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Control Chart for Power Rayleigh Distribution Through Sampling Technique

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Abstract-Control charts are widely used to track the manufacturing processes and find any odd changes in the target quality characteristic. To return the process to statistical control, prompt and thorough assessment of the process shift and the remedial actions provided by the control charts are helpful steps. The mean life of the product, whose lifetime follows the Power Rayleigh distribution, is to be monitored using an attribute control chart that uses repetitive sampling and a shortened life test. The tables and figures show the average run length and average sample size together with various shifts in the parameter value. By simulation study, the resulting control chart is created to evaluate performance. The process is more accurately detected by the power Rayleigh distribution than by the Rayleigh distribution, which is sensitive.

Keywords: Attributes, Average run length, Average sample size, Control chart, Distribution, Lower control Limit, Upper control limit.

INTRODUCTION

Quality control has been around since the advent of manufacturing and the competition that accompanied it, but its scientific foundation regarding the number of sample units to inspect, what inferences to make from the results, and the eventual extension to statistical quality control happened relatively late. The first control chart for fractional nonconforming units was introduced in 1924 by (1), marking the beginning of statistical quality control. His first control chart checks to see if, at the time of observation, the nonconforming portion of a product is still within the control limits. The variable characteristics and attribute characteristics can both be studied using the control charts. A chart used to track the attribute quality characteristic, such as the p, np, and u control charts, is referred to as an attribute control chart. Attribute data is data that is based on qualitative traits or categories. The use of variable control charts, such as the X-bar, R, and S control charts, is made in the event that the distribution of the data is continuous. On the other hand, continuous distributions of data require the use of continuous control charts. The statistical control strategy that is most frequently used to track the quantity of non-conforming items is the traditional Shewhart np control charts (2).

Control statisticians are highly motivated to create various control charts due to the significance of control charts in the market. In the literature, many of these control chart approaches were created under the presumption that the particular quality feature would follow a normal distribution. Several times, the distributional shape of a quality attribute is unknown or does not conform to the assumption of a normal distribution. Also, it has been noted that many studies emphasize control charts with non-normal distributions, such as (3,4,5).

Control charts are widely used to track the manufacturing processes and find any odd changes in the target quality characteristic. To return the process to statistical control, prompt and thorough assessment of the process shift and the remedial actions provided by the control charts are helpful steps. The control chart is a popular and efficient online monitoring method for this (6,7). In the literature on process control, two types of control charts are most frequently employed: one is used for monitoring the process mean or shape; example, $\bar{X}$-Chart), and the other is used for monitoring the variability; example R - Chart (8).

Average Run Length (ARL), which can be defined as the average number of samples before the process signals an out-of-control process, is used to evaluate the performance of any suggested chart (7). An essential instrument for process design and performance assessment is the ARL (9). To determine the in-control ARL (ARL_0) and the out-of-control (ARL_1) processes, various approaches, such as the Markov Chain approach, integral equation approach, and Monte Carlo simulations, are available (10). A smaller value of ARL_1 is believed to be better for the efficient monitoring of the process as the out-of-control process is signaled soon to avoid losses of scrap and/or rework. The value of ARL_0 is considered to be the greater as the process is in a state of in-control. The ARL computation was used by a number of researchers to assess the effectiveness of the suggested plan, including (11–14).

The ARL of a control chart can be calculated numerically utilizing a mix of individual observations and moving range charts based on two consecutive measurements, according to Crowder (15). He provided the precise integral form of ARL's equation as well as its numerical approximation. Additionally, he provided ARL values for various control limit settings as well as changes to the process mean and standard deviation. Amin and Wolff (16) investigated the ARL characteristics of a particular control approach for tracking a process' mean and variation. They calculated the ARL values for the X, R, and Extreme-value charts and demonstrate that the latter is the most effective of the three charts when it is intended to detect the presence of mixing alternatives in a scenario where the underlying distribution is a mixture of normal distributions.

It has been demonstrated that using the Multiple dependent state and exponentially distributed quality parameters together is an effective way to monitor the mean of the manufacturing process (17). By measuring the ARL, which is defined as the average number of samples falling inside control boundaries up to it shows the out-of-control process, the performance of the newly constructed chart may be assessed (7).

Distribution used in the process of Control chart

Author (Year)	Distribution	Conclusion
Aslam et al., (2016)	Pareto distribution	To track the lifespan of a product that follows the Pareto distribution of the second kind, a control chart under a truncated life test was proposed. (18)
Rao (2018)	Exponentiated half logistic distribution	Created an attribute control chart for the time-truncated life test's exponentiated half logistic distribution. (19)
Rosaiah et al. (2018)	Exponentiated Frechet distribution	Examined the attribute control chart for the exponentiated Frechet distribution under the time-truncated life test. (20)
Adeoti and Ogundipe (2021)	Generalized exponential distribution	Developed an attribute chart for the generalized exponential distribution under the time truncated life test. (21)
Jafarian-Namin et al. (2021)	Weibull distribution	Investigated the effective layout of attribute control charts for the Weibull distribution using a shortened life test. (22)
Rao et al. (9)	Dagum distribution	Introduced a control chart for the abbreviated life test's Dagum distribution. (23)

Olatunde and Gadde Srinivasa (2022)	Rayleigh Distribution	Attribute Control Chart for Rayleigh Distribution Using Repetitive Sampling under Truncated Life Test (24)

Designing of Proposed control chart

To enhance the flexibility of Rayleigh model by formulating an extended version of the model based on power transformation technique. The probability density function of the Power Rayleigh distribution (PRD) (25) is

$$f(x) = \frac{x}{\alpha^2} t^{2\beta-1} e^{\frac{-x^2}{2\alpha^2}} \quad (1)$$

The cumulative distribution function of PRD is

$$F(x) = 1 - e^{\frac{-x^2\beta}{2\alpha^2}} \quad (2)$$

The survival function of PRD is

$$S(x) = 1 - F(x) = e^{\frac{-x^2\beta}{2\alpha^2}} \quad (3)$$

The hazard rate function of PRD is

$$h(x) = \frac{f(x)}{S(x)} = \frac{x^{2\beta-1}}{\alpha^2}\beta \quad (4)$$

Considering X as lifetime product defined as PRD with $\propto, \beta$, representing the scale and shape parameter. The mean residual function of PRD is $2\alpha^2 x^{1-2\beta}$.

Assessing the probability when a product fails before the occurring of time x_0, the shift in time process which is in control, defied as

$$t_0 = 1 - e^{\frac{-x_0^2\beta}{2\alpha^2}} \quad (5)$$

The shape parameter β of PRD is fixed and the scale parameter $\propto$, is substituted in equation (5) is modified and defined as

$$t_0 = 1 - e^{-\left(\frac{\alpha^2\pi}{4}\right)} \quad (6)$$

np control chart is taken into consideration as the number of failure is noted reasonably than the defective plot p for the in control limit of PRD.

With the upper control limit (UCL) and lower control limit (LCL) of the derived control chart using the inner control limit is defined as

$$UCL = nt_0 + L\sqrt{nt_0(1-t_0)} \quad (7)$$

$$LCL = MAX\left[0, nt_0 - L\sqrt{nt_0(1-t_0)}\right] \quad (8)$$

Where

L = the coefficient of the derived control limit selected for in control Average Run Length (ARL).

t_0= derived from equation (6) and

n= sample size

The number of unsuccessful product in the control limit is derived when the probability of failed product t_0 which is obtained from equation (6) is unknown, then the UCL and LCL is

$$UCL = \bar{M} + L\sqrt{\bar{M}\left(1 - \frac{\bar{M}}{n}\right)} \quad (9)$$

$$LCL = MAX\left[0, \bar{M} - L\sqrt{\bar{M}\left(1 - \frac{\bar{M}}{n}\right)}\right] \quad (10)$$

Where, $\bar{M}$= Number of mean in the unsuccessful product.

The in control ARL of the control chart is derived as probability of the process to be out of control when the process is in control is given as

$$t^{(0)}_{out,0} = prob\left(M > UCL \mid t_0\right) + prob\left(M < LCL \mid t_0\right) \quad (11)$$

The number of failed products follows binominal distribution as

$$t_{out,0}^{(0)} = \sum_{m=UCL+1}^{n} \binom{n}{m} \left(1 - e^{-\frac{\alpha^2 \pi}{4}}\right)^m \left(e^{-\frac{\alpha^2 \pi}{4}}\right)^{n-m} + \sum_{m=0}^{LCL} \binom{n}{m} \left(1 - e^{-\frac{\alpha^2 \pi}{4}}\right)^m \left(e^{-\frac{\alpha^2 \pi}{4}}\right)^{n-m} \quad (12)$$

Therefore, the calculated performance for the control chart when the process is in control is estimated by the in control ARL.

$$ARL = \frac{1}{t_{out}^{(0)}}$$

The ARL derived in equation (13) is the mean number of sample to signal out of control when the process is in control.

The in control Average Sample Size (ASS) of the control chart is derived as

$$ASS = \frac{n}{1 - t_{out}^{(0)}}$$

RESULTS

Table 1. ARL and ASS of PRD for ARL_0=300; n=30, a=0.5

c	ARL	ASS
1.0	300	39.22
1.1	83.53	41.13
1.2	27.06	42.59
1.3	13.34	43.82
1.4	5.73	44.75
1.5	2.91	45.91
1.6	2.05	44.38
1.7	1.37	43.60
1.8	1.01	42.42
1.9	1.0	42.15
2.0	1.0	42.01

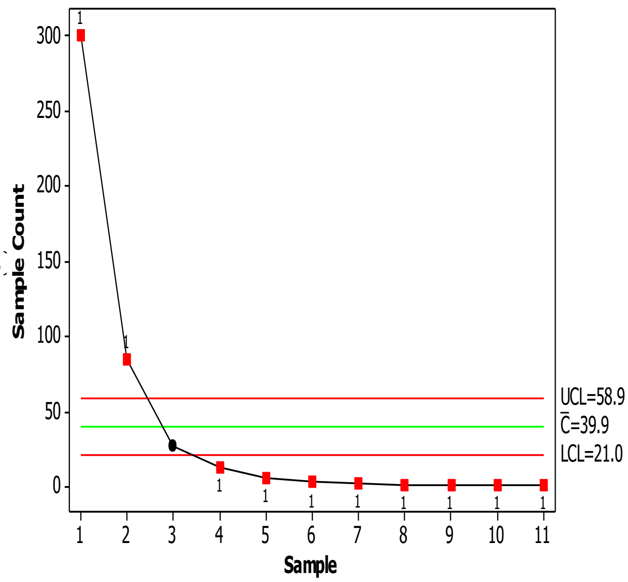

Figure 1. ARL control chart when LCL=21, UCL=58.9

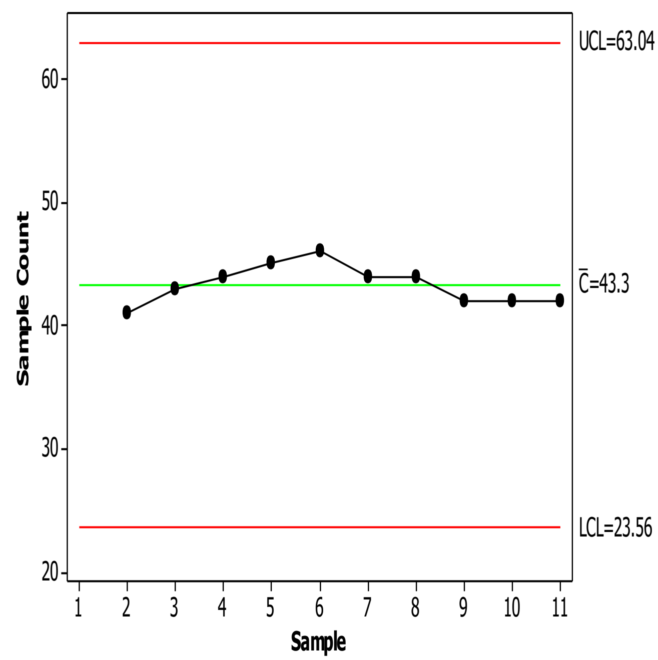

Figure 2. ASS control chart when LCL=23.56, UCL=63.04

Table 2. ARL and ASS of PRD for ARL_0=400; n=40, a=0.6

c	ARL	ASS
1.0	400.06	37.05
1.1	103.15	37.92
1.2	56.87	38.13

1.3	21.05	39.68
1.4	9.87	40.07
1.5	3.10	39.41
1.6	2.32	38.01
1.7	1.49	37.69
1.8	1.12	37.11
1.9	1.05	36.80
2.0	1.01	36.03

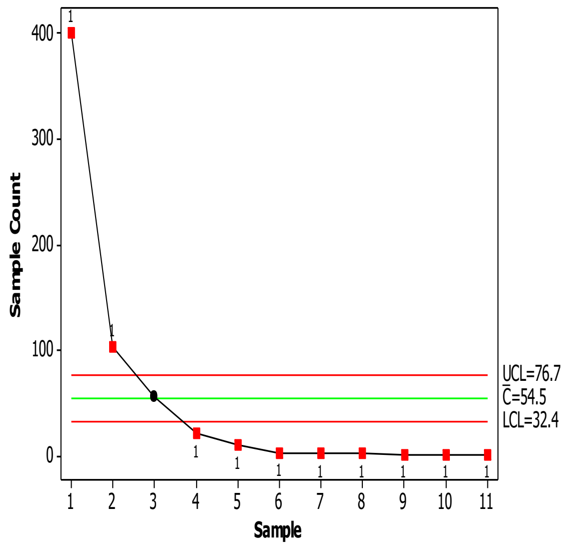

Figure 3. ARL control chart when LCL=32.4, UCL=76.7

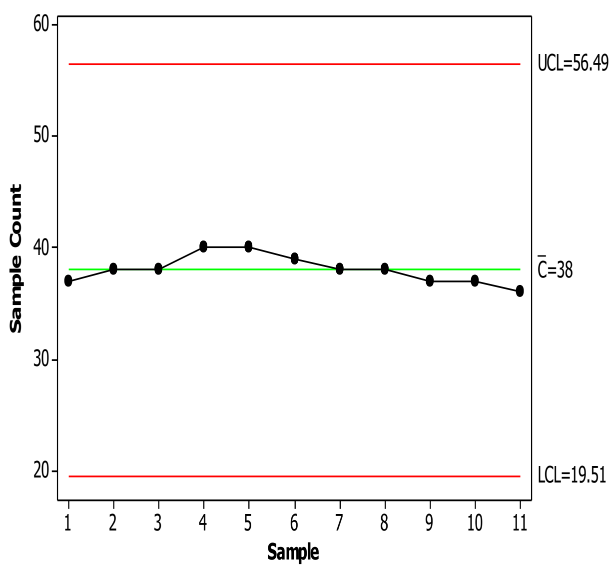

Figure 4. ASS control chart when LCL=19.51, UCL=56.49

Table 3. ARL and ASS of PRD for ARL_0=500; n=50, LCL=0, UCL=12, a=0.7

c	ARL	ASS
1.0	500.06	36.35
1.1	169.77	37.04
1.2	85.64	37.91
1.3	66.12	39.13
1.4	28.36	40.47
1.5	9.48	41.36
1.6	4.51	39.82
1.7	2.09	38.24
1.8	1.33	37.60
1.9	1.05	36.51
2.0	0.92	35.73

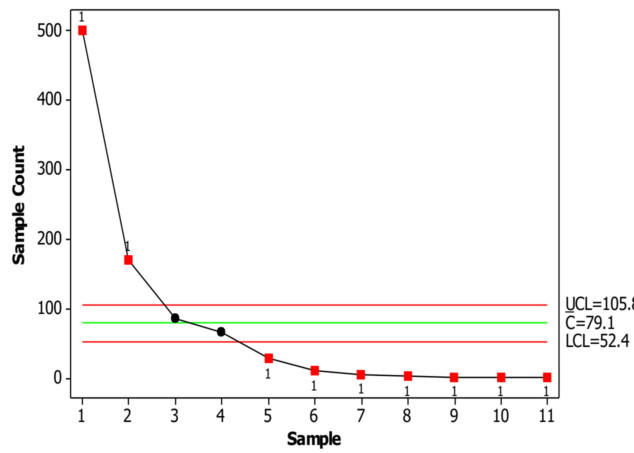

Figure 5. ARL control chart when LCL=52.4, UCL=105.8

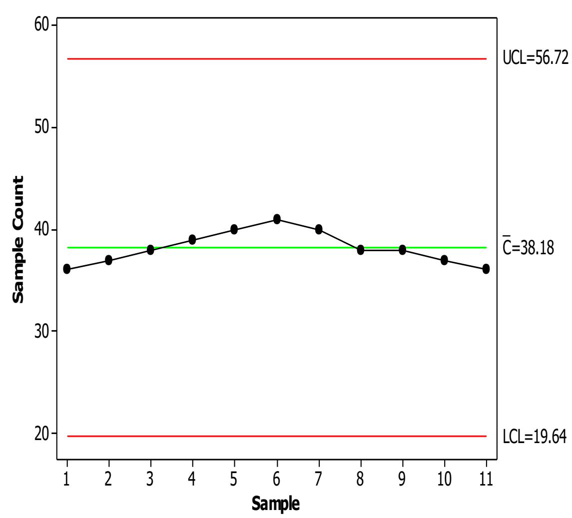

Figure 6. ASS control chart when LCL=19.64, UCL=56.72

Table 1 represents the values of ARL and ASS, for the shift increases from 1.0 to 2.0 there is a decreasing tendency in the values of ARL and for ASS in the initial state there is an increasing tendency till the shift reaches to 1.5, then there is a decreasing tendency till the shift reaches 2.0. In figure 1 of ARL the LCL=21 and UCL=58.9 was detected in the control chart. For figure 2 of ASS the LCL=23.56 and UCL=63.04.

Table 2 represents the values of ARL and ASS, for the shift increases from 1.0 to 2.0 there is a decreasing tendency in the values of ARL and for ASS in the initial state there is an increasing tendency till the shift reaches to 1.4, then there is a decreasing tendency till the shift reaches 2.0. In figure 3 of ARL the LCL=32.4 and UCL=76.7 was detected in the control chart. For figure 4 of ASS the LCL=19.51 and UCL=56.49.

Table 3 represents the values of ARL and ASS, for the shift increases from 1.0 to 2.0 there is a decreasing tendency in the values of ARL and for ASS in the initial state there is an increasing tendency till the shift reaches to 1.5, then there is a decreasing tendency till the shift reaches 2.0. In figure 5 of ARL the LCL=52.4 and UCL=105.8 was detected in the control chart. For figure 6 of ASS the LCL=19.64 and UCL=56.72.

CONCLUSION

The PRD represented for the attributed control chart is been seen. The derived chart is developed to assess the performance of ARL and ASS through simulation study The PRD distribution is sensitive and detects the process better than the Rayleigh distribution (RD). An extension of this research is the comparative study between RD and PRD can been seen as future study.

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