63,0.88]	[0.247,0.418]	[0.19,0.31]	[0.059,0.149]
1	[3.0,3.0]	[7.0,7.0]	[0.75,0.75]	[0.321,0.321]	[0.25,0.25]	[0.107,0.107]

By this example , we can conclude that L_s value at α=1 is 0.75 and impossible to fall outside [0.33,2.0].
Similarly L_q maximum value is 0.321 and impossible to fall outside [0.083,1.33].

GRAPHICAL REPRESENTATION

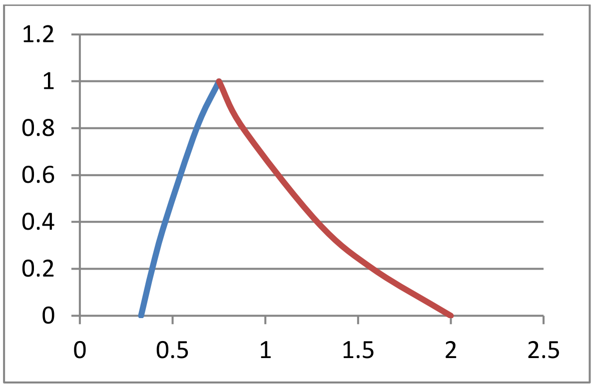

Figure 3.1 L_q

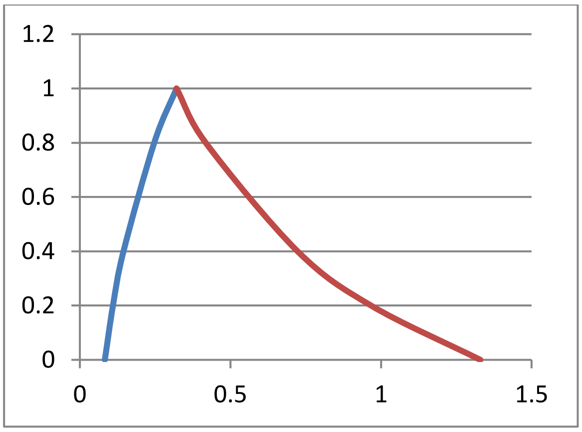

Figure 3.2 L_s

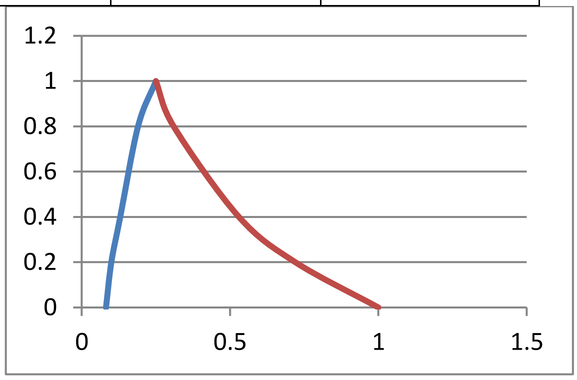

Figure 3.3 W_s

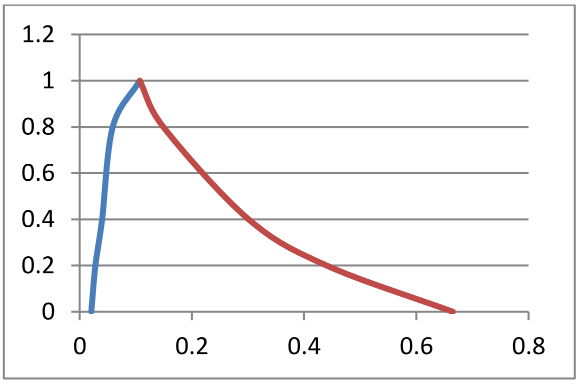

Figure 3.4 W_q

Performance Measures of Fuzzy M/M/2 : The M/M/2 model under fuzzy intervals allows for the calculation of various performance measures that provide insights into the system's behavior and efficiency. Some important performance measures include:

1. **Mean value of customer in system:**

$$L_s^\alpha = \frac{\lambda^\alpha \mu^\alpha (\frac{\lambda^\alpha}{\mu^\alpha})^2}{(2-1)!(2\mu^\alpha - \lambda^\alpha)^2} P_0 + \frac{\lambda^\alpha}{\mu^\alpha}$$

2.Mean value of customer in queue:

$$L_q^\alpha = \frac{\lambda^\alpha \mu^\alpha (\frac{\lambda^\alpha}{\mu^\alpha})^2}{(2-1)!(2\mu^\alpha - \lambda^\alpha)^2} P_0$$

3. Mean timespend in the Queue:

$$W_q^\alpha = \frac{\mu^\alpha (\frac{\lambda^\alpha}{\mu^\alpha})^2}{(2-1)!(2\mu^\alpha - \lambda^\alpha)^2} P_0$$

4. Mean time spend in the System:

$$W_s^\alpha = \frac{\mu^\alpha (\frac{\lambda^\alpha}{\mu^\alpha})^2}{(2-1)!(2\mu^\alpha - \lambda^\alpha)^2} P_0 + \frac{1}{\mu^\alpha}$$

Where P_0

$$P_0 = \frac{1}{1 + \frac{\lambda^\alpha}{\mu^\alpha} + \frac{\left(\frac{\lambda^\alpha}{\mu^\alpha}\right)^2}{2!} \cdot \frac{2\mu^\alpha}{2\mu^\alpha - \lambda^\alpha}}$$

Example: Taking same example as we take in M/M/1 model.(3.4.1)

Let's consider λ and μ to be fuzzy triangular numbers. We can represent $\lambda = (\lambda_1, \lambda_2, \lambda_3)$ and $\mu = (\mu_1, \mu_2, \mu_3)$.

The alpha-cut representation for a fuzzy triangular number $A = (a_1, a_2, a_3)$ is given as:

$A^\alpha = [a_1 + \alpha(a_2 - a_1), a_3 - \alpha(a_3 - a_2)]$ for $0 <= \alpha <= 1$

The alpha-cuts of λ and μ can be represented as:

$\lambda^\alpha = [\lambda_1 + \alpha(\lambda_2 - \lambda_1), \lambda_3 - \alpha(\lambda_3 - \lambda_2)]=[\alpha+2,4-\alpha]$

$\mu^\alpha = [\mu_1 + \alpha(\mu_2 - \mu_1), \mu_3 - \alpha(\mu_3 - \mu_2)]=[\alpha+6,8-\alpha]$

A	Λ	μ	L_q	L_s	W_s	W_q
0	[2.0,4.0]	[6.0,8.0]	[0.002,0.171]	[0.252,0.831]	[0.125,0.251]	[0.0001,0.085]
0.2	[2.2,3.8]	[6.2,7.8]	[0.003,0.112]	[0.285,0.724]	[0.129,0.211]	[0.0007,0.050]
0.4	[2.4,3.6]	[6.4,7.6]	[0.004, 0.07]	[0.319,0.632]	[0.132,0.185]	[0.0011,0.029]
0.8	[2.8,3.2]	[6.8,7.2]	[0.012, 0.03]	[0.401,0.501]	[0.142,0.157]	[0.0037,0.010]
1	[3.0,3.0]	[7.0,7.0]	[0.020,0.020]	[0.448,0.448]	[0.148,0.148]	[0.006, 0.006]

By this example we can conclude that L_s i.e. average number of customers at α=1 is 0.0448 and impossible to fall outside [0.252,0.831].

L_q i.e. mean value of customers in the queue at α=1 is 0.020 and its value impossible to fall outside [0.002,0.171]

Similarly $W_s$and W_q at α=1 is 0.148 and 0.006 respectively.

GRAPHICAL REPRESENTATION :

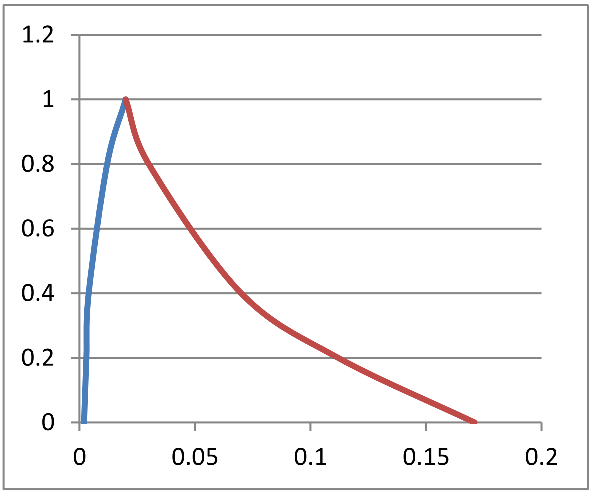

Figure 3.5 L_q

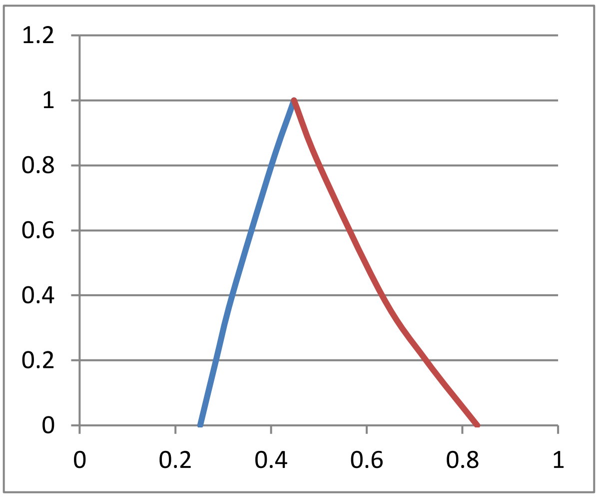

Figure 3.6 L_s

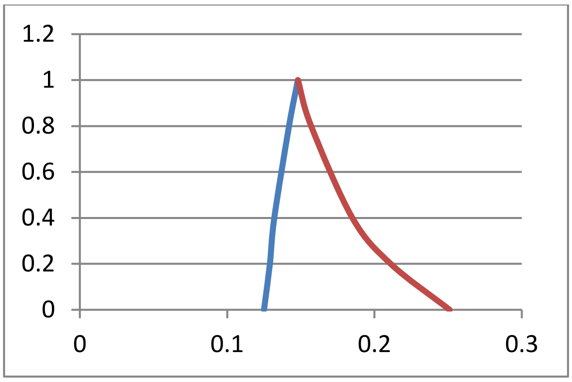

Figure 3.7 W_s

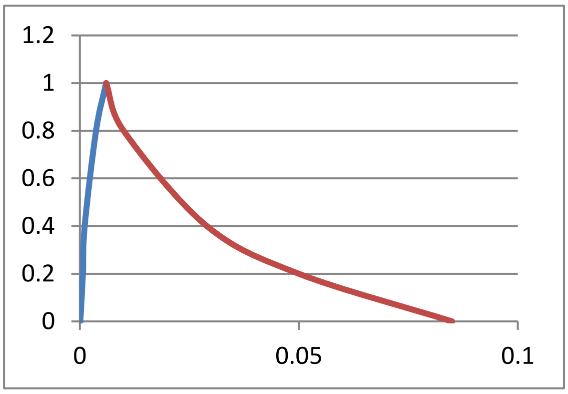

Figure 3.8 W_q

Comparison of M/M/1 &M/M/2 fuzzy models :Comparison by graphs of min. and max. value of different parameters of these models

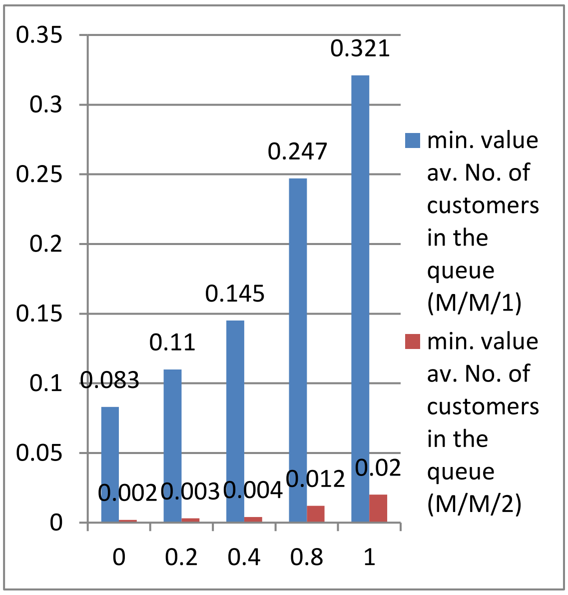

Figure 3.1 min. value L_q

This graph represents that comparison between min. value of average number of customers in the queue of Fuzzy M/M/1 and Fuzzy M/M/2 models. L_q value of M/M/2 model is lesser than M/M/1 fuzzy model with respect to different α-cut values.

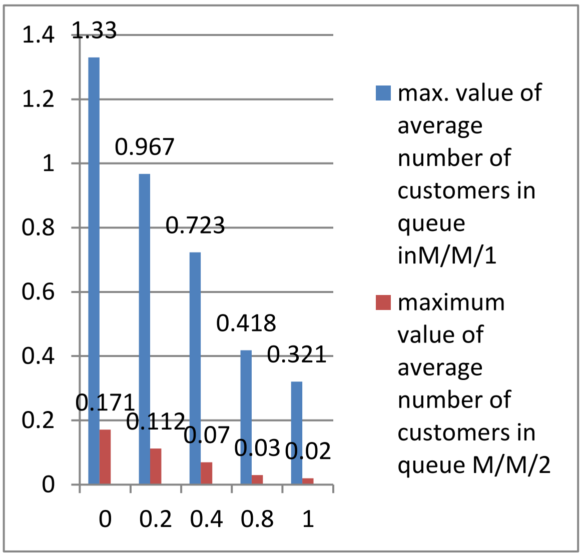

Figure:3.2 max. ofL_q

This graph represents comparison between the maximum value of mean value of customers in the queue of fuzzy M/M/1 & M/M/2 models. This shows that the maximum value of L_q M/M/1 is higher values than that of fuzzy M/M/2 model with respect to different α –cut values.

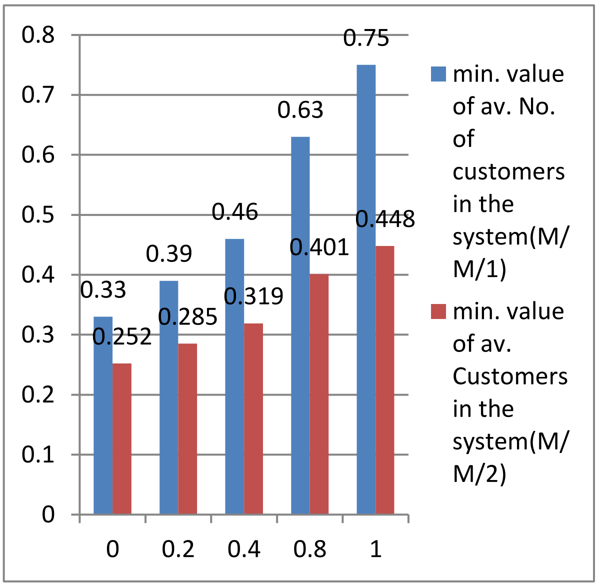

Figure 3.3 min. value of L_s

This graph shows the comparison bwteen minimum values of mean value of customers in system of fuzzy models M/M/1 and M/M/2 model. Minimum Values of L_S M/M/1 model is higher than M/M/2 model at different values of α-cut.

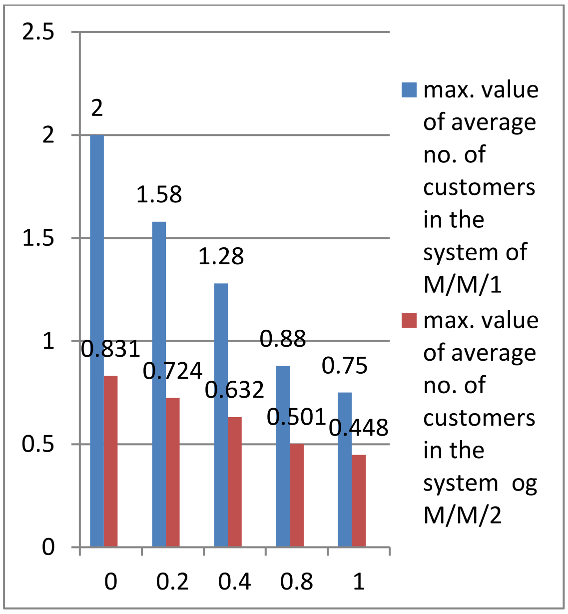

Figure 3.4 max. value of L_s

This graph represents comparison between maximum value of mean value of customers in the system of M/M/1 and M/M/2 fuzzy models. This shows that the value of customers is more in M/M/1 in comparison to M/M/2 fuzzy model with respect to α-cut value.

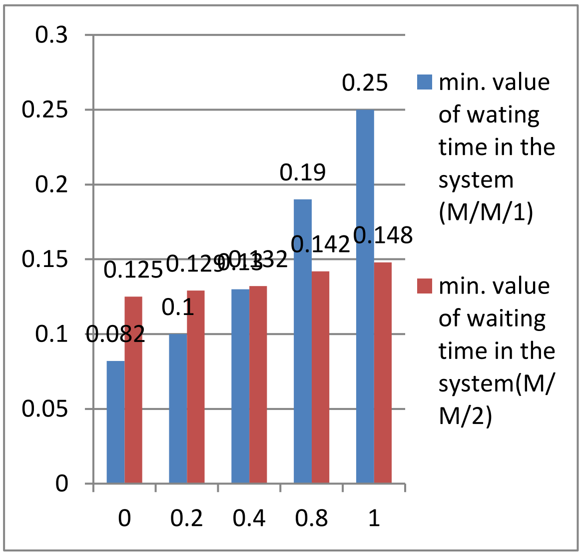

Figure 3.5 min. value of W_s

This graph shows the comparison between minimum value of average time spend in the system of M/M/1 and M/M/2 fuzzy models. Firstly the minimum value of M/M/2 model is greater but further it is lesser than M/M/1 fuzzy model. So we taking mean value it is lesser than M/M/1 with respect to α-cut values.

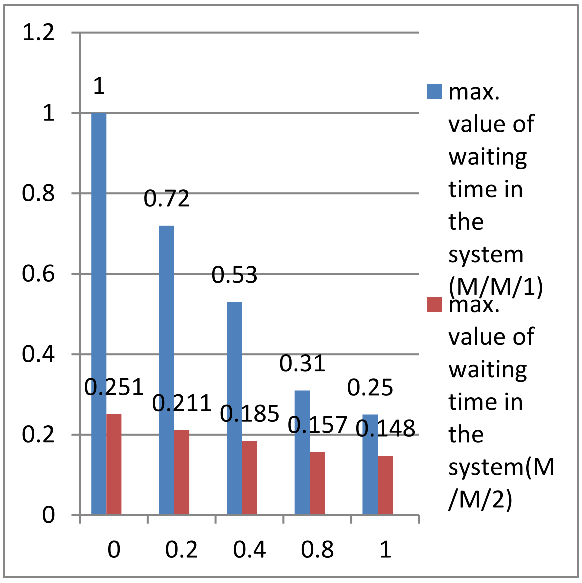

Figure 3.6 max. value of W_s

In this comparison of maximum value of time spend in the system of fuzzy queue models M/M/1 and M/M/2 models respectively. This graph shows that the time spend is less in the M/M/2 fuzzy queue model with respect to α-cut values.

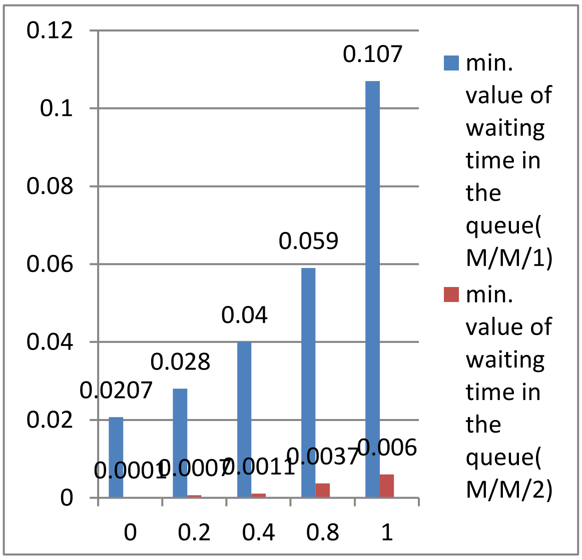

Figure 3.7 min. value W_q

This graph represents comparison between minimum values of average waiting time in the queue of M/M/1 and M/M/2 fuzzy models. In this value of waiting time in M/M/1 is greater than M/M/2.

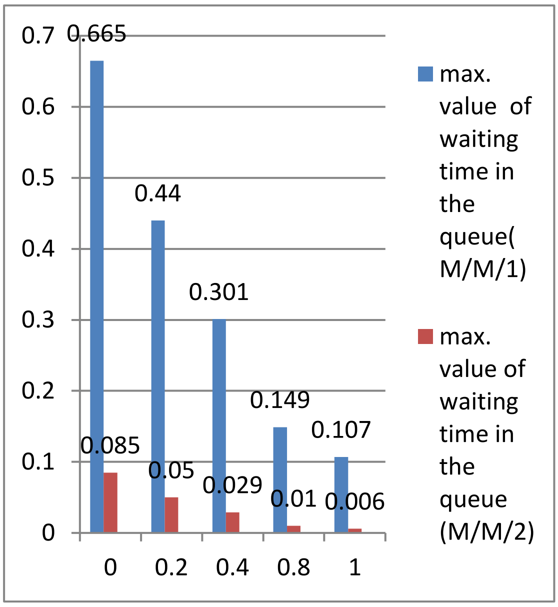

Figure 3.8 max. value of W_q

This graph shows the comparison between the maximum values of average number of customers waiting time in the queue of M/M/1 and M/M/2 fuzzy models. In this graph time spend in queue is lesser in M/M/2 fuzzy models comparison to M/M/1 fuzzy models.

Conclusion:

In this chapter we find mean value of customers in system and queue .Also find the value of mean time spend in the system and in the queue by the model M/M/1 and M/M/2 under fuzzy logic by using α-cut method. Comparing these two fuzzy models we conclude that when number of server increase there is decrease in waiting time of customers. This is shown by graphs with max. and min. value of different parameters of queuing theory. Similarly we can find different parameters of queuing theory under fuzzy set by increasing number of serversupto k.

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