

A Detailed investigation of Pre-Coder Design in Massive MIMO and Comparing Different Precoder Schemes

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Abstract

The practical implementation of massive MIMO will enhance the performance of applications according to requirements. But, in order to obtain an optimal performance from the massive MIMO system lies on various different factors such as precoding scheme, resource scheduling and others. In this paper, we are investigating how the different precoding techniques are influencing the performance of massive MIMO system's performance. Therefore, a keen review on existing and recent literature has been conducted to identify the recent trends and techniques for adopting the precoding techniques. Next, the investigation involves the employment and experimental analysis of the two classical and popular precoders namely Zero Forcing (ZF) precoder and Wiener filter (WF) precoder. In addition, a recently available precoding technique namely Matched-Filter (MF) has also been evaluated experimentally. The simulation has been done and the trade off between signal to noise ratio (SNR) and bit error rate (BER) has been reported. According to the performed experimental evaluation the BER is reducing with the increasing SNR.

Keywords: Influence of Precoder, Massive MIMO, Need of Precoder, Precoder, Performance Evaluation, Quality of Service.

1. Introduction

The increasing data demand in personal and commercial use, motivate the cellular network providers to develop dense networks. These dense networks are promising to deliver target quality of service requirements in terms of efficiency and flexibility [1]. Massive multiple-input multiple-output (MIMO) is a concept to offer such kind of

service delivery. The massive MIMO offers larger bandwidth to deliver high quality experience and provides uniform service quality even in high-mobility based application scenarios [2]. This system involves Base Stations (BS) with multiple antenna arrays to serve large number of users, at the same time and/or frequency resources [3]. An overview of the massive MIMO system is demonstrated in figure 1.

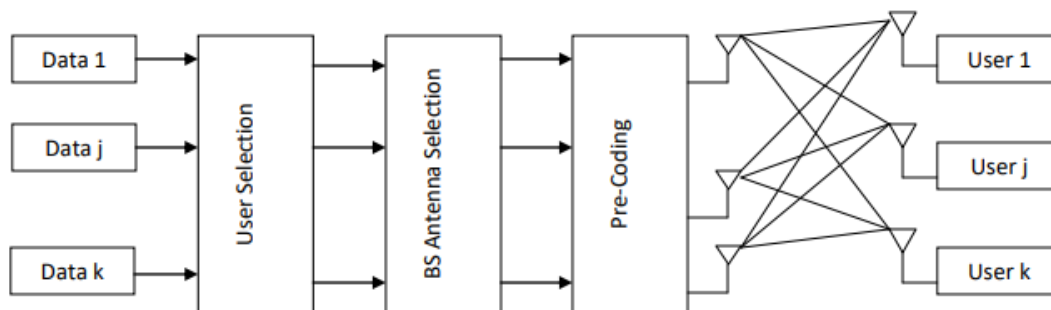


Figure 1: An overview of massive MIMO System

According to the given diagram the data which is required by users has passes through two initial constraints i.e. user selection and BS antenna

selection. In our recent study [4], we have discussed the aspects of user selection and Base Station (BS) antenna selection for enhancing the

performance of the massive MIMO system. Basically, these two components has used for scheduling the resources to enhance the capacity and capability of the massive MIMO base stations. This kind of scheduling has a positive impact on energy consumption of the massive MIMO systems. After selection of both the initial parameters user and antenna the precoding of the data has been done using different kinds of filters or precoders. The precoding techniques are responsible to combine the input signals in a predefined way and deliver them in a right proportion to the multiple antennas. Such precoding algorithms are crucial for the design of the massive MIMO system. In this paper the key aim is to investigate the precoding schemes and their influence on the performance of massive MIMO. Thus, the paper has been organized as follows.

This section provides an overview of the presented work involved in this paper. The next section

discusses the recent progress on the massive MIMO technology. Thus a review has been carried out and the key highlights have been discussed in the next section. In the third section the different precoding techniques has been discussed and then simulation has been performed. Next, the simulation results have been prepared and the conclusion has been drowned.

2. Related Work

This section reports the recent studies and development in the massive MIMO by improving the precoding techniques. Therefore, recently contributed research articles have been studied and a highlight has been described in this section.

A. Abbreviations

This section provides a list of abbreviations which are used in the remaining study.

Table 1 List of abbreviations

Keywords	Full form
AN	artificial noise
BSs	base stations
CEO	cross-entropy optimization
CSI	channel state information
DPC	dirty-paper coding
GMD	geometric mean decomposition
HR	high-resolution
IoTs	Internet of Things
MD-HP	matrix decomposition based hybrid processing
MIMO	multi-input multi-output
MM	minorize-maximize
mmWave	millimeter-wave
MT	mobile terminal
MUMIMO	multi-user MIMO
MMSE	minimum mean squared error
OFDM	orthogonal frequency division multiplexing
OMP	orthogonal matching pursuit
RF	radio frequency
SLNR	signal-to-leakage-plus-noise ratio
SNRs	signal-to-noise-ratios
SVD	singular value decomposition
ZF	zero-forcing

B. Related work

A hybrid precoding strategy for massive MIMO that makes use of spatial channel covariance matrices in the analog precoder design is proposed by S. Park et al. [1]. Applying a regularized zero-driving precoder for the baseband precoding network, they find an unconstrained simple precoder that expands SLNR while disregarding simple stage shifter requirements. They foster a method for an obliged simple precoder that emulates the unconstrained simple precoder under stage shifter requirements. The thought is to take on a baseband precoding lattice, which is known as a pay framework. They compare the loss from SLNR to that from fully digital precoding to figure out which factors affect loss. They demonstrate that, in comparison to fully digital precoding, the SLNR loss is negligible if the channel is spatially correlated and the number of users is smaller than the number of RF chains. The use of spatial channel matrices in a way that takes into account both the desired signal of each user and inter-user interference is what gives the method its advantage.

A. A. Lu et al [2] plan strong direct precoders for huge MIMO downlinks with blemished CSI explored. The effects of channel estimation error, channel aging, and spatial correlation are all taken into account when modeling the imperfect CSI for each UE at the BS as statistical CSI in a correlated channel with both the mean and variance of the channel. The goal is to expand the weighted aggregate rate. A robust linear precoder design algorithm is derived by combining the MM algorithm with the deterministic equivalent method. The expected point of the weighted sum-rate maximization problem is met by the algorithm. To lessen the computational intricacy, two low-intricacy calculations are determined. One for the general case, and the other for the situation when all the channel implies are zeros. Precoder design reduces power allocation optimization, and it is demonstrated that beam domain transmission is optimal. Results show that these hearty direct precoder plans apply to different situations and accomplish high effectiveness.

Hybrid processing has the potential to reduce the expense incurred by a large number of RF chains for the practical massive MIMO. It is carried out using analog RF and low-dimensional digital baseband. By breaking down a pre-designed, unconstrained digital precoder with a large dimension, W. Ni et al. [3] propose hybrid RF and baseband precoders for multi-stream transmission. The non-convex matrix decomposition, which is based on the optimization method, is solved by limiting the phase increment of each entry, which results in a series of convex subproblems. The SVD-based method is utilized to get an underlying point near the arrangement of the first non-raised issue. Performance is demonstrated and the MD-HP scheme's convergence is examined.

To save money, the number of RF chains at the BSs may be smaller than the number of BS antennas in actual massive MIMO deployments. A security-enhancing framework for joint data and AN precoding in massive MIMO with a limited number of RF chains is proposed by J. Zhu et al. [4]. For both analog and hybrid precoding, they establish a lower bound on the ergodic secrecy rate of any MT using imperfect CSI. The shut structure lower bound is utilized to decide the ideal power parting among information and AN, which expands the mystery rate through a straightforward hunt. Although the hybrid precoder has a lower secrecy rate than the theoretical full-dimensional precoder, the results show that it is free of high computational complexity and has lower hardware costs.

A crucial strategy for expanding the network's capacity and coverage is to make use of massive MIMO and mmWave. For super thick mmWave heterogeneous situations with various clients, one ought to address both between and intra-level obstructions in spite of the low-thickness situations. D. Castanheira et al [5] propose low complex crossover simple advanced get and communicate shaft framing for super thick uplink huge MIMO frameworks to relieve obstructions. Where the analog processing takes place, the hybrid analog-digital receive beam-forming/equalizer is computed. Consider the Frobenius norm's distance from the fully digital

counterpart as a metric for optimizing the analog portion of the user terminal precoders. Aside from the simple requirements typically viewed as inhomogeneous frameworks, they force imperatives intrinsic to the conveyed idea of the passageways. The digital precoders used to cancel inter-tier interference place it in a low-dimensional subspace. The outcomes show that the presentation of the half-and-half simple advanced precoder/adjuster conspires is near the completely computerized partner and can effectively eliminate obstructions.

Half and half precoding can diminish the quantity of required RF chains in mmWave huge MIMO. The existing hybrid precoding based on SVD necessitates complex bit allocation to match the various sub-channel SNRs. A GMD-based hybrid precoding is proposed by T. Xie et al. [6] to avoid complicated bit allocation. They are looking for two analog and two digital precoders that are close to the free digital GMD precoder. They design the digital precoder from the analog one, and vice versa. The OMP algorithm is used to create the analog precoder, while GMD is used to create the digital precoder. Reproductions show that this GMD-based crossover precoding accomplishes preferred execution over SVD-based half-breed precoding.

In real life, it is necessary to know what advantages can be gained with a large number of antennas but a limited number in realistic settings. The BS has 128 antennas, so G. Xiang et al. [7] evaluate the properties of residential-area channels. Because this property reveals the MUMIMO pre-coding schemes we require in the downlink, the orthogonality between channels and users is taken into consideration. They demonstrate that orthogonality improves as the number of antennas increases, but only for two users with a single antenna. The sum-rate performance of ZF and MMSE, two linear precoding schemes, is also evaluated in relation to the number of BS antennas. As of now at 20 BS receiving wires, these direct pre-coding plans reach 98% of the ideal DPC limit with respect to the deliberate channels.

Massive MIMO presents brand-new difficulties that have piqued the interest of researchers and

businesses alike: CSI securing, channel input, momentary correspondence, measurable correspondence, models, and equipment impedances. D. C. Araújo et al [8] present an outline of the fundamental ideas of enormous MIMO, with an emphasis on the difficulties and open doors.

R. C. de Lamare et al [9] present an instructional exercise on enormous MIMO. Signal-handling difficulties and future patterns in monstrous MIMO are introduced, and key applications are nitty gritty. The system and data models of massive MIMO are described using a linear algebraic method. In addition to time-division duplexing, resource allocation, and calibration, massive MIMO's operational requirements are discussed. Transmitter and collector handling calculations are analyzed considering the particular requirements of gigantic MIMO. Results show the presence of transmitter and collector handling calculations.

T. Mir et al [10] explore an energy-proficient crossover precoding plan involving the slightest bit PSs for commonsense recurrence-specific wideband mmWave huge MIMO. They provide an analysis of the energy consumed to demonstrate that the array gain loss caused by using one-bit PSs is negligible and that the energy consumed by one-bit PSs is significantly lower than that of HR-PSs. Besides, roused by the President's calculation for AI, they propose a Chief based half and half precoding plan to expand the aggregate rate. In CEO-based hybrid precoding, the elements' probability distributions are updated to reduce cross-entropy between the two probability distributions in order to produce a final solution. They show that the CEO-based hybrid precoding scheme can be used in other situations by extending it from the case with one-bit PSs to the general case with HR-PSs. The presentation shows the way that the plan can acquire an ideal total rate and higher energy effectiveness.

S. Malkowsky et al [11] offer a structure for a huge MIMO testbed by examining equipment and framework-level necessities. Taking into account a conventional and it is proposed to deal with bunching. That makes processing distribution and scaling possible. In light of the HW imperatives, for

example, a most extreme number of connections and greatest throughput joined with handling capacities, it empowers estimating HW parts. To check the plan, they present the Lund College Huge MIMO testbed.

To decrease the power utilization of monstrous MIMO, half-breed handsets, and low-goal application converters are utilized. L. N. Ribeiro et al [12] explore the energy proficiency of quantized crossover transmitters with a completely/to some extent associated stage moving organization and contrast them with quantized computerized precoders. Based on additive quantization noise estimates, they offered quantized single-user MIMO. Real power expenditure and loss models were used to evaluate the energy efficiency of the precoding methods. To some degree associated crossover precoders are energy-productive when contrasted with advanced precoders. Likewise, completely associated cross-breed precoders show unfortunate proficiency. An energy-spectral tradeoff is provided by component topology.

Recent nonlinear precoding algorithms for a downlink transmit signal is constrained by nonlinearity. These methods use iterative search algorithms to design the signal that is transmitted from each antenna. Since the dimension of the search space equals the number of antennas, the computational complexity of these approaches can be high. Thus, *A. L. Swindlehurst et al [13]* pose the problem in a smaller dimensional space by constraining the signal prior to the nonlinearity to be the output of a linear precoder. The search is then over the vector of pre-distorted symbols at the input to the linear precoder, which is smaller than the number of antennas. They focus on algorithms that minimize the bit error rate.

The massive MIMO requires a trade-off between cost and power. Thus need to maintain balance between the complexity and performance. Recent design split the pre-coding at transmitter and combining at receiver. *R Corvaja et al [14]* study effect of phase noise in the design of hybrid alternatives to massive MIMO. Its effect depending on the number of RF chains, oscillators, and groups of antennas.

G. Anjos et al [15] design channel precoder and security scheme for massive MIMO. They ensure the precoding operation does not reduce the degree of secrecy offered by the security technique. The concept of the technique is to use selective random cycles in transmitted signal at the antenna level to get a compromise between legitimate and malicious channel capacities. These rotations use the reciprocal wireless channel as random source. To assess the security, the scheme is compared with a recent approach. The number of antennas does not influence malicious channel capacity.

3. Precoding Techniques

This section provides the discussion about the different precoding techniques and their initial concept. Thus first we need to understand the system model of massive MIMO system. This system includes a BS with B antennas. These antennas are used for serving U single-antenna user-equipments (UEs). We consider a block-fading and narrowband scenario as:

$$y[k] = Hx[k] + n[k], \quad k = 1, 2, \dots, K \dots \dots \dots (1)$$

Where, $y[k]$ is a U-dimensional vector such that:

$$y[k] = [y_1[k], \dots, y_U[k]]^T$$

It contains the signals received at all UEs with $y_u[k] \in \mathcal{C}$ corresponding to the signal received in time slot k. The matrix $H \in \mathcal{C}^{U \times B}$ represents the downlink MIMO channel and assumed to remain constant. The vector $n[k] \in \mathcal{C}^U$ models additive noise and assumed to be independent and identically distributed (i.i.d.). The channel matrix H and noise variance N_0 is needed to be known at the BS. The pre-coded vector $x[k] \in \mathcal{C}^B$ at time slot k is denoted using:

$$x[k] = P(s[k], H, N_0, \rho^2)$$

It is depends on transmit signal vector $s[k] \in \mathcal{O}^U$, where \mathcal{O} is the constellation set, the channel matrix H, the noise variance N_0 , and the power constraint ρ^2 . The precoded vector is needed to satisfy the average power constraint

$$E_s[\|x[k]\|^2] \leq \rho^2, \quad k = 1, 2, \dots, K \dots \dots \dots (2)$$

And the vector $s[k] = [s_1[k], \dots, s_U[k]]^T$ contains the information symbols $s_u[k] \in \mathcal{O}$ to be transmitted to UE.

A. Wiener filter

As UEs are unable to perform signal processing, BS has to precode the information symbols with justifying multi-user interference (MUI) and power. However, there are a number of optimization criteria for precoding, such as sum-rate or error. The linear precoders $x = Ps$ minimize the mean-square error (MSE) between the estimated symbol vector \hat{s} and the transmit signal vector s . Since coherent transmission with a multi-antenna BS leads to an array gain, we assume that the UEs are able to scale the received signals $y_u, u = 1, \dots, U$, by a precoding factor $\beta_u \in \mathbb{C}$, i.e., the UEs compute estimates of the transmit symbols as follows:

$$\hat{s}_u = \beta_u y_u$$

While each UE u would be able to estimate their own precoding factor β_u , we design precoders that minimize the MSE for a joint precoding factor $\beta \in \mathbb{C}$ defined as:

$$\begin{aligned} \text{MSE} &= E_{s,n}[\|s - \hat{s}\|^2] = E_{s,n}[\|s - \beta y\|^2] \\ &= E_s[\|s - \beta Hx\|^2] + |\beta|^2 UN_0 \end{aligned}$$

The MSE-optimal linear precoding matrix $P \in \mathbb{C}^{B \times U}$ can be designed by solving the following optimization problem:

$$\begin{cases} \{P^{WF}, \beta^{WF}\} \\ \text{minimize}_{P, \beta} E_s[\|s - \beta HP_s\|^2] + |\beta|^2 UN_0 \\ \text{subject to } E_s[\|P_s\|^2] \leq \rho^2 \end{cases} \dots \dots \dots (3)$$

The solution to this optimization problem is known as the Wiener filter (WF) precoder. The WF precoding matrix P^{WF} is given by

$$P^{WF} = \frac{1}{\beta^{WF}} Q^{WF}$$

Where we define the matrix

$$Q^{WF} = (H^H H + \kappa^{WF} I_B)^{-1} H^H \dots \dots \dots (4)$$

The associated regularization parameter κ^{WF} and precoding factor β^{WF} are defined as

$$\kappa^{WF} = \frac{UN_0}{\rho^2}$$

And

$$\beta^{WF} = \sqrt{\frac{\text{tr}((Q^{WF})^H Q^{WF}) E_s}{\rho^2}} \dots \dots \dots (5)$$

The simple calculation of the precoding factor β^{WF} in (5) involves a $B \times B$ matrix followed by a number of matrix to matrix multiplications.

B. Zero Forcing

Zero-forcing precoding is a method by which a multiple antenna transmitter can null the multiuser interference in a multi-user MIMO system. When the channel state information is known at the transmitter, the zero-forcing precoder is given by the pseudo-inverse of the channel matrix. The signal model which is defined as:

$$y = H^T W D_s + n$$

Where, y is the $k \times 1$ receiver signal vector, $H = [h_1, h_2, \dots, h_k]$ is $N_t \times K$ channel matrix, $D = \text{diag}(\sqrt{p_1}, \sqrt{p_2}, \dots, \sqrt{p_k})$ is a $K \times K$ diagonal power matrix and $s = [s_1, s_2, \dots, s_k]^T$ is a $K \times 1$ transmit signal.

Then a zero-forcing precoder is defined as a precoder where w_i intended for user i is orthogonal to every channel vector h_j associated with users j where $j \neq i$.

$$w_i \perp h_j \text{ if } i \neq j$$

Thus, the interference caused by the signal meant for one user is effectively nullified for the users via zero-forcing precoder. Each beam generated by zero-forcing precoder is orthogonal to all the other user channel vectors, we can rewrite the received signal as:

$$\begin{aligned} y_k &= h_k^T \sum_{i=1}^K \sqrt{P_i} s_i w_i + n_k = h_k^T w_k \sqrt{P_k} s_k + n_k, \quad k \\ &= 1, 2, \dots, K \dots \dots \dots (6) \end{aligned}$$

The orthogonality can be expressed as:

$$H^T W = Q$$

Where Q is a $K \times K$ diagonal matrix, Q is selected as identity matrix. This makes W the right Moore-Penrose pseudo-inverse of H^T given by:

$$W = (H^T)^+ = H(H^T H)^{-1}$$

Given this zero-forcing precoder design, the received signal at each user is decoupled from each other as:

$$y_k = \sqrt{P_k} s_k + n_k, \quad k = 1, 2, \dots, K \dots \dots \dots (7)$$

C. Matched Filter Precoding

Transmit matched filter (TxMF) maximizes the signal-to interference ratio (SIR) at the receiver and promising to deliver optimum results for high signal-to-noise-ratio scenarios. The TXMF is also used for non-cooperative cellular wireless network. The MF is derived by maximizing the ratio between the power of the desired signal portion in the received signal and the signal power under the transmit power constraint.

$$T = arg \max_T \frac{E(\|X^H \tilde{y}\|^2)}{E(\|n\|^2)} \dots \dots \dots (8)$$

$$subject \ to: EE(\|Tx\|^2) \leq \rho$$

Where \tilde{y} is the noiseless received signal $\tilde{y} = Tx$
The solution of (8) is given by:

$$T = \beta H^H$$

With

$$\beta = \sqrt{\frac{1}{tr(H^H H)}} \dots \dots \dots (9)$$

The received signal is given by:

$$y = HH^H x + n = \beta U \Lambda U^H x + n \dots \dots \dots (10)$$

4. Simulation and Results

The simulation of three popular precoders has been evaluated and compared in this section. In this context the required simulation parameters has given in table 2.

Table 2 simulation parameters

Parameters	Values
Precoders	MF, ZF and WF
No of BS antennas	32, 64, 128
No of Users	8, 16, 32

By using the above discussed simulation parameters a simulation using MATLAB has been performed. Additionally the tread off between SNR and BER has been measured and reported with the fixed number of antenna at the base station and increasing number of user equipments. There are two type of experiments has been done, first performance has been compared between precoder algorithms by varying number of user devices. Second experiment has been performed based on comparing same algorithm's performance with increasing number of users.

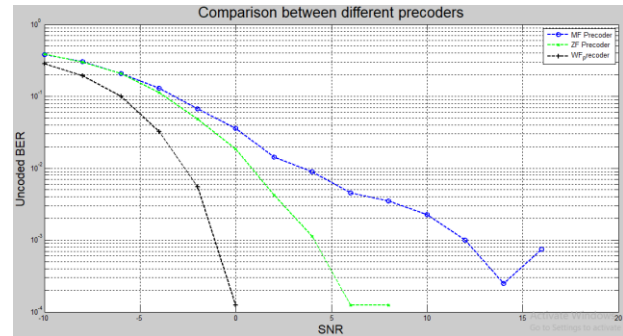


Figure 2: Comparative BER of different precoding algorithms with the number of users 8

In figure 2 the comparative BER with respect to SNR has been demonstrated for the implemented algorithms namely MF, WF and ZF. The figure contains the performance of MIMO system for different precoding scheme for a number of 8 users. According to the comparison the WF algorithm provides much better performance as compared to other two algorithms. Next, figure 3 shows the performance of algorithms with the total of 16 users. Here, we can see the difference between the performances of MIMO system when we increase the number of users. According to comparing both the scenarios we found the when the number of user increasing the BER is increasing for all the algorithms. Further for the total number of 32 users the performance has been measured and reported in figure 4. We can also observe using the results the BER is reducing with the increasing SNR. In both the algorithms ZF and MF the increasing SNR slightly reduces BER. Additionally with the increasing SNR it is start being parallel to the SNR. Therefore, if the BS has fewer resources to serve the network the performance of user experience has been degraded because when we increasing number of users and keep limited number of BS antennas.

On the other hand the performance of three precoding algorithms is also measured. Figure 5 shows the performance of MIMO system using MF precoding technique. Based on the performance of the given in figure the MF algorithm is producing higher BER among all the algorithms. Additionally, as the SNR has increasing the BER become worst and producing low quality results.

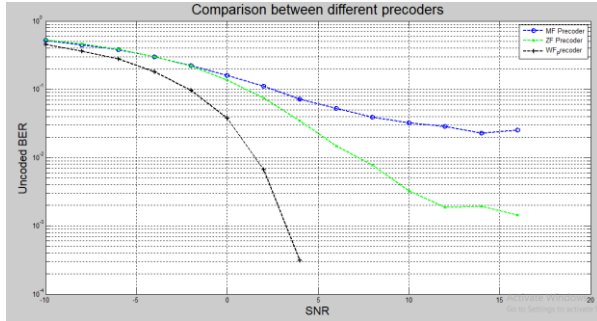


Figure 3: Comparative BER of different precoding algorithms with the number of users 16

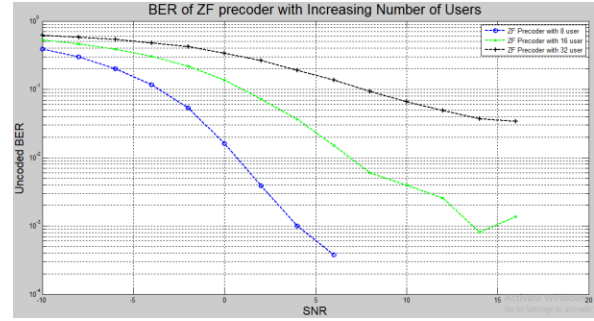


Figure 6: Influence of increasing number of users on MIMO performance for ZF Precoder

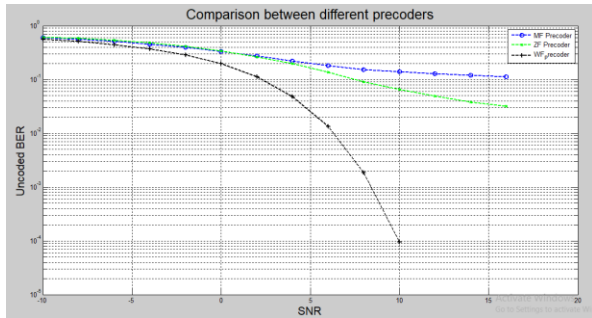


Figure 4: Comparative BER of different precoding algorithms with the number of users 32

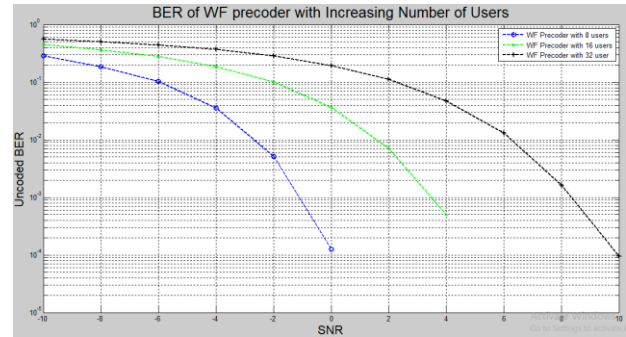


Figure 7: Influence of increasing number of users on MIMO performance for WF Precoder

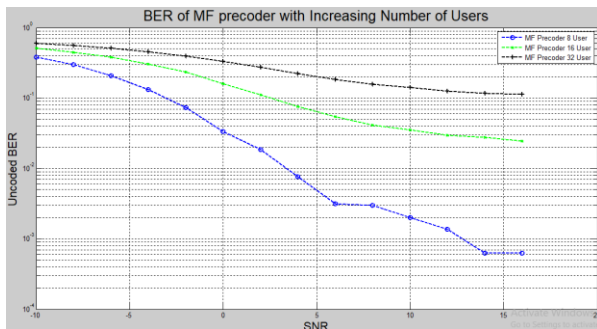


Figure 5: Influence of increasing number of users on MIMO performance for MF Precoder

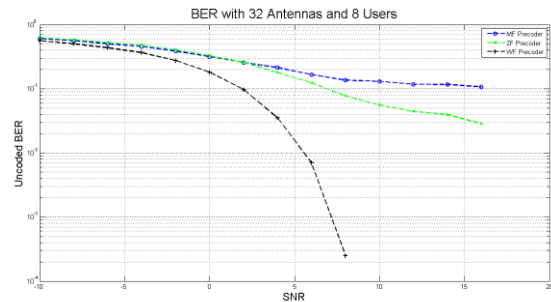


Figure 8: Performance of WF precoder with 32 BS antennas and 8 users

Next the performance of ZF algorithm is demonstrated using figure 6. When we comparing the performance MF and ZF algorithm we found the ZF is far better than MF algorithm and reports the low BER as compared to MF algorithm. Next, the figure 7 shows the WF algorithm's performance. This algorithm provides better results and able to cancel the noise even in higher SNR.

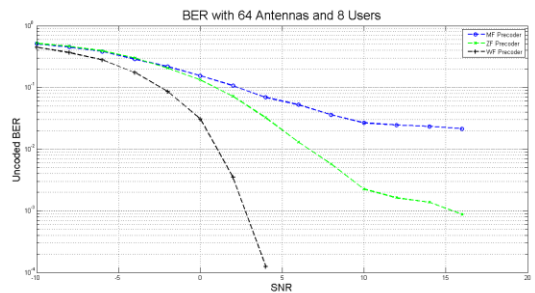


Figure 9: Performance of WF precoder with 64 BS antennas and 8 users

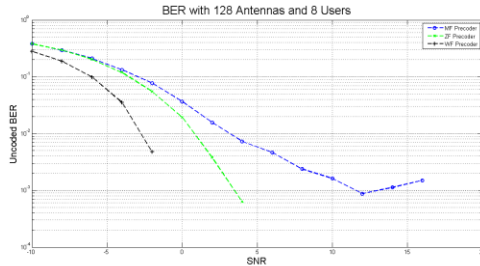


Figure 10: Performance of WF precoder with 128 BS antennas and 8 users

The performance of massive MIMO system has also investigated with varying the amount of base station antenna. In this context, the number of antennas is increasing with the same number of users. Here, a total of 8 fixed set of users are considered for measuring the results additionally the number of antennas are increased i.e. 32, 64 and 128. Figure 8 demonstrate the performance of MIMO system for 8 users and 32 antennas, which shows higher BER as compared to other two scenarios as demonstrated in figure 9 and figure 10. Basically, when the number of antennas in base station has increasing the resource availability has been improved significantly therefore with the increasing number of antennas the BER is reducing more frequently as compared to low resource scenarios.

5. Conclusion and Future work

This paper reports a comparative performance study between three popular massive MIMO precoding algorithms. The precoding schemes are used to maintain service quality of application. That highly influences the performance of massive MIMO systems. Therefore, in order to understand how the performance is fluctuating with increasing number of users with the limited resources (BS antennas). Thus, a simulation of massive MIMO system has been developed additionally three algorithms namely MF, ZF and WF has been implemented to demonstrate the tread off between SNR and BER. According to the results the WF is providing more effective results as compared to ZF and MF algorithm. Based on the results we also found if the resources are limited and number of users is increases then the performance of quality of service is degraded significantly. Therefore

appropriate use of precoding is necessary for achieving high quality of service in massive MIMO systems. In this paper we investigate the effect of increasing number of users in massive MIMO precoding schemes. Next we planned to discuss the energy consumption of massive MIMO base stations.

References

- [1] R. Dangi, P. Lalwani, G. Choudhary, I. You, G. Pau, "Study and Investigation on 5G Technology: A Systematic Review", *Sensors* 2022, 22, 26
- [2] R. Chataut, R. Ak, "Massive MIMO Systems for 5G and Beyond Networks—Overview, Recent Trends, Challenges, and Future Research Direction", *Sensors* 2020, 20, 2753;
- [3] E. G. Larsson, O. Edfors, F. Tufvesson, T. L. Marzetta, *Massive MIMO for Next Generation Wireless Systems*, 2014, IEEE Communications Magazine, (52), 2, 186-195.
- [4] Nikhil Rathore and Dr. Santosh Pawar, "THE ROLE OF OPTIMIZATION IN POWER ALLOCATION OF MASSIVE MIMO SYSTEMS", *Journal of Data Acquisition and Processing* Vol. 38, issue 1, pages 5596-5610, 2023
- [5] S. Park, J. Park, A. Yazdan, R. W. Heath, "Exploiting Spatial Channel Covariance for Hybrid Precoding in Massive MIMO Systems", *IEEE Transactions on Signal Processing*, Vol. 65, No. 14, July 15, 2017
- [6] A. A. Lu, X. Gao, W. Zhong, C. Xiao, X. Meng, "Robust Transmission for Massive MIMO Downlink with Imperfect CSI", 0090-6778, 2019 IEEE
- [7] W. Ni, X. Dong, W. S. Lu, "Near-Optimal Hybrid Processing for Massive MIMO Systems via Matrix Decomposition", arxiv, (2015)
- [8] J. Zhu, W. Xu, N. Wang, "Secure Massive MIMO Systems with Limited RF Chains", (2016)
- [9] D. Castanheira, P. Lopes, A. Silva, A. Gameiro, "Hybrid Beamforming Designs for Massive MIMO Millimeter-Wave Heterogeneous Systems", Vol. 5, 2017
- [10] T. Xie, L. Dai, X. Gao, M. Z. Shakir, J. Li, "Geometric Mean Decomposition Based Hybrid Precoding for Millimeter-Wave Massive MIMO", *China Communications*, pages 229- 238, May 2018
- [11] G. Xiang, E. Ove, R. Fredrik, T. Fredrik, "Linear pre-coding performance in measured very-large

MIMO channels”, Proc. of the 74th IEEE Vehicular Technology Conference, pages 6, 2011

[12] D. C. Araújo, T. Maksymyuk, A. L.F. de Almeida, T. Maciel, J. C. M. Mota, M. Jo, “Massive MIMO: survey and future research topics”, IET Communications, Special Issue: Evolution and Development of 5G Wireless Communication Systems, pages 1938-1946, Vol. 10, Iss. 15, 2016

[13] R. C. de Lamare, “Massive MIMO Systems: Signal Processing Challenges and Future Trends”, The Radio Science Bulletin, No 347, pages 8-20, Dec 2013

[14] T. Mir, M. Z. Siddiqi, U. Mir, R. Mackenzie, M. Hao, “Machine Learning Inspired Hybrid Precoding for Wideband Millimeter-Wave Massive MIMO Systems”, IEEE, Vol. 7, 2019

[15] S. Malkowsky, J. Vieira, L. Liu, P. Harris, K. Nieman, N. Kundargi, I. C. Wong, F. Tufvesson, V. Öwall, O. Edfors, “The World’s First Real-Time Testbed for Massive MIMO: Design, Implementation, and Validation”, IEEE Access, Vol. 5, pages 9073-9088, 2017

[16] L. N. Ribeiro, S. Schwarz, M. Rupp, A. L. F. de Almeida, “Energy Efficiency of mmWave Massive MIMO Precoding with Low-Resolution DACs”, 2018 IEEE

[17] A. L. Swindlehurst, H. Jedda, I. Fijalkow, “Reduced Dimension Minimum BER PSK Precoding for Constrained Transmit Signals in Massive MIMO”, ICASSP, IEEE, pages 3584-3588, 2018

[18] R. Corvaja, A. G. Armada, M. A. Vazquez, A. P. Neira, “Design of Pre-coding and Combining in Hybrid Analog-Digital Massive MIMO with Phase Noise”, EURASIP, 25th European Signal Processing Conference (EUSIPCO), pages 2527-2531, 2017

[19] G. Anjos, D. Castanheira, A. Silva, A. Gameiro, M. Gomes, J. Vilela, “Joint Design of Massive MIMO Precoder and Security Scheme for Multiuser Scenarios under Reciprocal Channel Conditions”, Hindawi Wireless Communications and Mobile Computing, Vol. 2017, Article ID 5396092, pages 10