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Boros Integral Associated with Generalized Galué Type Struve and K-Struve Functions

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Abstract:
The aim of this paper is to investigate Boros integral with three parameters, involving generalized Galué- type Struve function and k-Struve function. The outcomes of results are expressed in terms of the generalized Wright hypergeometric function. Several interesting corollaries of various Struve functions and generalized functions are deduced as special cases. The applications of the obtained results are useful in applied mathematical sciences.

Keywords: Gamma function, Generalized hypergeometric function${}_pF_q$, Generalized (Wright) hypergeometric functions ${}_p\Psi_q$, Galué type Struve function, k-Struve function, Boros integral formula.
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1. Introduction and Preliminaries

Numerous integral formulae associated with generalized special functions have been derived in the last decade due to their potential applications in science and engineering [1-4]. A lot of research has been carried out on generalized Struve function and its applications [5-9]. In this sequel, we focus our work on two important special functions, Galué type Struve and k-Struve functions.

Non-homogeneous Bessel differential equation is defined as

$$z^2u^{``}(z) + zu^{`}(z)+(z^2 - p^2)u(z) = \frac{4\left(\frac{z}{2}\right)^{p+1}}{\Gamma(p+\frac{1}{2})\sqrt{\pi}} \quad (1.1)$$

The particular solution of the non-homogeneous Bessel differential equation is known as Struve function [10].It is defined as follows:

$$S_n(z) = \sum_{k=0}^{\infty} \frac{(-1)^k\left(\frac{z}{2}\right)^{2k+n+1}}{\Gamma(k+\frac{3}{2})\Gamma(k+n+\frac{3}{2})} \quad (1.2)$$

where $\Gamma(z) = \int_0^\infty t^{z-1}e^{-t}dt, \Re(z) > 0$

Furthermore, Orhan and Yagmur have introduced several generalizations of Struve function [11] and [12], as below:

$$S_{n,b,c}(z) \sum_{k=0}^{\infty} \frac{(-c)^k\left(\frac{z}{2}\right)^{2k+n+1}}{\Gamma(k+\frac{3}{2})\Gamma(k+n+\frac{b+2}{2})}, n, b, c \in \mathbb{C} \quad (1.3)$$

A new generalization of Galué-type Struve function is introduced by Nissar et al. [13] as follows: For $n, b, c \in \mathbb{C}$ and $q \in \mathbb{N}$

$${}_q W_{n,b,c,\xi}^{\alpha,\mu}(z) = \sum_{k=0}^{\infty} \frac{(-c)^k\left(\frac{z}{2}\right)^{2k+n+1}}{\Gamma(\alpha k+\mu)\Gamma(qk+\frac{n}{\xi}+\frac{b+2}{2})} \quad (1.4)$$

The k-Struve function, introduced by Nisar et al. [14] is another generalization of the Struve function.

$$S_{\xi,c}^k(z) = \sum_{r=0}^{\infty} \frac{(-c)^r\left(\frac{z}{2}\right)^{2r+\frac{\xi}{k}+1}}{\Gamma_k\left(rk+\xi+\frac{3k}{2}\right)\Gamma\left(r+\frac{3}{2}\right)} \quad (1.5)$$

In 2007, Diaz and Pariguan [15] introduced the k-Pochhammer symbol and k-gamma function as follows:

$$(\gamma)_{n,k}=\begin{cases}\frac{\Gamma_k(\gamma+nk)}{\Gamma_k(\gamma)} & (k\in\Re;\ \gamma\in\mathbb{C}\setminus\{0\})\\ \gamma(\gamma+k)\dots(\gamma+(n-1)k) & (n\in\mathbb{N};\ \gamma\in\mathbb{C}),\end{cases} \tag{1.6}$$

The following relation holds

$$\Gamma_k(\gamma)=k^{\frac{\gamma}{k}-1}\Gamma\left(\frac{\gamma}{k}\right) \tag{1.7}$$

Clearly, for $k=1$, (1.7) reduces to the classical Pochammer symbol and Euler's gamma function, respectively [16].

A generalization of the generalized hypergeometric series ${}_pF_q$(1.7) is due to Wright [17-18] and Fox [19] gave the generalized (Wright) hypergeometric function

$${}_p\Psi_q\left[\begin{matrix}(\alpha_1,A_1),\dots,(\alpha_p,A_p)\,;z\\(\beta_1,B_1),\dots,(\beta_q,B_q);\end{matrix}\right]=\sum_{k=0}^{\infty}\frac{\prod_{j=1}^{p}\Gamma(\alpha_j+A_jk)}{\prod_{j=1}^{q}\Gamma(\beta_j+B_jk)}\frac{z^k}{k!} \tag{1.8}$$

Where the coefficients $A_1,\dots,A_q$ and $B_1,\dots,B_q$are real positive numbers such that

$$1+\sum_{j=1}^{q}B_j-\sum_{j=1}^{p}A_j\geqq 0 \tag{1.9}$$

A special case of (1.4) is

$${}_p\Psi_q\left[\begin{matrix}(\alpha_1,1),\dots,(\alpha_p,1)\,;z\\(\beta_1,1),\dots,(\beta_q,1);\end{matrix}\right]=\frac{\prod_{j=1}^{p}\Gamma(\alpha_j)}{\prod_{j=1}^{q}\Gamma(\beta_j)}\,{}_pF_q\left[\begin{matrix}\alpha_1,\dots,\alpha_p\,;z\\\beta_1,\dots,\beta_q\,;\end{matrix}\right] \tag{1.10}$$

Where ${}_pF_q$ is the generalized hypergeometric series [20].

$${}_pF_q\left[\begin{matrix}\alpha_1,\dots,\alpha_p\,;z\\\beta_1,\dots,\beta_q\,;\end{matrix}\right]=\sum_{n=0}^{\infty}\frac{(\alpha_1)_n,\dots,(\alpha_p)_n\,;z^n}{(\beta_1)_n,\dots,(\beta_q)_n n!} \tag{1.11}$$

Lemma 1. For $a>-\sqrt{lm}, l>0, m\geq 0$ *and* $p>\frac{1}{2}$, the Boros integral [21] which depends on three parameters is

$$\int_0^{\infty}\left(\frac{x^2}{lx^4+2ax^2+m}\right)^p dx=\frac{\sqrt{\pi}\Gamma(p-\frac{1}{2})}{\Gamma(p)\sqrt{l}2^{p+\frac{1}{2}}(a+\sqrt{lm})^{p-\frac{1}{2}}} \tag{1.12}$$

2. Relation of generalized Galué type Struve function ${}_qW^{\alpha,\mu}_{n,b,c,\xi}(z)$ with some generalized special functions:

In this section, the connection of generalized Galué type Struve function with some generalized special functions will be enumerated as follows:

(i) On substituting $\xi=\alpha=q=1$ and $\mu=\frac{3}{2}$ in (1.4), it reduces to generalized Struve function $S_{n,b,c}(z)$ as defined by Yagmur and Orhan [11] and [12].

$${}_1w^{1,\frac{3}{2}}_{n,b,c,1}(z)\rightarrow S_{n,b,c}(z) \tag{2.1}$$

(ii) On substituting $\xi=\alpha=q=b=c=1$ and $\mu=\frac{3}{2}$ in (1.4), it reduces to Struve function $S_n(z)$ of order n as defined in [10].

$${}_1w^{1,\frac{3}{2}}_{n,1,1,1}(z)\rightarrow S_n(z) \tag{2.2}$$

(iii) On substituting $\xi=\alpha=q=b=1, c=-1$ and $\mu=\frac{3}{2}$ in (1.4), reduces to modified Struve function $S_n(z)$ of order n defined in [22].

$${}_1w^{1,\frac{3}{2}}_{n,1,1,1}(z)\rightarrow H_n(z) \tag{2.3}$$

(iv) For $\xi=q=1, b=2, c=n=-1$ and $z=2\sqrt{z}$ in (1.4), reduces to Wright function $\phi(\mu,\alpha;z)$ as defined in [23],

$${}_1w^{\alpha,\mu}_{-1,2,-1,1}(2\sqrt{z})\rightarrow\phi(\mu,\alpha;z) \tag{2.4}$$

(v) On putting $\xi=\mu=1, \alpha=b=0, c=-1, n=n-1$ and $z=2\sqrt{z}$ in (1.4),we get

$${}_qw^{0,1}_{n-1,0,-1,1}(2\sqrt{z})\rightarrow z^{n/2}E_{q,n}(z) \tag{2.5}$$

Where $E_{q,n}(z)$generalized Mittag-Leffler function defined in [24]

(vi) On putting $\xi=\mu=\alpha=c=1, b=2$ *and* $n=n-1$ in (1.4),we get

$${}_{q}w^{1,1}_{n-1,1,-1,1}\ (z) \to z^{n/2}\,{}_{q}I_{n}(z) \qquad (2.6)$$

Where ${}_{q}I_{n}(z)$ is Galué type generalization of modified Bessel function defined in [25].

(vii) On putting q= $\xi=\mu=\alpha=c=1, b=2$ *and* $n=n-1$ in (1.4),we get

$${}_{q}w^{1,1}_{n-1,1,-1,1}\ (z) \to I_{n}(z) \qquad (2.7)$$

Where $I_n(z)$ is the modified Bessel function of first kind of order n [26].

(viii)Relation with Fox-Wright function [17-19] is

$${}_{q}w^{\alpha,\mu}_{n,b,c,\xi}\ (z) \to \left(\frac{z}{2}\right)^{n+1} {}_{1}\Psi_{2}\left[\begin{matrix}(1,1) & ; \frac{-cz^2}{4}\\ (\mu,\alpha),\left(\frac{n}{\xi}+\frac{b+2}{2},q\right) & \end{matrix}\right] \qquad (2.8)$$

3. Main Results

Theorem 1.
$$\int_0^\infty \left(\frac{x^2}{lx^4+2ax^2+m}\right)^p {}_{q}\mathrm{W}^{\alpha,\mu}_{n,b,c,\xi}\left(\frac{zx^2}{lx^4+2ax^2+m}\right)dx = \sqrt{\frac{\pi}{l}}\,\frac{\left(\frac{z}{2}\right)^{n+1}}{2[2(a+\sqrt{lm})]^{p+n+\frac{1}{2}}}\ {}_{2}\Psi_{3}\left[\begin{matrix}\left(p+n+\frac{1}{2},2\right),(1,1) & ; \frac{-cz^2}{16\left(a+\sqrt{lm}\right)^2}\\ (\mu,\alpha),\left(\frac{n}{\xi}+\frac{b+2}{2},q\right),(p+n+1,2) & \end{matrix}\right]$$

Under the conditions $a>-\sqrt{lm}, l>0, m\geq 0$ *and* $p>\frac{1}{2}$ (3.1)

Proof. Let $\mathcal{I}_1$ be the left-hand side of (3.1) and applying (1.12) to the integrand of (3.1)

$$\mathcal{I}_1=\int_0^\infty \left(\frac{x^2}{lx^4+2ax^2+m}\right)^p \sum_{k=0}^{\infty}\frac{(-\mathrm{c})^k\left(\frac{\mathrm{z}}{2}\right)^{2k+n+1}\left(\frac{x^2}{lx^4+2ax^2+m}\right)^{2k+n+1}}{\Gamma(\alpha k+\mu)\Gamma\left(\mathrm{qk}+\frac{\mathrm{n}}{\xi}+\frac{\mathrm{b}+2}{2}\right)}dx$$

On interchanging the order of integration and summation above equation gives

$$\mathcal{I}_1=\sum_{k=0}^{\infty}\frac{(-\mathrm{c})^k\left(\frac{\mathrm{z}}{2}\right)^{2k+n+1}}{\Gamma(\alpha k+\mu)\Gamma\left(\mathrm{qk}+\frac{\mathrm{n}}{\xi}+\frac{\mathrm{b}+2}{2}\right)}\int_0^\infty \left(\frac{x^2}{lx^4+2ax^2+m}\right)^{p+2k+n+1}dx$$

From the condition given in (3.1) since $a>-\sqrt{lm}, l>0, m\geq 0$ *and* $(p+2k+n+1)>\frac{1}{2}$ and using (1.12) we get

$$\frac{\sqrt{\frac{\pi}{l}}\left(\frac{\mathrm{z}}{2}\right)^{n+1}}{2^{p+n+\frac{3}{2}}\left(a+\sqrt{lm}\right)^{p+n+\frac{1}{2}}}\sum_{k=0}^{\infty}\frac{(-\mathrm{c})^k\left(\frac{\mathrm{z}}{4}\right)^{2k}\Gamma(p+2k+n+1/2)}{\Gamma(\alpha k+\mu)\Gamma(p+2k+n+1)\Gamma\left(\mathrm{qk}+\frac{\mathrm{n}}{\xi}+\frac{\mathrm{b}+2}{2}\right)\left(a+\sqrt{lm}\right)^{2k}}$$

In view of definition in (1.8) required result can be obtained.

Theorem 2.
$$\int_0^\infty \left(\frac{x^2}{lx^4+2ax^2+m}\right)^p S^{k}_{\xi,c}\left(\frac{zx^2}{lx^4+2ax^2+m}\right)dx = \sqrt{\frac{\pi}{l}}\,\frac{\left(\frac{z}{2}\right)^{\frac{\xi}{k}+1}}{2k^{\frac{1}{2}+\frac{\xi}{k}}[2(a+\sqrt{lm})]^{p+\frac{\xi}{k}+\frac{1}{2}}}\ {}_{2}\Psi_{3}\left[\begin{matrix}\left(p+\frac{\xi}{k}+\frac{1}{2},2\right),(1,1); & \frac{-cz^2}{16k\left(a+\sqrt{lm}\right)^2}\\ \left(p+\frac{\xi}{k}+1,2\right),\left(\frac{\xi}{k}+\frac{3}{2},1\right),\left(\frac{3}{2},1\right) & \end{matrix}\right]$$

Under the conditions $a>-\sqrt{lm}, l>0, m\geq 0$ *and* $p>\frac{1}{2}$ (3.2)

Proof. Let $\mathcal{I}_2$ be the left-hand side of (3.2) and applying (1.12) to the integrand of (3.2)

$$\mathcal{I}_2=\int_0^\infty \left(\frac{x^2}{lx^4+2ax^2+m}\right)^p \sum_{r=0}^{\infty}\frac{(-\mathrm{c})^r\left(\frac{\mathrm{z}}{2}\right)^{2r+\frac{\xi}{\mathrm{k}}+1}\left(\frac{x^2}{lx^4+2ax^2+m}\right)^{2r+\frac{\xi}{\mathrm{k}}+1}}{\Gamma_k\left(\mathrm{rk}+\xi+\frac{3\mathrm{k}}{2}\right)\Gamma\left(\mathrm{r}+\frac{3}{2}\right)}dx$$

On interchanging the order of integration and summation above equation gives

$$\mathcal{I}_2=\sum_{r=0}^{\infty}\frac{(-\mathrm{c})^r\left(\frac{\mathrm{z}}{2}\right)^{2r+\frac{\xi}{\mathrm{k}}+1}}{\Gamma_k\left(\mathrm{rk}+\xi+\frac{3\mathrm{k}}{2}\right)\Gamma\left(\mathrm{r}+\frac{3}{2}\right)}\int_0^\infty \left(\frac{x^2}{lx^4+2ax^2+m}\right)^{p+2r+\frac{\xi}{\mathrm{k}}+1}dx$$

From the condition give in (3.2) since $a>-\sqrt{lm}, l>0, m\geq 0$ *and* $(p+2r+\frac{\xi}{\mathrm{k}}+1)>\frac{1}{2}$ and using (1.12) we get

$$\mathcal{I}_2 = \sqrt{\frac{\pi}{l}}\ \frac{\left(\frac{z}{2}\right)^{\frac{\xi}{k}+1}}{2[2(a+\sqrt{lm})]^{p+\frac{\xi}{k}+\frac{1}{2}}}\sum_{r=0}^{\infty}\frac{(-c)^r\left(\frac{z}{4}\right)^{2r}\Gamma\left(p+2r+\frac{\xi}{k}+1/2\right)}{\Gamma_k\left(rk+\xi+\frac{3k}{2}\right)\Gamma\left(p+2r+\frac{\xi}{k}+1\right)\Gamma\left(r+\frac{3}{2}\right)\left(a+\sqrt{lm}\right)^{2r}}$$

In view of definition in (1.8) and (1.7) we get the required result.

4. Special Cases

For $X = \frac{x^2}{lx^4+2ax^2+m}$ following are some corollaries of theorem 3.1:

Corollary 3.1 for $\xi = \alpha = q = 1$ and $\mu = \frac{3}{2}$ theorem 3.1 reduces in following form where $S_{n,b,c}(z)$ is generalized Struve function given in (1.3) defined by Yagmur and Orhan.

$$\int_0^\infty (X)^p S_{n,b,c}(zX)dx = \sqrt{\frac{\pi}{l}}\ \frac{\left(\frac{z}{2}\right)^{n+1}}{2[2(a+\sqrt{lm})]^{p+n+\frac{1}{2}}}\ {}_2\Psi_3\left[\begin{matrix}\left(p+n+\frac{1}{2},2\right),(1,1); & \frac{-cz^2}{16\left(a+\sqrt{lm}\right)^2}\\ \left(n+\frac{b+2}{2},1\right),\left(\frac{3}{2},1\right),(p+n+1,2) & \end{matrix}\right] \quad (4.1)$$

Corollary 3.2 for $\xi = \alpha = q = b = c = 1$ and $\mu = \frac{3}{2}$ theorem 3.1 reduces in following form where $S_n(z)$ is Struve function of order n given in (1.2).

$$\int_0^\infty (X)^p S_n(zX)dx = \sqrt{\frac{\pi}{l}}\ \frac{\left(\frac{z}{2}\right)^{n+1}}{2[2(a+\sqrt{lm})]^{p+n+\frac{1}{2}}}\ {}_2\Psi_3\left[\begin{matrix}\left(p+n+\frac{1}{2},2\right),(1,1); & \frac{-z^2}{16\left(a+\sqrt{lm}\right)^2}\\ \left(n+\frac{3}{2},1\right),\left(\frac{3}{2},1\right),(p+n+1,2) & \end{matrix}\right] \quad (4.2)$$

Corollary 3.3 On putting $\xi = \alpha = q = b = 1, c = -1$ and $\mu = \frac{3}{2}$ theorem 3.1 reduces in following form where $H_n(z)$ is modified Struve function [22] of order n.

$$\int_0^\infty (X)^p H_n(zX)dx = \sqrt{\frac{\pi}{l}}\ \frac{\left(\frac{z}{2}\right)^{n+1}}{2[2(a+\sqrt{lm})]^{p+n+\frac{1}{2}}}\ {}_2\Psi_3\left[\begin{matrix}\left(p+n+\frac{1}{2},2\right),(1,1); & \frac{z^2}{16\left(a+\sqrt{lm}\right)^2}\\ \left(n+\frac{3}{2},1\right),\left(\frac{3}{2},1\right),(p+n+1,2) & \end{matrix}\right] \quad (4.3)$$

Corollary 3.4 for $\xi = q = 1, b = 2, c = n = -1$ and $z = 2\sqrt{z}$ theorem 3.1 reduces in following form where $\phi(\mu, \alpha; z)$ is Wright function [23].

$$\int_0^\infty (X)^p \phi(\mu, \alpha; zX)dx = \sqrt{\frac{\pi}{l}}\ \frac{}{2^{p+\frac{1}{2}}\left(a+\sqrt{lm}\right)^{p-\frac{1}{2}}}\ {}_1\Psi_2\left[\begin{matrix}\left(p-\frac{1}{2},1\right); & \frac{z}{2\left(a+\sqrt{lm}\right)}\\ (p,1),(\mu,\alpha) & \end{matrix}\right] \quad (4.4)$$

Corollary 3.5 for $\xi = \mu = 1, \alpha = 0, n = n-1$ and $z = 2\sqrt{z}$ theorem 3.1 reduces in following form where $E_{q,n}(zX)$ is generalized Mittag-Leffler function [24].

$$\int_0^\infty (X)^p E_{q,n}(zX)dx = \sqrt{\frac{\pi}{l}}\ \frac{}{2^{p+\frac{1}{2}}\left(a+\sqrt{lm}\right)^{p-\frac{1}{2}}}\ {}_2\Psi_2\left[\begin{matrix}\left(p-\frac{1}{2},1\right),(1,1); & \frac{z}{2\left(a+\sqrt{lm}\right)}\\ (p,1),(n,q) & \end{matrix}\right] \quad (4.5)$$

Corollary 3.6 for $\xi = \mu = \alpha = c = 1, b = 2,\ n = n-1$ and $z = 2\sqrt{z}$ theorem 3.1 reduces in following form where ${}_qI_n(zX)$ is Galué type generalization of modified Bessel function [25].

$$\int_0^\infty (X)^p\, {}_qI_n(zX)dx = \sqrt{\frac{\pi}{2l}}\ \frac{\left(\frac{z}{2}\right)^{n}}{2^{p+n}\left(a+\sqrt{lm}\right)^{p+n-\frac{1}{2}}}\ {}_1\Psi_2\left[\begin{matrix}\left(p+n-\frac{1}{2},2\right); & \frac{z^2}{16\left(a+\sqrt{lm}\right)^2}\\ (p+n,2),(n+1,q) & \end{matrix}\right] \quad (4.6)$$

Corollary 3.7 for q= $\xi = \mu = \alpha = c = 1, b = 2$ *and* $n = n-1$ theorem 3.1reduces in following form where $I_n(z)$ is modified Bessel function of first kind of order n [26].

$$\int_0^\infty (X)^p\, I_n(z)(zX)dx = \sqrt{\frac{\pi}{2l}}\ \frac{\left(\frac{z}{2}\right)^{n}}{2^{p+n}\left(a+\sqrt{lm}\right)^{p+n-\frac{1}{2}}}\ {}_1\Psi_2\left[\begin{matrix}\left(p+n-\frac{1}{2},2\right); & \frac{z^2}{16\left(a+\sqrt{lm}\right)^2}\\ (p+n,2),(n+1,1) & \end{matrix}\right] \quad (4.7)$$

5. Concluding Remark

Boros integral with three parameters for the generalized Galué-type Struve function and k-Struve function has been obtained in this article. Additionally, relations of the generalized Galué-type Struve function ${}_qW^{\alpha,\mu}_{n,b,c,\xi}(z)$ with some generalized special functions are also derived which makes these results more significant. Furthermore, using these relations more forms of Boros integral can be found. Derived results can also be obtained in Beta integral.

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