$nIgsemi^*$-Locally Closed Sets in Nano Ideal Topological Spaces

Dr. J. Arul Jesti[1]
[1]Assistant Professor, PG and Research Department of Mathematics, St. Mary's College (Autonomous) Thoothukudi-628001, India. Email: aruljesti@gmail.com

J. Joycy Renuka[2]
[2]Research Scholar, PG and Research Department of Mathematics, St. Mary's College (Autonomous) Thoothukudi-628001, India. Email: renukajoycy@gmail.com

***Corresponding Author:** Dr. J. Arul Jesti
Email: aruljesti@gmail.com

Abstract

In this paper, we introduce and characterize some new generalized locally closed sets known as $nIgsemi^*$-locally closed sets and lightly $nIgsemi^*$-locally closed sets in nano ideal topological spaces. Also we discuss some of its properties. Additionally, we define and study certain new classes of continuous mappings namely $nIgsemi^*$-LC continuous functions and L $nIgsemi^*$-LC continuous functions in nano ideal topological spaces. Comparison and characterisation of these functions are explored.

Keywords- Nano Ideal Topology, locally closed sets, $nIgsemi^*$-locally closed sets, lightly $nIgsemi^*$-locally closed sets, $nIgsemi^*$-LC continuous mappings and L $nIgsemi^*$-LC continuous mappings.

1. Introduction

In topological spaces, locally closed sets were studied more by Bourbaki [11] in 1966, which is the intersection of an open set and a closed set. Kuratowski [3] was introduced the local function in ideal spaces. The notion of nano topology was introduced by Lellis Thivagar[4] which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it. M. Parimala et al. [5,6,7] introduced the concept of nano ideal topological spaces and investigated some of its basic properties. In 2018, [12] R. Asokan, O. Nethaji and I. Rajasekaran, introduced lightly nano I -locally closed set in nano ideal topological spaces. In 2021, Selvaraj Ganesan [14] introduced μ-nI-locally closed sets set in nano ideal topological spaces. Recently we [1] defined $nIgsemi^*$-closed sets and $nIgsemi^*$-open sets in nano ideal topological spaces. The purpose of this paper is to introduce the new notions of locally closed sets and locally closed continuous functions in nano ideal topological spaces. We study the concept of $nIgsemi^*$-locally closed sets, lightly $nIgsemi^*$locally closed sets, $nIgsemi^*$-LC continuous mappings and L $nIgsemi^*$-LC continuous mappings in nano ideal topological spaces.

2. Preliminaries

Throughout this paper $(U,\tau_R(X))$ represent nano topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space $(U, \tau_R(X))$, $ncl(A)$ and $nint(A)$ denote the nano closure of A and the nano interior of A respectively. We recall the following definitions, which will be used in the sequel.

Definition 2.1 [1] A subset A of a nano ideal topological space $(U, \mathcal{N}, I)$ is said to be ***nano ideal generalized semi*-closed*** (briefly, $nIgsemi^*$-closed) if $A_n^* \subseteq V$ whenever $A \subseteq V$ and V is *nano semi**-open.

Definition 2.2 [1] A subset A of a nano ideal topological space $(U, \mathcal{N}, I)$ is said to be ***nano ideal generalized semi*-open*** (briefly, $nIgsemi^*$-open) if X – A is $nIgsemi^*$-closed.

Theorem 2.3. [1] Consider a space $(U, \mathcal{N}, I)$, then the following implications are true and reverse implications are may not be true.
i) A nano closed is always a $nIgsemi^*$-closed set.
ii) A ng-closed is always a $nIgsemi^*$-closed set.
iii) A $nIgsemi^*$-closed is always a nIg-closed set.

Definition 2.4 [1] Consider a subset A of a nano ideal topological space $(U,\mathcal{N},I)$. Then ***nano I generalized semi******-closure** of A is defined as the intersection of all *nano I generalized semi**-closed sets containing A and it is denoted by *nI generalized semi**-$cl(A)$. i.e., $nIgsemi^*\text{-}cl(A) = \cap\{F: A \subseteq F \in nIgsemi^*c(U)$. *nI generalized semi**-cl(A) is the smallest *nano* ideal *generalized semi***-closed* set containing A.
The following two propositions are easy consequences from definitions.

Proposition 2.5 [1] For any $M \subseteq U$, the following holds.
1. $nIgsemi^*\text{-}cl(M)$ is the smallest $nIgsemi^*$-closed set containing M.

2. M is $nIgsemi^*$-closed if and only if $nIgsemi^*cl(M) = M$.

Proposition 2.6 [1] For any two subsets M and P of (U, N, I), the following holds.
1. If $M \subseteq P$, then $nIgsemi^*cl(M) \subseteq nIgsemi^*\text{-}cl(P)$.
2. $nIgsemi^*cl(M \cap P) \subseteq nIgsemi^*cl(M) \cap nIgsemi^*cl(P)$.

Definition: 2.7 [2] Let $(\mathcal{U}, \tau_R(X))$ be a nano topological space and $A \subseteq \mathcal{U}$. Then A is said to be a ***nano locally closed*** if $A = U \cap F$, where U is nano open and F is a nano closed in $\mathcal{U}$.

Definition 2.8. [2] A subset A of $(U, \tau_R(X))$, is called ***Nano generalized locally closed set*** (briefly NGLC) if $A = G \cap F$, where G is Ng-open in $(U, \tau_R(X))$ and F is Ng-closed in $(U, \tau_R(X))$. The collection of all nano generalized locally closed sets of $(U, \tau_R(X))$ will be denoted by $NGLC(U, \tau_R(X))$.
The following two collections of subsets of $(U, \tau_R(X))$ $(i.e)$ $NGLC^*(U, \tau_R(X))$ and $NGLC^{**}(U, \tau_R(X))$ are defined as follows.

Definition 2.9.[2] For a subset A of $(U, \tau_R(X))$, $A \in$ $\boldsymbol{NGLC^*}$ $(U, \tau_R(X))$, if there exist a Ng- open set G and a nano closed set F of $(U, \tau_R(X))$, respectively, such that $A = G \cap F$.

Definition 2.10. [2] For a subset A of $(U, \tau_R(X))$, $A \in$ $\boldsymbol{NGLC^{**}}(U, \tau_R(X))$, if there exist a nano open set G and a Ng-closed set F of $(U, \tau_R(X))$, respectively, such that $A = G \cap F$.

Definition 2.11. [2] A function $f: (U, \tau_R(X)) \to (V, \tau_{R0}(Y))$ is called **n*ano locally closed continuous function*** [shortly, NLC continuous] if $f^{-1}(B)$ is a nano locally closed set of $(U, \tau_R(X))$ for each nano open set B of $(V, \tau_{R_0}(Y))$.

Definition 2.12. [2] A function $f: (U, \tau_R(X)) \to (V, \tau_{R_0}(Y))$ is called ***Nano Generalized locally closed continuous function*** [shortly, $NGLC$ continuous], (resp. $NGLC^*$-continuous, resp. $NGLC^{**}$-continuous) if $f^{-1}(B) \in NGLC((U, \tau_R(X))$ (resp. $f^{-1}(B) \in NGLC^*((U, \tau_R(X))$, $f^{-1}(B) \in NGLC^{**}((U, \tau_R(X))$ for each $B \in \tau_{R_0}(Y)$.

Definition 2.13. [12] A subset A of an nano ideal topological space (U, N, I) is called an **lightly nano I -locally closed** (briefly L-nI-LC) if $A = M \cap N$ where M is n-open and N is n^*-closed.

Definition 2.14 [8]. A subset A of a nano ideal topological space $(U, \mathcal{N}, I)$ is n^***-dense** in itself (resp. n^* **-perfect and** n^* **-closed**) if $A \subseteq A_n^*$ (resp. $A = A_n^*$ and $A_n^* \subseteq A$.

3. $nIgsemi^*$-Locally Closed Sets

Definition 3.1. A subset A of an nano ideal topological space $(U, \mathcal{N}, I)$ is called $\boldsymbol{nIgsemi^*}$ **-locally closed set** (briefly $nIgsemi^*LC$) if $A = G \cap F$ where G is $nIgsemi^*$-open and F is $nIgsemi^*$ -closed in $(U, \mathcal{N}, I)$.

Notation 3.2. The class of all $nIgsemi^*$-locally closed sets in $(U, \mathcal{N}, I)$ is denoted by $nIgsemi^*LC$ $(U, \mathcal{N}, I)$ or simply $nIgsemi^*LC$.

Definition 3.3. For a subset A of $(U, \mathcal{N}, I)$, $\boldsymbol{A} \in \boldsymbol{nIgsemi^*LC^*(U, \mathcal{N}, I)}$ if there exist a $nIgsemi^*$-open set G and a nano closed set F of $(U, \mathcal{N}, I)$ such that $A = G \cap F$.

Definition 3.4. For a subset A of $(U, \mathcal{N}, I)$, $\boldsymbol{A} \in \boldsymbol{nIgsemi^*LC^{**}(U, \mathcal{N}, I)}$ if there exist an nano open set G and a $nIgsemi^*$ -closed set F of $(U, \mathcal{N}, I)$ such that $A = G \cap F$.

Proposition 3.5. Let A be a subset of an ideal space $(U, \mathcal{N}, I)$. Then the following holds.
(i) If $A \in nIgsemi^*LC$, then $A \in NLC$
(ii) If $A \in nIgsemi^*LC^*$, then $A \in NLC$
(iii) If $A \in nIgsemi^*LC$, then $A \in NGLC$
(iv) If $A \in nIgsemi^*LC^*$, then $A \in NGLC^*$
(v) If $A \in nIgsemi^*LC^{**}$, then $A \in NLC$
(vi) If $A \in nIgsemi^*LC^{**}$, then $A \in NGLC^{**}$

Proof.
i) Assume $A \in nIgsemi^*LC$. Let G be nano open set and F be nano closed set in nano ideal topological spaces. We know the fact that every nano open set is $nIgsemi^*$-open set and every nano closed set is $nIgsemi^*$-closed set. Then G is $nIgsemi^*$-open set and F is $nIgsemi^*$-closed set. Since $A \in nIgsemi^*LC$, $A = G \cap F$. Hence $A \in NGLC$.
ii) Assume $A \in nIgsemi^*LC^*$. Let G be nano generalised open set and F be nano closed set in nano ideal topological spaces. Then clearly G is $nIgsemi^*$-open set and F is nano closed set. Since $A \in nIgsemi^*LC^*$, $A = G \cap F$. Hence $A \in NGLC$.
iii) Assume $A \in nIgsemi^*LC$. Let G be nano generalised open set and F be nano generalised closed set in nano ideal topological spaces. We know the fact that every nano generalised open set is $nIgsemi^*$-open set and every nano generalised closed set is $nIgsemi^*$-closed set. Then G is $nIgsemi^*$-open set and F is $nIgsemi^*$-closed set. Since $A \in nIgsemi^*LC$, $A = G \cap F$. Hence $A \in NGLC$.
iv) Assume $A \in nIgsemi^*LC^*$. Let G be nano open set and F be nano closed set in nano ideal topological spaces. Then clearly G is $nIgsemi^*$-open set and F is nano closed set. Since $A \in nIgsemi^*LC^*$, $A = G \cap F$. Hence $A \in NGLC^*$
v) Assume $A \in nIgsemi^*LC^{**}$. Let G be nano open set and F be nano closed set in nano ideal topological spaces. Then clearly G is nano open

set and F is $nIgsemi^*$-closed set. Since $A \in nIgsemi^*LC^{**}$, $A = G \cap F$. Hence $A \in NGLC$.

vi) Assume $A \in nIgsemi^*LC^{**}$. Let G be nano open set and F be nano generalised closed set in nano ideal topological spaces. We know the fact that every nano generalised closed set is $nIgsemi^*$-closed set. Then clearly G is nano open set and F is $nIgsemi^*$-closed set. Since $A \in nIgsemi^*LC^{**}$, $A = G \cap F$. Hence $A \in NGLC^{**}$

Remark 3.6. The following examples shows that the converse of the above proposition is not always true.

Example 3.7. Consider the universal set $U = \{a, b, c, d\}$, the approximation space $U/\mathcal{R}$ = $\{\{a\}, \{c\}, \{b, d\}\}$, $X = \{a, b\} \in U$ with the ideal $I = \{\varphi, \{a\}\}$. The nano topology defined by U is $\tau_R(X) = \{U, \varphi, \{a\}, \{b, d\}, \{a, b, d\}\}$ and $nIgsemi^*$-open sets are $\{\{a\}, \{b\}, \{d\}, \{a, b\}, \{b, d\}, \{a, d\}, \{b, c, d\}, \{a, b, d\}, U, \varphi\}$. Nano generalised closed sets are $\{\{c\}, \{b, c\}, \{c, d\}, \{a, c\}, \{b, d\}, \{a, b, c\}, \{b, c, d\}, \{a, c, d\}, U, \varphi, \}$. Let $G = \{b, d\}$, $F = \{b, c, d\}$. Then $A = \{b, d\} \in NLG, NGLC, NGLC^*$ but $A \notin nIgsemi^*LC^{**}$.

Example 3.8. Let $U = \{a, b, c, d\}$, with $U/R = \{\{b\}, \{d\}, \{a, c\}\}$ and $X = \{a, d\}$. Then the Nano topology $\mathcal{N} = \{\phi, U, \{d\}, \{a, c\}, \{a, c, d\}\}$ and $I = \{\phi, \{d\}\}$.Then $nIgsemi^*$-closed sets are $\{\phi, U, \{b\}, \{d\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{b, c, d\}, \{a, b, d\}\}$ and ng-closed sets are $\{\phi, U, \{b\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{b, c, d\}, \{a, b, d\}$. Let $G = \{b, d\}$, $F = \{a, b, c\}$, Then $A = \{b\} \in NGLC^{**}$ but not in $nIgsemi^*LC$ and $nIgsemi^*LC^*$.

Theorem 3.9. For a subset A of nano ideal space $(U, \mathcal{N}, I)$, If $A \in nIgsemi^*LC^*$ then $A = G \cap n\text{-}cl(A)$ for some $nIgsemi^*$-open set G.

Proof. Let $A \in nIgsemi^*LC^*$, then there exist a $nIgsemi^*$-open set U and a nano closed set F in $(U, \mathcal{N}, I)$ such that $A = G \cap F$. Since $A \in G$ and $A \in n\text{-}cl(A)$, we have $A \subseteq G \cap n\text{-}cl(A)$.
Since $A \subseteq F$, $n\text{-}cl(A) \subseteq n\text{-}cl(F)$. Since F is nano closed, $n\text{-}cl(F) = F$. Therefore, $n\text{-}cl(A) \subseteq F$ and $A = G \cap F \supseteq G \cap n\text{-}cl(A)$. Hence $A = G \cap n\text{-}cl(A)$.

Theorem 3.10. For a subset A of an nano ideal space (U, N, I), $(n\text{-}cl(A))-A$ is $nIgsemi^*$-closed if and only if $A \cup U - (n\text{-}cl(A))$ is $nIgsemi^*$-open.

Proof. Necessity- Let $F = (n\text{ - }cl(A)) - A$. By hypothesis, F is $nIgsemi^*$-closed and $U - F =$ $U \cap (U - F) = U \cap (U - (n\text{-}cl(A)) - A) = A \cup (U - (n\text{-}cl(A))$. Since $U - F$ is $nIgsemi^*$-open, $A \cup (U - (n\text{-}cl(A))$ is $nIgsemi^*$-open.
Sufficiency- Let $G = A \cup (U - (n\text{ - }cl(A))$. By hypothesis G is $nIgsemi^*$-open. Then $U - G$ is $nIgsemi^*$-closed and $U - G = U - (\cup (U - (n\text{-}cl(A))) = n\text{-}cl(A) \cap (U - A) = n\text{-}cl(A)) - A$. Hence proved.

Theorem 3.11. For a subset A of an nano ideal space $(U, \mathcal{N}, I)$, the following are equivalent.

(i) $A \in nIgsemi^*LC$
(ii) $A = G \cap nIgsemi^*cl(A)$ for some $nIgsemi^*$-open set G and $nIgsemi^*$-closed set A.
(iii) $nIgsemi^*cl(A) - A$ is $nIgsemi^*$-closed.
(iv) $A \cup (U - nIgsemi^*cl(A))$ is $nIgsemi^*$-open.

Proof. (i) ⇒(ii). Let $A \in nIgsemi^*LC$, then there exist a $nIgsemi^*$-open set G and $nIgsemi^*$-closed set F in $(U, \mathcal{N}, I)$ such that $A = G \cap F$. Since $A \subseteq G$ and $A \subseteq nIgsemi^*cl(A)$, we have $A \subseteq G \cap nIgsemi^*cl(A)$. Also, since
$A \subseteq F$, $nIgsemi^*cl(A) \subseteq nIgsemi^*cl(F)$. Since F is $nIgsemi^*$-closed, $nIgsemi^*cl(F) = F$.
Therefore, $nIgsemi^*cl(A) \subseteq F$ and $A = G \cap F \supseteq G \cap nIgsemi^*cl(A)$.
This proves (ii).
(ii) ⇒ (i). Since G is $nIgsemi^*$-open and A is $nIgsemi^*$-closed we have
$G \cap nIgsemi^*cl(A) = G \cap A \in nIgsemi^*LC$.
(iii)⇒ (iv). Let $F = nIgsemi^*cl(A) - A$. By hypothesis, F is $nIgsemi^*$-closed and $U - F = U \cap (U - F) = U \cap (U - (nIgsemi^*cl(A) - A)) = A \cup (U - nIgsemi^*cl(A))$. Since $U - F$ is $nIgsemi^*$-open, $A \cup (U - nIgsemi^*cl(A))$ is $nIgsemi^*$-open.
(iv)⇒ (iii). Let $G = A \cup (U - nIgsemi^*cl(A))$. By hypothesis G is $nIgsemi^*$-open. Then $U - G$ is $nIgsemi^*$ -closed and $U - G = U - (A \cup (U - nIgsemi^*cl(A)) = nIgsemi^*cl(A) \cap (U - A) = nIgsemi^*cl(A) - A$.
This proves (iii).
(ii) ⇒ (iv). Let $A = G \cap nIgsemi^*cl(A)$ for some $nIgsemi^*$ -open set G . Now, $A \cup (U - nIgsemi^*cl(A)) = (G \cap nIgsemi^*cl(A)) \cup (U - nIgsemi^*cl(A)) = (G \cup U - nIgsemi^*cl(A)) \cap (nIgsemi^*cl(A) \cup (U - nIgsemi^*cl(A)) = (G \cup U - nIgsemi^*cl(A)) \cap U = (G \cup (U - nIgsemi^*cl(A)))$ is $nIgsemi^*$-open.
(iv)⇒(ii) Let $G = A \cup \big(U - nIgsemi^*cl(A)\big)$. Then G is $nIgsemi^*$ -open. Now, $G \cap nIgsemi^*cl(A) = (A \cup (U - nIgsemi^*cl(A))) \cap nIgsemi^*cl(A) = (nIgsemi^*cl(A) \cap A) \cup (nIgsemi^*cl(A) \cap (U - nIgsemi^*cl(A))) = A \cup \phi = A$. Therefore $A = G \cap nIgsemi^*cl(A)$ for some $nIgsemi^*$-open set G.

Theorem 3.12. For a subset A of $(U, \mathcal{N}, I)$, If $A \in nIgsemi^*LC^{**}$ then there exist an nano open set G such that $A = G \cap nIgsemi^*cl(A)$.
Proof. Let $A \in nIgsemi^*LC^{**}$. Then there exists an nano open set G and a $nIgsemi^*$-closed set F in $(U, \mathcal{N}, I)$ such that $A = G \cap F$. Since $A \subseteq G$ and $A \subseteq nIgsemi^*cl(A)$, we have $A \subseteq G \cap nIgsemi^*cl(A)$-------(a). Also, since $A \subseteq F$, $nIgsemi^*cl(A) \subseteq nIgsemi^*cl(F)$. But $nIgsemi^*cl(F) = F$, since F is $nIgsemi^*$ -closed. Therefore ,$nIgsemi^*cl(A) \subseteq F$. Hence $A = G \cap F \supseteq G \cap nIgsemi^*cl(A)$-----------(b). From (a) and (b), $A = G \cap nIgsemi^*cl(A)$.

Theorem 3.13. Let A and B be any two subsets of $(U, \mathcal{N}, I)$. If $A \in nIgsemi^*LC^*$ and B is nano closed, then $A \cap B \in nIgsemi^*LC^*$.

Proof. If $A \in nIgsemi^*LC^*$, then there exists a $nIgsemi^*$-open set G and a nano closed set F in $(U, \mathcal{N}, I)$ such that $A = G \cap F$. Now, $A \cap B = (G \cap F) \cap B = G \cap (F \cap B) \in nIgsemi^*LC^*$.

Theorem 3.14. Let A and B be two subsets of$(U, \mathcal{N}, I)$. If $A \in nIgsemi^*LC^{**}$ and B is nano closed then $A \cap B \in nIgsemi^*LC^{**}$

Proof. If $A \in nIgsemi^*LC^{**}$, then there exist an nano open set G and a $nIgsemi^*$-closed set F such that $A = G \cap F$. Then $A \cap B = (G \cap F) \cap B = G \cap (F \cap B) \in nIgsemi^*LC^{**}$, since every nano open set is an $nIgsemi^*$-closed set.

Theorem 3.15. Let $(U, \mathcal{N}, I)$ be nano ideal space and A, B are subsets of U. Then the following hold.

(i) If$A, B \in nIgsemi^*LC^*$ then $A \cap B \in nIgsemi^*LC^*$

(ii) If $A, B \in nIgsemi^*LC$ then $A \cap B \in nIgsemi^*LC$.

(iii) If $A, B \in nIgsemi^*LC^{**}$then $A \cap B \in nIgsemi^*LC^{**}$

Proof. (i) Since $A, B \in nIgsemi^*LC^*$, there exist $nIgsemi^*$-open sets G, H and closed sets E, F such that $A = G \cap E$ and $B = H \cap F$. Now, $A \cap B = (G \cap E) \cap (H \cap F) = (G \cap H) \cap (E \cap F) \in nIgsemi^*LC^*$.

The proof of (ii) and (iii) are similar to the proof of (i).

4. Lightly $nIgsemi^*$-Locally Closed Sets

Definition 4.1. A subset A of an nano ideal topological space $(U, \mathcal{N}, I)$ is called an **Lightly $nIgsemi^*$-Locally Closed Sets** (briefly $L\ nIgsemi^*$-LC) if $A = M \cap N$ where M is *nano semi** -open and N is n^* -closed.

Example 4.2. Consider the universal set $U = \{a, b, c, d\}$, the approximation space $U/\mathcal{R} = \{\{a\}, \{c\}, \{b, d\}\}, X = \{a, b\} \subseteq U$ with the ideal $I = \{\varphi, \{a\}\}$. The nano topology defined by U is $\tau_R(X) = \{U, \varphi, \{a\}, \{b, d\}, \{a, b, d\}\}$ and $nIgsemi^*$ -open sets are $\{\{a\}, \{b\}, \{d\}, \{a, b\}, \{b, d\}, \{a, d\}, \{b, c, d\}, \{a, b, d\}, U, \varphi\}$. Nano $semi^*$ -open sets are $\{\{a\}, \{a, c\}, \{b, d\}, \{a, b, d\}, U, \varphi\}$. n^*-closed sets are $\{\{a\}, \{c\}, \{a, c\}, \{b, c, d\}, U, \varphi\}$ Then $A = \{b, d\} \in L\ nIgsemi^*$-$LC$.

Proposition 4.3. Let $(U, \mathcal{N}, I)$ be an nano ideal topological space and A be a subset of U. Then the following statements are hold.

(1) If A is nano $semi^*$-open, then A is $L\ nIgsemi^*$-LC set.

(2) If A is n^*-closed, then A is $L\ nIgsemi^*$-LC set.

(3) If A is a L-nI-LC-set, then A is an $L\ nIgsemi^*$-LC set.

Proof. It is obvious from definitions 2.13 and 4.1.

The converse of the above proposition 4.3 need not be true as shown in the following examples.

Example 4.4. Let U, N and I be defined as an example 4.2. Here $A = \{b, d\} \in L\ nIgsemi^*$-$LC$ but A is not n^*-closed.

Example 4.5. Let $U = \{a, b, c, d\}$ be the universe, $X = \{a, d\} \subset U$, $U/R = \{\{b\}, \{d\}, \{a, c\}\}$ and $\mathcal{N} = \{U, \varphi, \{d\}, \{a, c\}, \{a, c, d\}\}$ and the ideal $I = \{\varphi, \{d\}\}$ Then nano $semi^*$ -open sets are $\{\{a, c\}, \{d\}, \{b, d\}, \{a, b, c\}, \{a, c, d\}, U, \varphi\}$. n^* -closed sets are $\{\{b\}, \{d\}, \{b, d\}, \{a, b, c\}, U, \varphi\}$. Here $A = \{b\} \in L\ nIgsemi^*$-$LC$ but $A \notin L$-nI-LC-set.

Theorem 4.6. Let $(U, \mathcal{N}, I)$ be an nano ideal topological space. If A is an $L\ nIgsemi^*$-LC set and B is a n^*-closed set, then $A \cap B$ is an $L\ nIgsemi^*$-LC set.

Proof. Let B be n^* -closed. Then $A \cap B = (O \cap P) \cap B = O \cap (P \cap B)$, where $P \cap B$ is n^* -closed. Hence $A \cap B$ is an $L\ nIgsemi^*$-LC set.

Theorem 4.7. A subset A of an nano ideal topological space $(U, \mathcal{N}, I)$ is n^*-closed if and only if it is $L\ nIgsemi^*$-LC set and $nIgsemi^*$ -closed.

Proof. Necessity is trivial.

We prove only sufficiency. Let A be $L\ nIgsemi^*$-LC set and $nIgsemi^*$ -closed set. Since A is $L\ nIgsemi^*$-LC set, $A = O \cap P$, where O is $nsemi^*$ -open and P is n^*-closed. So we have $A = O \cap P \subseteq O$. Since A is $nIgsemi^*$-closed, $A_n^* \subseteq O$. Also since $A = O \cap P \subseteq P$ and P is n^*-closed, $A_n^* \subseteq P$. Consequently, $A_n^* \subseteq O \cap P = A$ and hence A is n^*-closed.

Remark 4.8. The notions of $L\ nIgsemi^*$-LC set and $nIgsemi^*$ -closed set are independent.

Example 4.9. Let $U = \{a, b, c, d\}$ be the universe, $X = \{c\} \subseteq U, U/\mathcal{R} = \{\{a\}, \{b\}, \{c, d\}\}$ and $\mathcal{N} = \{U, \varphi, \{c, d\}\}$ and the ideal $I = \{\varphi, \{c\}\}$. $nIgsemi^*$ -closed $= \{U, \varphi, \{a\}, \{b\}, , \{a, b\}, \{b, c\}, \{a, d\}, \{a, c\}, \{b, d\}, \{a, b, c\}, \{b, c, d\}, \{a, b, d\}, \{a, c, d\}\}$. Nano $semi^*$-open sets are $\{\{c, d\}`, U, \varphi\}$. n^*-closed sets are $\{\{b\}, \{a, b\}, U, \varphi\}$. $L\ nIgsemi^*$-LC sets are $\{U, \varphi\}$. It is clear that $\{a\}$ is an $nIgsemi^*$-closed but it is not $L\ nIgsemi^*$-LC set.

Example 4.10. Let $U = \{a, b, c, d\}$ be the universe, $X = \{a, d\} \subset U$, $U/R = \{\{b\}, \{d\}, \{a, c\}\}$ and $\mathcal{N} = \{U, \varphi, \{d\}, \{a, c\}, \{a, c, d\}\}$ and the ideal $I = \{\varphi, \{d\}\}$. $nIgsemi^*$ -closed sets are $\{\{b\}, \{d\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{b, c, d\}, \{a, b, d\}\}$ .Then nano $semi^*$ -open sets are $\{\{a, c\}, \{d\}, \{b, d\}, \{a, b, c\}, \{a, c, d\}, U, \varphi\}$. n^* -closed sets are $\{\{b\}, \{d\}, \{b, d\}, \{a, b, c\}, U, \varphi\}$ .Here $A = \{a, c\} \in L\ nIgsemi^*$-$LC$ but $A \notin nIgsemi^*$-closed.

Remark 4.11. The notions of $L\ nIgsemi^*$-LC set and $nIgsemi^*LC$ set are independent.

Example 4.12. Let U, N and I be defined as an Example 3.8, Then $nIgsemi^*$ -open sets are $\{\{a\}, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}, U, \varphi\}$

.$nIgsemi^*LC$ sets are $\{\{a\},\{b\},\{c\},\{d\},\{a,c\},\{a,d\},\{c,d\},\{a,b\},\{b,c\},\{a,b,c\},U,\varphi\}$. $L\ nIgsemi^*$-LC sets are $\{\{b\},\{d\},\{b,d\},\{a,c\},\{a,b,c\},U,\varphi\}$. Here $\{a\}\in nIgsemi^*$LC but $\{a\}\notin L\ nIgsemi^*$-LC . Also $\{b,d\}\in L\ nIgsemi^*$-LC but $\{b,d\}\notin nIgsemi^*$LC. Thus $L\ nIgsemi^*$-LC set and $nIgsemi^*LC$ set are independent.

5. $nIgsemi^*$-LC continuous and $L\ nIgsemi^*$-LC continuous mappings

Definition 5.1. A function $f:(U,N,I)\to(V,M,J)$ is said to be ***nIgsemi*-LC* continuous** if $f^{-1}(A)$ is $nIgsemi^*LC$ set in (U,N,I) for every n-closed set A of (V,M,J).

Example 5.2. Let $U=\{a,b,c,d\}$, the approximation space $U/\mathcal{R}=\{\{a\},\{b\},\{c,d\}\}$, $X=\{\{b\},\{d\}\}\subseteq U$ with the ideal $I=\{\varphi,\{a\}\}$. The nano topology defined by U is $\tau_R(X)=\{U,\varphi,\{b\},\{c,d\},\{b,c,d\}\}$and $nIgsemi^*$-open sets are $\{\{b\},\{c\},\{d\},\{c,d\},\{b,d\},\{b,c\},\{b,c,d\},U,\varphi\}$, and $nIgsemi^*$-closed sets are $\{\{a\},\{a,b\},\{a,c\},\{a,d\},\{a,b,c\},\{a,c,d\},\{a,b,d\},U,\varphi\}$. $nIgsemi^*LC$ sets are $\{\{b\},\{c\},\{d\},\{c,d\},\{b,d\},\{b,c\},\{c,d\},U,\varphi\}$. Let $V=\{a,b,c,d\}$, the approximation space $V/\mathcal{R}=\{\{a\},\{d\},\{b,c\}\}$, $Y=\{\{a\},\{d\}\}\subseteq V$ with the ideal $J=\{\varphi,\{a\}\}$. The nano topology defined by V is $\Omega_R(Y)=\{V,\varphi,\{a,d\}\}$. The function $f:(U,N,I)\to(V,M,J)$ is defined as $f(a)=c,f(b)=b,f(c)=a,f(d)=d$ is a $nIgsemi^*$-LC continuous function.

Definition 5.3. A function $f:(U,N,I)\to(V,M,J)$ is said to be ***L nIgsemi*-LC* continuous** if $f^{-1}(A)$ is $L\ nIgsemi^*$-LC set in (U,N,I) for every n-closed set A of (V,M,J).

Example 5.4. Consider the universal set $U=\{a,b,c,d\}$, the approximation space $U/\mathcal{R}=\{\{a\},\{c\},\{b,d\}\},X=\{a,b\}\subseteq U$ with the ideal $I=\{\varphi,\{a\}\}$. The nano topology defined by U is $\tau_R(X)=\{U,\varphi,\{a\},\{b,d\},\{a,b,d\}\}$ and $nIgsemi^*$ -open sets are $\{\{a\},\{b\},\{d\},\{a,b\},\{b,d\},\{a,d\},\{b,c,d\},\{a,b,d\},U,\varphi\}$. nano $semi^*$ -open sets are $\{\{a\},\{a,c\},\{b,d\},\{a,b,d\},U,\varphi\}$. n^*-closed sets are $\{\{a\},\{c\},\{a,c\},\{b,c,d\},U,\varphi\}$, $L\ nIgsemi^*$-LC sets are $\{\{a\},\{c\},\{a,c\},\{b,d\},U,\varphi\}$. Let $V=\{a,b,c,d\}$, the approximation space $V/R=\{\{a\},\{d\},\{b,c\}\}$, $Y=\{\{a\},\{d\}\}\subseteq V$ with the ideal $J=\{\varphi,\{a\}\}$. The nano topology defined by V is $\Omega_R(Y)=\{V,\varphi,\{a,d\}\}$ and $nIgsemi^*$-open sets are $\{\{a\},\{b\},\{c\},\{a,b\},\{b,c\},\{c,d\},\{a,c\},\{b,d\}\{b,c,d\},\{a,c,d\},\{a,b,d\},V,\varphi\}$. Then nano $semi^*$-open sets are $\{\{a,d\},V,\varphi\}$. n^*-closed sets are $\{\{a\},\{b,c\},V,\varphi\}$. The function $f:(U,N,I)\to(V,M,J)$ is defined as $f(a)=b,f(b)=a,f(c)=c,f(d)=d$ is a $L\ nIgsemi^*$-LC continuous function.

Theorem 5.5. A function $f:(U,N,I)\to(V,M,J)$ is n^*-continuous if and only if it is $L\ nIgsemi^*$-LC continuous and $nIgsemi^*$-continuous.

Proof. It is an immediate consequence of Theorem 4.7.

Remark 5.6 The notions of $L\ nIgsemi^*$-LC continuous and $nIgsemi^*$-LC continuous are independent.

Example 5.7. Let $U=\{a,b,c,d\}$, the approximation space $U/\mathcal{R}=\{\{a\},\{b\},\{c,d\}\}$, $X=\{\{b\},\{d\}\}\subseteq U$ with the ideal $I=\{\varphi,\{a\}\}$. The nano topology defined by U is $\tau_R(X)=\{U,\varphi,\{b\},\{c,d\},\{b,c,d\}\}$ and $nIgsemi^*$-open sets are $\{\{b\},\{c\},\{d\},\{c,d\},\{b,d\},\{b,c\},\{b,c,d\},U,\varphi\}$, and $nIgsemi^*$ -closed sets are $\{\{a\},\{a,b\},\{a,c\},\{a,d\},\{a,b,c\},\{a,c,d\},\{a,b,d\},U,\varphi\}$. Nano $semi^*$ -open sets are $\{\{b\},\{c,d\},\{b,c,d\},U,\varphi\}$ and n^*-closed sets are $\{\{a\},\{a,b\},\{a,c,d\},U,\varphi\}$. $L\ nIgsemi^*$-LC sets are $\{\{b\},\{c,d\},U,\varphi\}$ and $nIgsemi^*LC$ sets are $\{\{b\},\{c\},\{d\},\{c,d\},\{b,d\},\{b,c\},\{c,d\},U,\varphi\}$. Let $V=\{a,b,c,d\}$, the approximation space $V/\mathcal{R}=\{\{a\},\{d\},\{b,c\}\}$, $Y=\{\{a\},\{d\}\}\subseteq V$ with the ideal $J=\{\varphi,\{a\}\}$. The nano topology defined by V is $\Omega_R(Y)=\{V,\varphi,\{a,d\}\}$. The function $f:(U,N,I)\to(V,M,J)$ is defined as $f(a)=c,f(b)=b,f(c)=a,f(d)=d$ is a $nIgsemi^*$-LC continuous function. But the function f is not $L\ nIgsemi^*$-LC continuous since $f^{-1}(\{b,c\})=\{a,b\}\notin L\ nIgsemi^*$-$LC$ in (U,N,I).

Example 5.8. Consider the universal set $U=\{a,b,c,d\}$, the approximation space $U/\mathcal{R}=\{\{a\},\{c\},\{b,d\}\},X=\{a,b\}\subseteq U$ with the ideal $I=\{\varphi,\{a\}\}$. The nano topology defined by U is $\tau_R(X)=\{U,\varphi,\{a\},\{b,d\},\{a,b,d\}\}$and $nIgsemi^*$-open sets are $\{\{a\},\{b\},\{d\},\{a,b\},\{b,d\},\{a,d\},\{b,c,d\},\{a,b,d\},U,\varphi\}$. $nIgsemi^*$-closed sets are $\{\{a\},\{c\},\{c,d\},\{a,c\},\{b,c\},\{a,b,c\},\{b,c,d\},\{a,c,d\},U,\varphi\}$. Nano $semi^*$-open sets are $\{\{a\},\{a,c\},\{b,d\},\{a,b,d\},U,\varphi\}$. n^*-closed sets are $\{\{a\},\{c\},\{a,c\},\{b,c,d\},U,\varphi\}$, $L\ nIgsemi^*$-LC sets are $\{\{a\},\{c\},\{a,c\},\{b,d\},U,\varphi\}$. $nIgsemi^*LC$ sets are $\{\{a\},\{b\},\{c\},\{d\},\{a,b\},\{b,c\},\{c,d\},\{a,d\},\{b,d\},\{b,c,d\},U,\varphi\}$ Let $V=\{a,b,c,d\}$, the approximation space $V/\mathcal{R}=\{\{a\},\{d\},\{b,c\}\}$, $Y=\{\{a\},\{d\}\}\subseteq V$ with the ideal $J=\{\varphi,\{a\}\}$. The nano topology defined by V is $\Omega_R(Y)=\{V,\varphi,\{a,d\}\}$ and $nIgsemi^*$-open sets are $\{\{a\},\{b\},\{c\},\{a,b\},\{b,c\},\{c,d\},\{a,c\},\{b,d\}\{b,c,d\},\{a,c,d\},\{a,b,d\},V,\varphi\}$. Then nano $semi^*$-open sets are $\{\{a,d\},V,\varphi\}$. n^*-closed sets are $\{\{a\},\{b,c\},V,\varphi\}$.

The function $f : (U, N, I) \to (V, M, J)$ is defined as $f(a) = b, f(b) = a, f(c) = c, f(d) = d$ is a $L\ nIgsemi^*$-LC continuous function. But the function f is not $nIgsemi^*$-LC continuous since $f^{-1}(\{b, c\}) = \{a, c\} \notin nIgsemi^*LC$ in (U, N, I).

Conclusion

In this present work, we introduced the new concept namely $nIgsemi^*$ -locally closed sets, lightly $nIgsemi^*$ -locally closed sets, $nIgsemi^*$ - LC continuous mappings and $L\,nIgsemi^*$ - LC continuous mappings in nano ideal topological space (U, N, I). Also we studied and proved the basic properties of these locally closed sets.

References

[1] J. Joycy Renuka and Dr. J. Arul Jesti, "Nano Ideal Generalised $Semi^*$ -Closed Sets in Nano Ideal Topological Spaces" Turkish Journal of Computer and Mathematics Education, Vol 13, No.02 (2022), pp 385-393.
[2] K. Bhuvaneswari and K. Mythili Gnanapriya, Nano Generalized locally closed sets and NGLC-Continuous Functions in Nano Topological Spaces, Int . J.Math and its Applications,4(1-A)(2016)101-106.
[3] K. Kuratowski, Topology, Vol.I, Academic press, New York, 1966.
[4] M. LellisThivagar and Carmel Richard, "On Nano forms of weakly open sets" International Journal of Mathematics and Statistics Invention 1(1)(2013), pp 31-37
[5] M. Parimala and S.Jafari, "On Some New notions in nano ideal topological spaces", International Balkan Journal of mathematics, 1 (3) (2018), pp 85-92.
[6] M. Parimala, S. Jafari and S. Murali, "Nano ideal generalized closed sets in nano ideal topological spaces", Annales Univ.Sci. Budapest, 60(2017), pp 3-11.
[7] M. Parimala, S. Jafari, "On Some New Notions in Nano Ideal Topological Spaces", Eurasian Bulletin Of Mathematics Ebm, vol. 1, no. 3 (2018), pp 85-93
[8] M. Parimala, T.Noiri and S. Jafari, "New types of nano topological spaces via nano ideals", https://www.researchgate.net/publication/315892279.
[9] M.L. Thivagar and C. Richard, "On Nano Continuity, Mathematical Theory and Modelling" ,3 (7)(2013), pp 32-37.
[10] M.L. Thivagar and V.S. Devi, "New sort of operator in nano ideal topology", Ultra Scientist,28(1)(2016), pp 51-64.
[11] N. Bourbaki, General Topology, Part I, Addison – Wesley, Reading, Mass, 1966
[12] R. Asokan, O. Nethaji and I. Rajasekaran, "On nano generalized ⋆-closed sets in an ideal nano topological space", Asia Mathematika, 2(3), (2018), pp 50-58.
[13] R. Vaidyanathaswamy, Set topology, Chelsea Publishing Company, 1946
[14] Selvaraj Ganesan, "$nIg\mu$-Closed Sets In Nano Ideal Topological Spaces", Palestine Journal of Mathematics Vol. 10(1)(2021), pp 340–348